Varimax Rotation, the Simple Visual Story

An ultra-simple companion to Lesson 3 (PCA and EFA)

Prof. Dr. Jörg Osterrieder

School of Engineering and Computer Sciences

A Puzzle Before We Start

You ran a factor analysis on 6 survey questions and asked for 2 factors. Every single question comes back loading on **both** factors at once.

If a question is half Factor 1 and half Factor 2, which group does it belong to?

That confusing table is the problem Varimax rotation was invented to fix. Keep that question in mind: we will answer it visually, with 6 variables and no proofs.

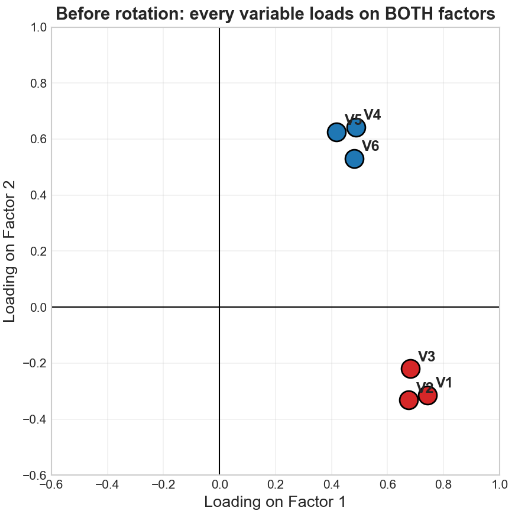
What makes a loadings table hard to read, and what would an easy one look like?

1. The Problem: Loadings Smeared Across Both Factors

The 6 variables (V1 to V6) each load on Factor 1 *and* Factor 2:

	Factor 1	**Factor 2**
V1	+0.74	−0.31
V2	+0.67	−0.33
V3	+0.68	−0.22
V4	+0.49	+0.64
V5	+0.42	+0.62
V6	+0.48	+0.53

No variable is clearly "a Factor 1 variable" or "a Factor 2 variable". This is called a *complex* (hard to interpret) structure.



Looking at the cloud of points, why can you not yet name the two factors?

2. What We Actually Want: Simple Structure

Simple structure (the goal): each variable loads *high* on one factor and *near zero* on the other. Then naming the factors is easy.

- A variable near an axis belongs clearly to that one factor.
- A variable stuck in the middle (like ours now) belongs to no one.

Predict before the next slide: the 6 points sit in a tilted band. What single move could line that band up with the axes?

2. What We Actually Want: Simple Structure

Simple structure (the goal): each variable loads *high* on one factor and *near zero* on the other. Then naming the factors is easy.

- A variable near an axis belongs clearly to that one factor.
- A variable stuck in the middle (like ours now) belongs to no one.

Predict before the next slide: the 6 points sit in a tilted band. What single move could line that band up with the axes? Answer: **rotate the axes**. We do not touch the data; we just look at the same points from a better angle.

Why does "high on one factor, near zero on the other" make a factor easy to name?

3. The Trick: Spin the Axes, Not the Data

The 6 points stay exactly where they are. We only rotate the Factor 1 and Factor 2 axes around the origin until the points line up with them.

- Each variable's distance from the origin does not change.
- So each *communality* h^2 (how much of the variable the 2 factors explain together, the row's sum of squared loadings) stays fixed.
- Only the *split* between Factor 1 and Factor 2 changes.

Predict: if we spin the axes by the right angle, roughly how many degrees do you think it takes here?

3. The Trick: Spin the Axes, Not the Data

The 6 points stay exactly where they are. We only rotate the Factor 1 and Factor 2 axes around the origin until the points line up with them.

- Each variable's distance from the origin does not change.
- So each *communality* h^2 (how much of the variable the 2 factors explain together, the row's sum of squared loadings) stays fixed.
- Only the *split* between Factor 1 and Factor 2 changes.

Predict: if we spin the axes by the right angle, roughly how many degrees do you think it takes here? About **30 degrees**. Varimax finds that angle automatically.

What quantity is guaranteed NOT to change when we only rotate the axes?

4. How Varimax Chooses the Angle

Varimax tries every rotation angle and keeps the one that makes the squared loadings as *spread out* as possible (some near 0, some near 1). That spread is measured by a single number V.

	Before	**After Varimax**
Spread score V	0.042	0.156
Rotation applied	none	about 30°
V improvement	$3.8\times$ larger	

A bigger V means loadings are pushed toward 0 or toward 1, which is exactly the simple structure we wanted. Here V grows from 0.042 to 0.156.

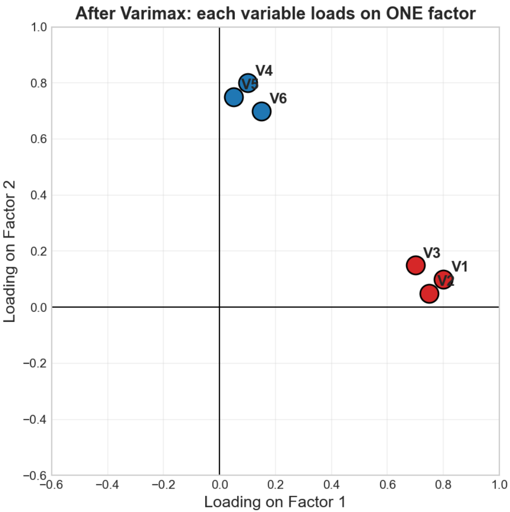
In one sentence: what is Varimax maximising, and why does that help reading?

5. After Varimax: The Same 6 Variables, Now Clear

Same data, rotated axes. Now the split is obvious:

	Factor 1	Factor 2
V1	+0.80	+0.10
V2	+0.75	+0.05
V3	+0.70	+0.15
V4	+0.10	+0.80
V5	+0.05	+0.75
V6	+0.15	+0.70

V1, V2, V3 are clearly Factor 1. V4, V5, V6 are clearly Factor 2.



Now you can name the factors: which three variables define each one?

6. What Changed, What Stayed the Same

Predict: did the variables' communalities h^2 go up, go down, or stay the same after rotation?

6. What Changed, What Stayed the Same

Predict: did the variables' communalities h^2 go up, go down, or stay the same after rotation?

	h^2 **before**	h^2 **after**	**match?**
V1	0.65	0.65	yes
V2	0.56	0.56	yes
V4	0.65	0.65	yes

- **Stayed:** every communality h^2 (the rotation only turned the axes).
- **Changed:** the Factor 1 vs Factor 2 split, and the spread score V ($0.042 \rightarrow 0.156$).

Same information, clearer picture: what exactly did rotation buy us?

The One Big Idea

Varimax **rotates the axes** to turn a smeared loadings table into a clean "one variable, one factor" table, **without changing** how much each variable is explained.

- Before: every variable loaded on both factors (uninterpretable).
- Rotate about 30°: V grows 3.8× (0.042 → 0.156).
- After: V1 to V3 are Factor 1, V4 to V6 are Factor 2. Communalities unchanged.

Could you now explain Varimax to a friend using only the before and after pictures?