Embeddings: From One-Hot to RAG

The Complete Journey — All Concepts, All Formulas

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When All You Have Is Linear Algebra...

"You pour the data into this big pile of linear algebra, then collect the answers on the other side."

XKCD #1838 "Machine Learning" by Randall Munroe (CC BY-NC 2.5)

What Will You Learn Today?

- **Analyze** why one-hot encoding fails for NLP tasks
- **Evaluate** trade-offs between static and contextual embeddings
- **Apply** cosine similarity to measure semantic closeness
- **Design** an embedding-based sentiment classifier for financial text
- **Critique** embedding approaches for specific finance use cases

Bloom's levels 4–5: Analyze, Evaluate, Apply, Design, Critique

Roadmap

From one-hot encoding to RAG: the complete embeddings journey

Why Can't Machines Read Text?

- **The problem:** text is categorical, not numeric — models need numbers
- An earnings call transcript has 10,000 words — how do we feed that to a model?
- **Sparse representation:** each word is an isolated symbol with no relationships
- **High dimensionality:** vocabulary sizes of 50,000–100,000 are common
- **No similarity:** "bullish" and "optimistic" look equally different as "bullish" and "penguin"

The fundamental challenge: converting discrete symbols into continuous vectors

What Is One-Hot Encoding?

Motivate: A vocabulary of 10,000 words means 10,000-dimensional sparse vectors.

Visualize:

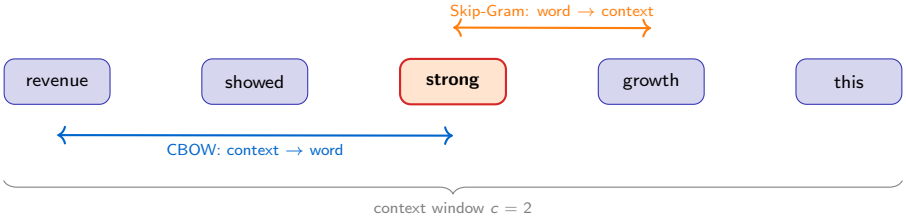
$$\mathbf{e}_{\text{stock}} = [\quad 0,\quad 0,\ \cdots,\ \underset{\text{position } i}{\overset{\downarrow}{1}},\ \cdots,\ 0,\quad 0\quad]$$

Formalize: $\mathbf{e}_i \in \{0,1\}^{|V|}$ where $e_{ij} = 1$ iff $j = i$

One-hot: simple but wasteful. No similarity between any two words.

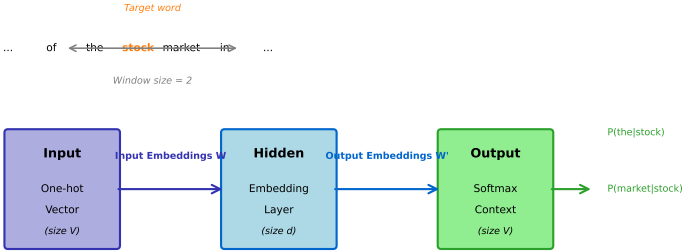
What Does 'Context' Tell Us About Meaning?

- **Distributional hypothesis:** "You shall know a word by the company it keeps" (Firth, 1957)
- Finance: "bullish" appears near "growth", "revenue", "upgrade"



The distributional hypothesis: the foundation of all word embedding methods (Firth, 1957)

How Does the Skip-Gram Network Look?



Skip-gram Architecture: Predict Context from Target

- **Input:** one-hot vector $\mathbf{e}_w \in \mathbb{R}^{|V|}$, **Hidden:** embedding $\mathbf{v}_w = W^\top \mathbf{e}_w \in \mathbb{R}^d$
- **Output:** context matrix W' produces softmax over vocabulary

Skip-Gram: predict context words from the center word. Two embedding matrices: W (input) and W' (output).

What Is the Skip-Gram Objective?

Motivate: We want to maximize the probability of seeing the actual context words.

