# Lesson 4.2 Exercises: Derivatives, Options, and Risk Transfer

## Module 4: The Risk Problem

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Digital Finance — BSc Course

# Exercise 1: Option Payoff Calculations

**Scenario:** You consider the following four option positions (all synthetic, illustrative):

| Position | Type | Strike | Premium Paid |
|---|---|---|---|
| A | Long call | \$100 | \$6 |
| B | Long put | \$100 | \$4 |
| C | Long call | \$110 | \$2 |
| D | Long put | \$90 | \$3 |

**Tasks:**

a. For each position, calculate the **payoff at expiration** and **net profit** if the stock closes at \$85, \$95, \$100, \$105, and \$115.
b. For each position, determine the **break-even price** (where net profit = 0).
c. Draw the payoff diagram for positions A and B on the same chart. At which stock price does the call profit exactly equal the put profit?
d. A friend says: "I can never lose more than the premium on an option." Is this true for a buyer? For a seller? Explain.

*Difficulty: Introductory — tests arithmetic and payoff logic.*

# Exercise 2: Using the BSM Calculator

**Scenario:** Use the BSM model as a calculator. You are given a European call option with:

| **Input** | **Value** |
|---|---|
| Stock price ($S$) | \$100 |
| Strike price ($K$) | \$100 (ATM) |
| Time to expiration ($T$) | 0.25 years (3 months) |
| Risk-free rate ($r$) | 5% per year |
| Volatility ($\sigma$) | 20% per year |

**Tasks:** (Use an online BSM calculator or the formula provided by your instructor.)

a. Compute the BSM call price. (Expected answer: approximately \$4.62.)
b. If volatility doubles to 40%, what is the new call price? By how much (in dollars and percent) did it increase?
c. If time to expiration increases to 1 year (all else equal), what happens to the price? Why?
d. Which single input has the **largest impact** on the option price for this ATM option? Justify your answer by varying each input by 10%.

*Difficulty: Intermediate — uses BSM as a tool, not a derivation.*

# Exercise 3: Interpreting the Greeks

**Scenario:** Your broker provides the following Greeks for a call option you own (all values synthetic):

| **Greek** | **Value** |
|---|---|
| Delta ($\Delta$) | 0.55 |
| Gamma ($\Gamma$) | 0.04 |
| Vega ($\mathcal{V}$) | 0.18 |
| Theta ($\Theta$) | $-0.06$ |

**Tasks:**

a. The stock rises by \$3. Using Delta only, estimate the option price change.
b. Now use both Delta and Gamma for a more accurate estimate of the same \$3 move. (Hint: the Gamma correction is $\frac{1}{2} \times \Gamma \times (\Delta S)^2$.)
c. Implied volatility increases by 2 percentage points (e.g., from 20% to 22%). How much does the option gain?
d. Five trading days pass with no stock movement and no volatility change. How much value does the option lose?
e. A friend says: "I want an option with high Delta and low Theta." Explain why this is difficult to achieve.

*Difficulty: Intermediate — tests Greek interpretation and arithmetic.*

# Exercise 4: Delta Hedging Step-by-Step

**Scenario:** You have sold 10 call option contracts (each covering 100 shares) on stock XYZ. The current Delta of each option is 0.50.

**Tasks:**

a. How many shares of XYZ must you hold to be **Delta-neutral** right now? Show the calculation.
b. The stock rises by \$2 and the option's Delta increases to 0.60. How many *additional* shares must you buy to restore Delta neutrality?
c. The stock then drops by \$4 and Delta falls to 0.40. How many shares must you sell?
d. Over these two rebalancing steps, you bought shares at a higher price and sold at a lower price. Calculate the approximate dollar loss from rebalancing (assume initial stock price \$50).
e. Explain in 2–3 sentences why Delta hedging is often described as "buying high, selling low." What compensates the hedger for this cost?

*Difficulty: Intermediate–Advanced — requires multi-step tracking.*

# Exercise 5: FX Hedging Decision

**Scenario:** A Swiss watchmaker expects to receive \$2 million from a US distributor in 6 months. Today's USD/CHF rate is 0.90 (i.e., \$1 = CHF 0.90).

Two hedging options are available:

| Instrument | Details | Cost |
|---|---|---|
| 6-month forward | Lock in USD/CHF = 0.895 | Zero upfront |
| 6-month put option | Strike USD/CHF = 0.89, protects below 0.89 | CHF 30,000 premium |

**Tasks:**

a. If the company uses the forward, how many CHF will it receive in 6 months?
b. If USD/CHF drops to 0.82 at expiry, what does the company receive under each strategy?
c. If USD/CHF rises to 0.98 at expiry, what does the company receive under each strategy?
d. Calculate the exchange rate at which both strategies yield the **same CHF amount**. Show your work.
e. Write a 3–4 sentence recommendation to the CFO. Which strategy do you recommend and why?

*Difficulty: Advanced — requires comparison analysis and recommendation.*

# Exercise 6: Duration and Convexity

**Scenario:** A bond portfolio has the following characteristics:

| Bond | Market Value | Duration | Convexity |
|---|---|---|---|
| Bond A (2-year) | \$2,000,000 | 1.9 | 5 |
| Bond B (10-year) | \$3,000,000 | 8.2 | 85 |
| Bond C (30-year) | \$1,000,000 | 19.5 | 420 |

**Tasks:**

a. Calculate the **portfolio-weighted duration**. (Weight by market value.)
b. Calculate the **portfolio-weighted convexity**.
c. Using duration alone, estimate the portfolio value change if rates rise by 1%.
d. Now add the convexity adjustment. By how much does convexity improve the estimate? (Convexity adjustment = $\frac{1}{2} \times C \times (\Delta y)^2 \times V$.)
e. If rates rise by 2% instead, recalculate both estimates. Is the convexity correction larger or smaller? Why?

