

## Lesson 2.3 Exercises: Democratizing Investment and Lending

### Module 2: The Access Problem

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Digital Finance — BSc Course

## Exercise 1: Sharpe Ratio Calculation

**Scenario:** The risk-free rate is 2.0%. You are evaluating three investment options:

Investment	Expected Return	Std. Deviation
Government Bond Fund	3.5%	3.0%
Balanced ETF Portfolio	7.0%	11.0%
Growth Equity Fund	11.0%	22.0%

### Tasks:

- Calculate the Sharpe ratio for each investment.
- Rank the three investments by risk-adjusted performance (best to worst).
- An investor says: “The Growth Equity Fund is the best because it has the highest return.” Is this correct? Explain using the Sharpe ratio.
- Under what circumstances might an investor rationally choose the Growth Equity Fund despite its lower Sharpe ratio?

*Difficulty: Introductory — tests Sharpe ratio computation and interpretation.*

## Exercise 2: The Compounding Cost of Fees

**Scenario:** Three investors each deposit \$25,000 today. All portfolios earn 7.0% gross return annually. The only difference is fees:

Investor / Product	Annual Fee
Alice — DIY index fund	0.05%
Bob — Robo-advisor	0.25%
Carol — Traditional advisor + active fund	1.50%

### Tasks:

- Calculate each investor's portfolio value after 10, 20, and 30 years. Use:  $V = P \times (1 + r - f)^t$ .
- Calculate the total dollar fees “lost” by each investor after 30 years, relative to a zero-fee benchmark  $V_0 = 25,000 \times (1.07)^{30}$ .
- Carol's advisor claims: “My expertise adds at least 1% of value annually.” What gross return must Carol's advisor achieve to match Alice's net outcome after 30 years?
- Create a one-paragraph argument **for** and **against** paying 1.50% for a human advisor.

*Difficulty: Intermediate — requires compounding and financial reasoning.*

## Exercise 3: Rebalancing Decision

**Scenario:** A robo-advisor manages a portfolio with a target allocation of 60% equity / 40% bonds and a  $\pm 5$  percentage point threshold band. The portfolio started the year at target with \$100,000.

After market movements:

Asset Class	Start Value	Return	End Value
Equity	\$60,000	+15%	\$69,000
Bonds	\$40,000	+2%	\$40,800
<b>Total</b>	<b>\$100,000</b>		<b>\$109,800</b>

### Tasks:

- Calculate the new portfolio weights (equity % and bond %).
- Has the threshold been breached? Show the calculation.
- If rebalancing is triggered, calculate the dollar amounts to sell/buy to restore 60/40.
- What is the Sharpe ratio of the portfolio if the standard deviation is 9% and the risk-free rate is 2%? (Use the actual return earned this year as  $r_p$ .)

*Difficulty: Intermediate — combines rebalancing mechanics with Sharpe ratio.*

## Exercise 4: P2P Lending Portfolio Returns

**Scenario:** An investor allocates \$5,000 across 200 P2P loans (\$25 each) on a marketplace lending platform. The loans are distributed across risk grades:

Grade	Loans	Interest Rate	Default Rate	Recovery Rate
A (low risk)	80	5.0%	2%	50%
B (medium)	80	9.0%	6%	30%
C (high risk)	40	14.0%	12%	20%

### Tasks:

- Calculate the gross interest income from each grade (assume 1-year loans, interest on performing loans only).
- Calculate the loss from defaults for each grade (principal lost minus recovery).
- Calculate the net return for the entire portfolio (total interest – total losses) as a dollar amount and as a percentage.
- The platform charges a 1% annual servicing fee on the total invested amount. What is the investor's net return after the platform fee?

*Difficulty: Intermediate–Advanced — multi-step P2P return calculation.*

## Exercise 5: Loan Default Waterfall Analysis

**Scenario:** A P2P platform publishes the following loan pool statistics after 24 months (1,000 original loans, \$10 average principal each):

Status	Number of Loans
Performing (current)	870
Late 30 days	45
Late 60+ days	25
Defaulted (charged off)	60
Of defaulted: recovered	18
Of defaulted: total loss	42

### Tasks:

- What is the cumulative default rate after 24 months?
- What is the recovery rate on defaulted loans?
- Calculate the total dollar loss (net of recovery) for an investor who funded the entire pool.
- If the average interest rate was 8% annually, was the investor's net return positive or negative after 24 months? Show the calculation.