Finance example: predicting "revenue" appears near "earnings".

Formalize:

$$J = \frac{1}{T} \sum_{t=1}^{T} \sum_{\substack{-c \leq j \leq c \\ j \neq 0}} \log p(w_{t+j} \mid w_t)$$

where $p(w_O \mid w_I) = \dfrac{\exp({\mathbf{v}'_{w_O}}^{\top} \mathbf{v}_{w_I})}{\sum_{w=1}^{|V|} \exp({\mathbf{v}'_{w}}^{\top} \mathbf{v}_{w_I})}$

The softmax denominator sums over ALL words in the vocabulary — this will be a problem.

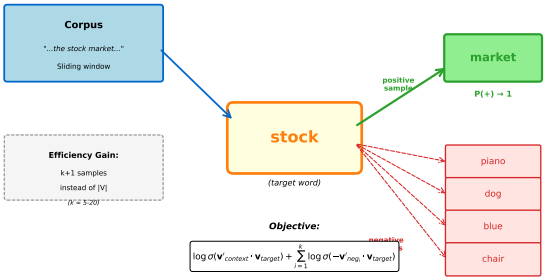
Why Is Full Softmax Intractable?

- $O(|V|)$ **per update** for $V = 100{,}000+$ — every gradient step touches the entire vocabulary
- The denominator $\sum_{w=1}^{|V|} \exp(\mathbf{v}'_w{}^\top \mathbf{v}_{w_I})$ requires computing all dot products
- **Two solutions:** hierarchical softmax (tree structure) and **negative sampling** (binary classification)

Computing the full softmax is the main computational bottleneck of Word2Vec.

How Does Negative Sampling Fix This?

Motivate: Replace V-class softmax with $K+1$ binary classifiers.



Negative Sampling: Binary Classification Instead of Full Softmax

Formalize:

$$J_{\text{NEG}} = \log \sigma({\mathbf{v}'_{w_O}}^{\top} \mathbf{v}_{w_I}) + \sum_{k=1}^{K} \mathbb{E}_{w_k \sim P_n} \left[\log \sigma(-{\mathbf{v}'_{w_k}}^{\top} \mathbf{v}_{w_I}) \right]$$

Push **positive pairs** together, push **negative pairs** apart.

Negative sampling: $O(K)$ **instead of** $O(|V|)$**. Typically** $K = 5$–20 **negatives per positive pair.**

How Do We Sample Negative Words?

1: Initialize W, W' randomly
2: **for** each word w_t in corpus **do**
3: **for** each context word w_c in window **do**
4: **Positive:** update (w_t, w_c) to increase σ
5: **for** $k = 1$ to K **do**
6: Sample $w_n \sim P_n(w)$
7: **Negative:** update (w_t, w_n) to decrease σ
8: **end for**
9: **end for**
10: **end for**

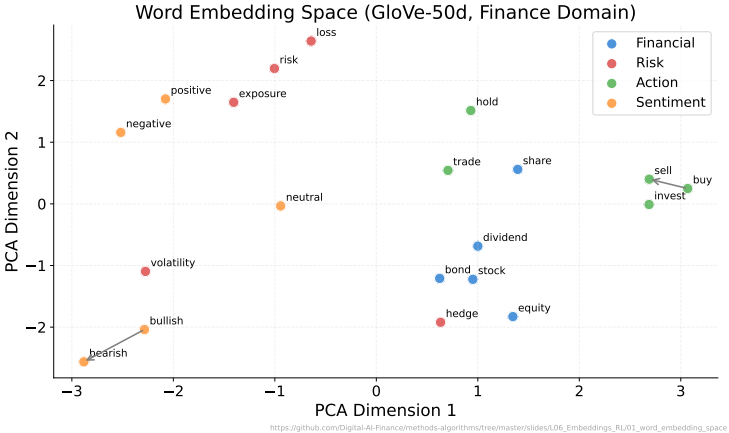
Noise distribution:

$$P_n(w) = \frac{f(w)^{3/4}}{\sum_{w'} f(w')^{3/4}}$$

The $3/4$ exponent **upweights rare words** relative to their raw frequency.

The 3/4 exponent upweights rare words relative to their frequency (Mikolov et al., 2013).

What Does the Embedding Space Look Like?

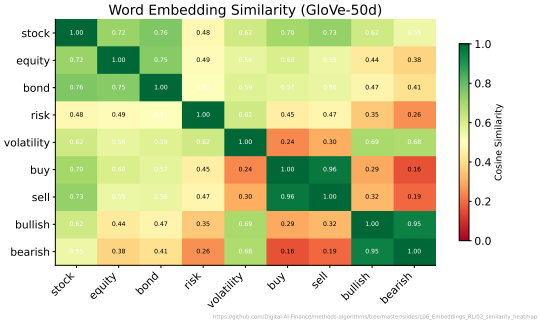


- Similar words cluster together in the learned embedding space
- Finance terms (stock, bond, equity) form a semantic neighborhood

t-SNE projection of 50-dimensional word vectors. Similar words cluster in embedding space.

How Do We Measure Semantic Closeness?

Motivate: Euclidean distance fails when vectors have different magnitudes.



Formalize:

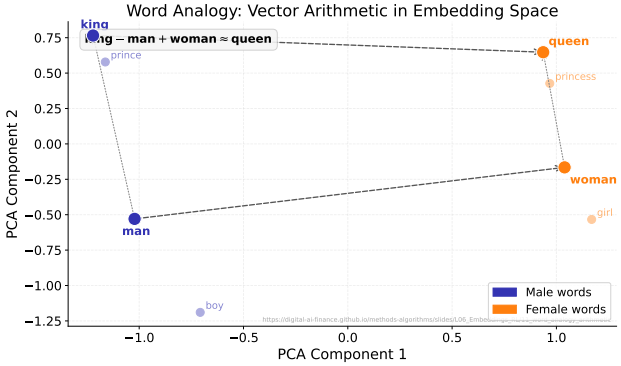
$$\cos(\mathbf{a}, \mathbf{b}) = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|}$$

Finance examples:

- $\cos(\text{bullish}, \text{positive}) = 0.82$
- $\cos(\text{bullish}, \text{negative}) = -0.31$

Cosine similarity: range $[-1, 1]$. Direction matters, not magnitude.

Can We Do Arithmetic with Words?



Classic:

$$\mathbf{v}_{\text{king}} - \mathbf{v}_{\text{man}} + \mathbf{v}_{\text{woman}} \approx \mathbf{v}_{\text{queen}}$$

Finance:

$$\mathbf{v}_{\text{stock}} - \mathbf{v}_{\text{equity}} + \mathbf{v}_{\text{debt}} \approx \mathbf{v}_{\text{bond}}$$

Vector arithmetic captures **relational structure** in embedding space.

Vector arithmetic captures relational structure. Success rate: 40–70% (Levy & Goldberg, 2014).

What If We Use Global Statistics Instead?

Motivate: Word2Vec only uses local context windows. **GloVe** uses global co-occurrence statistics.

Formalize:

$$J = \sum_{i,j=1}^{|V|} f(X_{ij}) \left(\mathbf{w}_i^\top \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij} \right)^2$$

where X_{ij} = co-occurrence count and the weighting function is:
$f(x) = \min\left((x/x_{\max})^{0.75},\ 1 \right)$

- Combines count-based (LSA) and prediction-based (Word2Vec) advantages
- Weighting caps frequent pairs, upweights informative rare pairs

GloVe: combines global co-occurrence statistics with embedding learning (Pennington et al., 2014).

How Do We Handle Unknown Words?

Motivate: The OOV problem — "bullish" unseen if only "bull" was trained.