*Difficulty: Intermediate — applies the waterfall framework with real-world-style data.*

## Exercise 6: Robo-Advisory Value Proposition

**Scenario:** A 30-year-old professional with \$15,000 to invest compares two options:

	<b>Robo-Advisor</b>	<b>Traditional Advisor</b>
Minimum investment	\$0	\$50,000
Annual fee	0.25%	1.00%
Portfolio	5 ETFs (MPT-optimized)	8 mutual funds
Rebalancing	Threshold (automated)	Quarterly (manual review)
Tax-loss harvesting	Automated (daily scan)	Manual (annual review)
Human contact	Chat/email only	Quarterly meetings

### Tasks:

- Can this investor use the traditional advisor? Why or why not?
- Estimate the value of tax-loss harvesting at 0.5% per year. Over 30 years with a 7% gross return, how much extra wealth does TLH generate on \$15,000?
- Identify two scenarios where the **traditional advisor** provides value that the robo-advisor cannot.
- Write a recommendation for this investor, considering both quantitative and qualitative factors.

*Difficulty: Advanced — requires multi-dimensional comparison and judgment.*

## Exercise 7: Microfinance Cost Analysis

**Scenario:** A microfinance institution (MFI) makes loans with the following cost structure (all values synthetic and annual):

Cost Component	Amount per Loan
Loan origination (fixed)	\$15
Credit assessment (fixed)	\$8
Disbursement and collection (fixed)	\$12
Cost of funds (variable, 6% of principal)	Variable
Expected credit losses (8% of principal)	Variable
Target profit margin (3% of principal)	Variable

### Tasks:

- Calculate the break-even interest rate for a \$100 loan (fixed costs + variable costs as % of principal).
- Calculate the break-even interest rate for a \$500 loan.
- Calculate the break-even interest rate for a \$5,000 loan.
- Explain why microfinance rates are high and what technological innovation could reduce them.

*Difficulty: Intermediate — combines fixed/variable cost analysis with access implications.*

## Exercise 8: Building a Democratized Investment Strategy

**Scenario:** Maria, a university graduate earning \$35,000/year, wants to start investing. She has \$2,000 saved and can add \$200/month. She has a 30-year time horizon (retirement) and moderate risk tolerance.

### Tasks:

- a. A traditional advisor requires \$50,000 minimum. How many months would Maria need to save \$200/month to meet this requirement (ignore interest)?
- b. Using a robo-advisor at 0.25% fee with a 60/40 portfolio (expected return 6.5%,  $\sigma = 10\%$ ,  $r_f = 2\%$ ), calculate the Sharpe ratio.
- c. If Maria invests \$2,000 now plus \$200/month at 6.25% net return ( $6.5\% - 0.25\%$ ) for 30 years, what is her projected portfolio value? Use:  $V = P(1 + r)^n + \text{PMT} \times \frac{(1+r)^n - 1}{r}$  where  $r$  is the monthly rate.
- d. Maria considers allocating 10% of her portfolio to P2P loans at 7% expected net return (after defaults). Discuss the trade-offs: diversification benefit vs. added risks.
- e. Write a three-sentence investment plan for Maria, covering platform choice, allocation, and risk management.

*Difficulty: Advanced–Integrative — combines all lesson concepts into a realistic scenario.*

## Exercise 1:

- (a) Bond:  $S = (3.5 - 2.0)/3.0 = 0.50$ . Balanced:  $S = (7.0 - 2.0)/11.0 = 0.45$ . Growth:  $S = (11.0 - 2.0)/22.0 = 0.41$ .
- (b) Rank: Bond (0.50) > Balanced (0.45) > Growth (0.41).
- (c) Incorrect. Growth has the highest *absolute* return but the lowest risk-adjusted return. Per unit of risk, the Bond Fund delivers the most excess return.
- (d) If the investor has a long horizon, high risk capacity, and needs high absolute returns (e.g., early career retirement saving), the higher expected return may justify the lower Sharpe ratio.

## Exercise 2:

- (a) After 10y: A:  $\$25k \times (1.0695)^{10} = \$48,837$ . B:  $\$25k \times (1.0675)^{10} = \$47,879$ . C:  $\$25k \times (1.055)^{10} = \$42,226$ .  
After 20y: A:  $\$95,435$ . B:  $\$91,593$ . C:  $\$71,321$ .  
After 30y: A:  $\$186,636$ . B:  $\$174,876$ . C:  $\$120,515$ .
- (b) Zero-fee:  $\$25k \times (1.07)^{30} = \$190,306$ . Fees lost: A:  $\$3,670$ . B:  $\$15,430$ . C:  $\$69,791$ .
- (c) Solve:  $25k(1 + r - 0.015)^{30} = 186,636$ .  $(1 + r - 0.015)^{30} = 7.465$ .  $1 + r - 0.015 = 1.0695$ .  $r = 8.45\%$ . Advisor must earn 8.45% gross vs. the market's 7%.