Example: "bullish" $\rightarrow$ [<bu, bul, ull, lli, lis, ish, sh>]

Formalize:

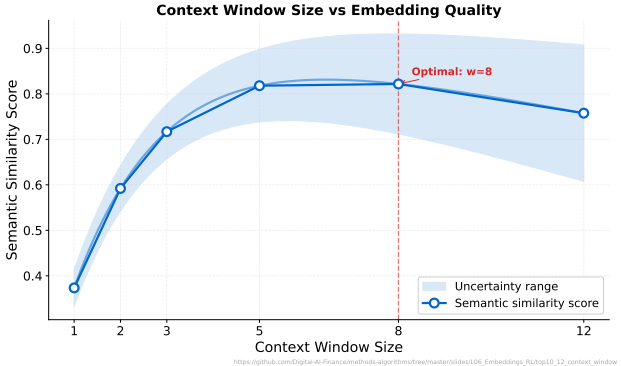
$$\mathbf{v}_w = \sum_{g \in \mathcal{G}(w)} \mathbf{z}_g$$

where $\mathcal{G}(w)$ is the set of character n-grams (typically $n = 3$–6).

- Any new word can be represented via its subword pieces
- Captures morphology: "un-" prefix, "-tion" suffix carry meaning

FastText: character n-grams solve the OOV problem and capture morphology (Bojanowski et al., 2017).

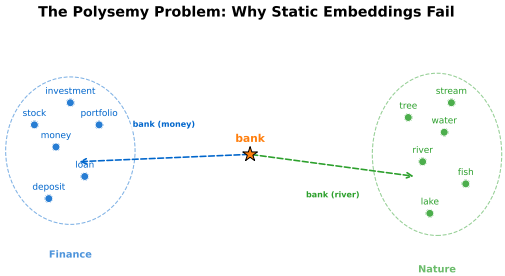
How Do Hyperparameters Affect Embeddings?



- **Small window** ($c = 2$): captures syntactic patterns (POS, grammar)
- **Large window** ($c = 10$): captures semantic/topical similarity
- **Dimensions:** 50–300 typical; diminishing returns beyond 300

Window size and dimensions are the two most important hyperparameters for static embeddings.

What Happens When Words Have Multiple Meanings?



Static: 1 vector for "bank". Contextual (BERT): different vectors based on meaning.

- "Bank" = financial institution OR riverbank
- "Interest" = financial vs. personal
- Static: **one vector per word**

The polysemy problem: static embeddings assign one vector per word regardless of context.

How Do Models Split Text into Tokens?

Example: "unaffordable" $\rightarrow$ [`un`, `##afford`, `##able`]

BPE (GPT family):
merge $= \arg\max_{(a,b)} \text{count}(ab)$
Frequency-based: merge the most common adjacent pair.

WordPiece (BERT family):
$$\text{merge} = \arg\max_{(a,b)} \frac{\text{freq}(ab)}{\text{freq}(a) \cdot \text{freq}(b)}$$
Likelihood-based: merge pairs that most increase corpus likelihood.

- Both reduce vocabulary to 30K–50K tokens while preserving subword meaning
- No more OOV — any word can be split into known subword tokens

BPE: frequency-based merging. WordPiece: likelihood-ratio merging. Both reduce vocabulary while keeping subword meaning.

How Do Transformers Know Word Order?

Motivate: Transformers process all tokens in parallel — they need position information injected.

Formalize:

$$PE_{(pos,2i)} = \sin\left(\frac{pos}{10000^{2i/d}}\right), \quad PE_{(pos,2i+1)} = \cos\left(\frac{pos}{10000^{2i/d}}\right)$$

- Each position gets a **unique frequency signature** — no two positions are alike
- Sinusoidal encoding allows the model to attend to relative positions
- Added directly to token embeddings: $\mathbf{x}_i = \mathbf{e}_{\text{token}} + \mathbf{e}_{\text{pos}}$

Sinusoidal position encoding: each position gets a unique frequency signature (Vaswani et al., 2017).

Which Words Should I Pay Attention To?

Motivate: Q/K/V analogy — Query = what I'm looking for, Key = what I offer, Value = what I contain.

The key formula:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^\top}{\sqrt{d_k}}\right) V$$

- $QK^\top$: how much should each token attend to every other token?
- $\sqrt{d_k}$: scaling prevents softmax from saturating for large d_k
- Result: each token is a **weighted sum** of all value vectors

Scaled dot-product attention: the core mechanism of all transformer models (Vaswani et al., 2017).

Why Use Multiple Attention Heads?

Motivate: One head captures syntax, another captures semantics — multiple heads in parallel.

Formalize:

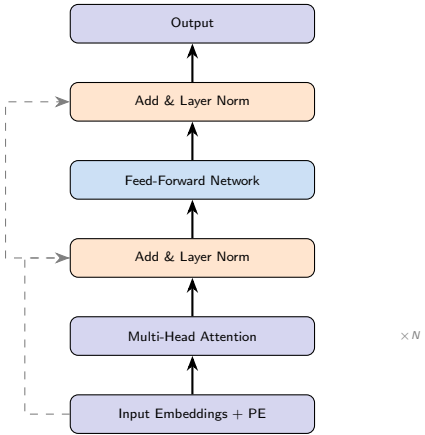
$$\text{MultiHead}(Q, K, V) = \text{Concat}(h_1, \ldots, h_H)\, W^O$$

where $h_i = \text{Attention}(QW_i^Q,\ KW_i^K,\ VW_i^V)$

- Each head projects Q, K, V to dimension $d_k = d_{\text{model}}/H$
- BERT uses $H = 12$ heads; GPT-3 uses $H = 96$ heads
- Concatenated outputs are projected back to d_{model}

Multi-head attention: H **parallel attention functions capture different relationship types.**

What Does a Transformer Block Look Like?



The transformer block: the building unit repeated N times in BERT (12×) and GPT (12–96×).

How Does BERT Read in Both Directions?

Motivate: GPT reads left-to-right only. BERT masks random words and predicts them from **both sides**.

Formalize — Masked Language Modeling:

$$\mathcal{L}_{\text{MLM}} = -\sum_{i \in \mathcal{M}} \log p(x_i \mid \mathbf{x}_{\setminus \mathcal{M}})$$

where $\mathcal{M}$ = set of masked positions (15% of tokens).

- Example: "The [MASK] reported strong earnings" $\rightarrow$ predict "company"
- Bidirectional context: uses both left and right words to predict
- Pre-trained on BookCorpus + Wikipedia (3.3B words)

BERT: Masked Language Modeling + Next Sentence Prediction. Pre-trained on BookCorpus + Wikipedia.

How Do We Fine-Tune for Classification?

Motivate: The `[CLS]` token representation feeds a linear layer for downstream tasks.

Finance: classifying an earnings call as positive/negative.

Formalize:

$$\hat{y}_c = \text{softmax}(\mathbf{W}\,\mathbf{h}_{[\text{CLS}]} + \mathbf{b})_c, \qquad \mathcal{L} = -\sum_{c=1}^{C} y_c \log(\hat{y}_c)$$

- **Pre-train** on large unlabeled corpus (expensive, done once)
- **Fine-tune** on small labeled dataset (cheap, task-specific)
- Only the classification head + top BERT layers are updated

Cross-entropy loss on the [CLS] representation: the standard approach for text classification.

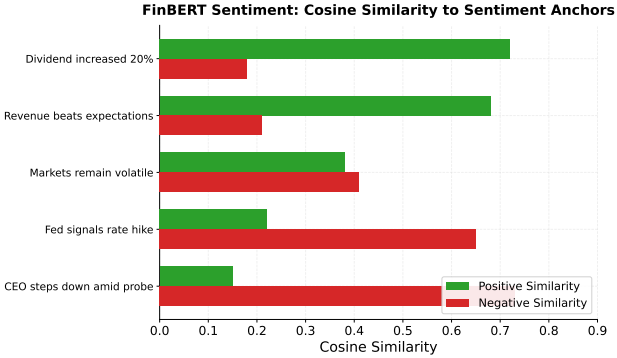
When Should You Use BERT vs GPT?