## Answer Key (continued)

### Exercise 3:

- (a) Equity:  $\$69,000/\$109,800 = 62.84\%$ . Bonds:  $\$40,800/\$109,800 = 37.16\%$ .
- (b) Drift =  $62.84\% - 60\% = 2.84\text{pp}$ . Threshold is 5pp. **Not breached** — no rebalancing triggered.
- (c) N/A (threshold not breached). If it were: target equity =  $60\% \times \$109,800 = \$65,880$ . Sell  $\$69,000 - \$65,880 = \$3,120$  equity, buy  $\$3,120$  bonds.
- (d) Portfolio return =  $(\$109,800 - \$100,000)/\$100,000 = 9.8\%$ .  $S = (9.8\% - 2\%)/9\% = 0.867$ .

### Exercise 4:

- (a) A:  $80 \times \$25 \times 0.98 \times 5\% = \$98$ . B:  $80 \times \$25 \times 0.94 \times 9\% = \$169.20$ . C:  $40 \times \$25 \times 0.88 \times 14\% = \$123.20$ . **Total interest: \$390.40.**
- (b) A:  $80 \times \$25 \times 2\% \times (1 - 50\%) = \$20$ . B:  $80 \times \$25 \times 6\% \times (1 - 30\%) = \$84$ . C:  $40 \times \$25 \times 12\% \times (1 - 20\%) = \$96$ . **Total losses: \$200.**
- (c) Net =  $\$390.40 - \$200 = \$190.40$ . Return =  $\$190.40/\$5,000 = 3.81\%$ .
- (d) Platform fee:  $\$5,000 \times 1\% = \$50$ . Net after fee:  $\$190.40 - \$50 = \$140.40$ . Return = **2.81%**.

### Exercise 5:

- (a) Default rate =  $60/1,000 = 6.0\%$ .
- (b) Recovery rate =  $18/60 = 30\%$ .
- (c) Net loss =  $(60 - 18) \times \$10 = \$420$ . Or:  $42 \times \$10 = \$420$ .
- (d) Interest (2y):  $870 \times \$10 \times 8\% \times 2 \approx \$1,392$  (approx., ignoring late loans). Loss:  $\$420$ . Net:  $\$1,392 - \$420 = \$972$ . On  $\$10,000$  invested, 2-year return  $\approx 9.7\%$ , annualized  $\approx 4.7\%$ . Positive.

## Answer Key (continued)

### Exercise 6:

- (a) No.  $\$15,000 < \$50,000$  minimum. She would need  $\$35,000$  more, which at  $\$200/\text{month}$  takes 175 months (14.6 years).
- (b) TLH value:  $\$15,000 \times (1.075)^{30} - \$15,000 \times (1.07)^{30} \approx \$129,531 - \$114,184 = \$15,347$  additional wealth.
- (c) (i) Complex tax/estate planning (inheritance, trusts). (ii) Behavioral coaching during market crashes (preventing panic selling).
- (d) Recommendation: Use the robo-advisor. At  $\$15,000$ , the traditional advisor is inaccessible. The robo delivers MPT-optimized allocation at 0.25%, includes automated rebalancing and TLH. Consider the traditional advisor later when assets exceed  $\$250,000$  and needs become more complex.

### Exercise 7:

- (a)  $\$100$  loan: Fixed =  $\$15 + \$8 + \$12 = \$35$ . Variable =  $(6\% + 8\% + 3\%) \times \$100 = \$17$ . Total =  $\$52$ . Rate =  $\$52/\$100 = 52\%$ .
- (b)  $\$500$  loan: Fixed =  $\$35$ . Variable =  $17\% \times \$500 = \$85$ . Total =  $\$120$ . Rate =  $\$120/\$500 = 24\%$ .
- (c)  $\$5,000$  loan: Fixed =  $\$35$ . Variable =  $17\% \times \$5,000 = \$850$ . Total =  $\$885$ . Rate =  $\$885/\$5,000 = 17.7\%$ .
- (d) Fixed costs ( $\$35$ ) are constant regardless of loan size. For a  $\$100$  loan, they represent 35% of principal; for  $\$5,000$ , only 0.7%. Digital origination (mobile apps, automated scoring) can reduce the  $\$35$  fixed cost to  $\$3$ – $\$5$ , making small loans viable at lower rates.

### Exercise 8:

- (a)  $\$50,000/\$200 = 250$  months = **20.8 years**.
- (b)  $S = (6.5\% - 2\%)/10\% = 0.45$ .
- (c) Monthly  $r = 6.25\%/12 = 0.5208\%$ .  $n = 360$ .  $V = 2,000(1.005208)^{360} + 200 \times \frac{(1.005208)^{360} - 1}{0.005208} = 2,000 \times 6.49 + 200 \times 1,054 = 12,980 + 210,802 = \mathbf{\$223,782}$ .