Property	BERT	GPT
Direction	Bidirectional	Left-to-right
Pre-training	MLM + NSP	Next token prediction
Best for	Classification, NER, QA	Text generation
Finance use	Sentiment, entity extraction	Report generation
Parameters	110M (base) / 340M (large)	117M (1) to 175B (3)

- **BERT for understanding**, **GPT for generation**
- Both use the same transformer architecture — the difference is the training objective
- Modern LLMs (GPT-4, Claude) use decoder-only architecture at massive scale

BERT for understanding, GPT for generation. Both use the same transformer architecture.

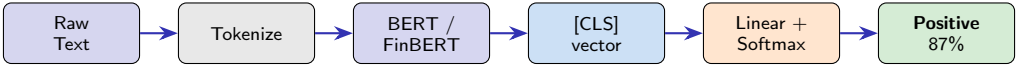
Why Does General BERT Fail on Financial Text?



- "Liability" = negative in general English, **neutral in finance**
- "Volatile" = negative generally, descriptive in markets
- FinBERT: BERT fine-tuned on financial news, earnings calls, analyst reports

FinBERT: BERT fine-tuned on financial text achieves 87% accuracy on sentiment (Araci, 2019).

How Does the Full Sentiment Pipeline Work?

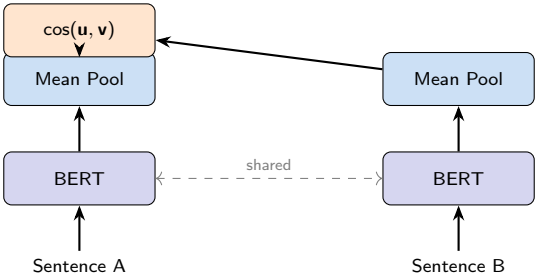


- Finance running example: "Q3 revenue exceeded expectations with 12% growth"
- The `[CLS]` token aggregates the full sentence meaning into one vector
- Linear layer maps 768-dim vector to 3 classes: positive / negative / neutral

End-to-end sentiment classification: text → tokens → BERT → [CLS] → softmax → label.

How Do We Get Sentence-Level Embeddings?

Motivate: BERT gives token-level embeddings, not sentence-level. Sentence-BERT uses a **siamese network**.

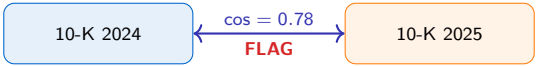


$\mathcal{L} = \frac{1}{N}\sum_{i=1}^{N}(\cos(\mathbf{u}_i, \mathbf{v}_i) - y_i)^2$ where $y_i \in [-1, 1]$

Sentence-BERT: cosine similarity MSE loss for contrastive sentence learning (Reimers & Gurevych, 2019).

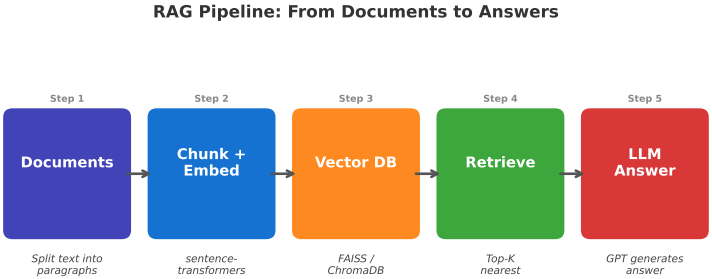
Can We Detect Changes in SEC Filings?

- Compare 10-K filings year-over-year using **document embeddings**
- If $\cos(\mathbf{d}_{2024}, \mathbf{d}_{2025}) < 0.85$, flag for manual review
- Material changes in risk factors, revenue recognition, or legal proceedings are detectable



Document similarity via cosine: a simple but powerful compliance monitoring tool.

How Does RAG Connect Embeddings to LLMs?



- Query → embed → search vector DB → retrieve top-k → augment prompt → generate
- Grounds LLM responses in **factual, up-to-date** documents — reduces hallucination

RAG: Retrieval-Augmented Generation. Combines embedding search with LLM generation.

What Entities Can We Extract from Financial Text?

- **Tickers:** AAPL, MSFT, TSLA — link mentions to structured databases
- **Monetary amounts:** $2.4B revenue, 500M write-down
- **Dates & events:** “Q3 2025 earnings”, “Fed meeting March 19”

BERT + token classification head: predict B-ORG, I-ORG, B-MONEY, etc. per token.

Named Entity Recognition: extract structured data from unstructured financial text.

Can Embeddings Generate Trading Signals?

- News sentiment → daily feature vector → ML model → buy/sell signal
- Aggregate across all articles for a given day and ticker:

$$s_t = \frac{1}{N} \sum_{i=1}^{N} \text{sentiment}(\text{article}_i) \cdot \text{relevance}(\text{article}_i)$$

- **Walk-forward validation** required — no peeking at future data
- Combine with price features, volume, and technical indicators

Sentiment-based trading signals: aggregate news embeddings into daily features for ML models.

Which Embedding Should You Choose?

Method	Best For	Limitation
One-Hot	Tiny vocab, baseline	No similarity
Word2Vec / GloVe	Static, fast, large corpus	No polysemy
FastText	Morphologically rich, OOV	Still static
BERT	Context-dependent, classification	Slow, 512 tokens
Sentence-BERT	Document comparison	Needs fine-tuning
FinBERT	Financial text	Domain-specific only

- Start with the **simplest method that works**
- Upgrade when you have evidence it will improve results
- Always benchmark against a bag-of-words baseline

Start with the simplest method that works. Upgrade when you have evidence it will improve results.

What Are the Key Takeaways?

- **Map** discrete text to continuous vectors with embeddings
- **Capture** distributional semantics with Word2Vec / GloVe
- **Add context** via transformer attention mechanisms
- **Specialize** for finance with FinBERT and domain fine-tuning
- **Connect** embeddings to LLMs via RAG for grounded generation

Five ideas that changed NLP: dense vectors, distributional learning, attention, fine-tuning, retrieval.

Key Terms

- **Embedding** — dense vector representing a discrete symbol
- **One-Hot** — sparse binary vector, one dimension per word
- **Skip-Gram** — predict context words from center word
- **Cosine Similarity** — direction-based similarity in $[-1, 1]$
- **Attention** — weighted sum using query–key–value
- **BERT** — bidirectional transformer, masked LM pre-training
- **Fine-Tuning** — adapt pre-trained model to downstream task
- **RAG** — retrieval-augmented generation for grounded LLM output

Master these 8 terms and you can discuss embeddings with any NLP practitioner.

What Should You Try Next?

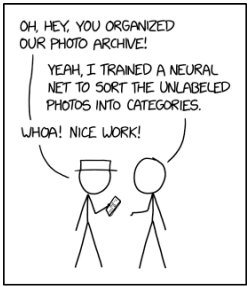
1. Load pre-trained Word2Vec vectors and find the 10 nearest neighbors to “inflation”
2. Compute cosine similarity between “bullish” and “bearish” — is it positive or negative?
3. Use FinBERT to classify 5 financial headlines as positive / negative / neutral

Colab notebook: `https://colab.research.google.com/`

See the L06 Colab notebook for starter code and pre-loaded data.

Hands-on: the best way to learn embeddings is to build with them. See the L06 Colab notebook.

From One-Hot to RAG: Journey Complete



"I trained a neural net to identify the most important part of any image, and it says it's this neural net."

XKCD #2173 by Randall Munroe (CC BY-NC 2.5). From one-hot to RAG: the journey is complete!