*Difficulty: Intermediate–Advanced — requires weighted calculations.*

# Exercise 7: Corporate Hedging Strategy

**Scenario:** An electronics manufacturer imports components priced in Japanese yen. Annual import bill: JPY 500 million. Current USD/JPY: 150 (so the bill is approximately $3.33 million).

Three strategies are under consideration:

1. **No hedge:** Accept the JPY exposure.
2. **Forward hedge (100%):** Lock in USD/JPY = 149 for the full amount.
3. **Option hedge (100%):** Buy a USD/JPY put at 148, premium = $80,000.

**Tasks:**

a. Calculate the USD cost under each strategy if USD/JPY is 140 at settlement (yen strengthened — adverse for the importer).
b. Calculate the USD cost under each strategy if USD/JPY is 160 at settlement (yen weakened — favorable for the importer).
c. Create a table comparing the three strategies across: cost if JPY strengthens, cost if JPY weakens, upfront cost, and maximum possible cost.
d. Under what circumstances would you recommend strategy 3 (options) over strategy 2 (forwards)?

*Difficulty: Advanced — integrates forward, option, and no-hedge analysis.*

# Exercise 8: Comprehensive Case – Tech Startup Equity Risk

**Scenario:** A venture capital fund holds 500,000 shares of a pre-IPO tech company currently valued at $40/share ($20M position). The IPO is expected in 6 months. The fund manager wants to protect the downside while keeping upside exposure.

Available instruments:

- 6-month put options: Strike $35, premium $2.50/share; Strike $30, premium $1.00/share
- 6-month forward: Sell at $39.50/share

**Tasks:**

a. Calculate the total cost of each put option hedge for the full 500,000 shares.
b. For each strategy (forward, $35 put, $30 put, no hedge), calculate the portfolio value if the stock is at $25, $35, $40, $50, and $60 at expiry. Include hedge costs.
c. Which strategy has the highest maximum gain? The lowest maximum loss?
d. The fund manager says: "The $30 put is cheap insurance — let's buy that." Explain the risk of choosing the cheaper put. When would it fail to protect the portfolio?
e. Write a 4–5 sentence recommendation. Consider the fund's need to report performance to investors.

*Difficulty: Advanced–Integrative — combines all lesson concepts in a realistic case.*

# Answer Key

**Exercise 1:**

(a) Position A (Long call K=100, prem=6): At \$85: payoff=0, profit=$-6$. At \$105: payoff=5, profit=$-1$. At \$115: payoff=15, profit=9.

(b) Break-even A: \$106. Break-even B: \$96. Break-even C: \$112. Break-even D: \$87.

(d) True for buyers (max loss = premium). **Not true for sellers** — the seller of a call has theoretically unlimited loss; the seller of a put can lose up to the strike price.

**Exercise 2:**

(a) BSM call price ≈ \$4.62 (exact value depends on calculator precision).

(b) At $\sigma = 40\%$: price ≈ \$8.95. Increase = \$4.33 (+94%). Volatility roughly doubles the price.

(c) At $T = 1$: price ≈ \$10.45. More time = more chance of a big move = higher value.

(d) Volatility has the largest impact for an ATM option. A 10% increase in $\sigma$ changes the price more than a 10% change in any other input.

**Exercise 3:**

(a) $\Delta S = 3$: Option change $\approx 0.55 \times 3 =$ \$1.65.

(b) With Gamma: $0.55 \times 3 + \frac{1}{2} \times 0.04 \times 9 = 1.65 + 0.18 =$ **\$1.83**.

(c) Vega effect: $0.18 \times 2 =$ **+\$0.36**.

(d) Theta: $-0.06 \times 5 =$ **−\$0.30**.

# Answer Key (continued)

**Exercise 4:**

(a) Total option exposure: $10 \times 100 \times 0.50 = 500$ shares short (from sold calls). Hold **500 shares** long.

(b) New: $10 \times 100 \times 0.60 = 600$ shares needed. Buy **100 additional shares** at \$52.

(c) New: $10 \times 100 \times 0.40 = 400$ shares needed. Sell **200 shares** at \$48.

(d) Bought 100 at \$52 = \$5,200. Sold 200 at \$48 = \$9,600 (cost basis of 200 shares was 100 at \$50 + 100 at \$52 = \$10,200). Loss ≈ \$600.

(e) Delta hedging requires buying when stock rises (Delta goes up) and selling when it falls (Delta goes down) — "buy high, sell low." The option premium collected compensates for this rebalancing cost.

**Exercise 5:**

(a) Forward: \$2M × 0.895 = **CHF 1,790,000**.

(b) At 0.82: Forward = CHF 1,790,000. Option: exercise put at 0.89, receive \$2M × 0.89 − 30,000 = **CHF 1,750,000**.

(c) At 0.98: Forward = CHF 1,790,000 (locked in). Option: let put expire, receive \$2M × 0.98 − 30,000 = **CHF 1,930,000**.

(d) Equal when: \$2M × rate − 30,000 = 1,790,000 ⇒ rate = 1,820,000/2,000,000 = **0.91**.

**Exercise 6:**

(a) Weighted duration $= (2M \times 1.9 + 3M \times 8.2 + 1M \times 19.5)/6M = (3.8 + 24.6 + 19.5)/6 =$ **7.98 years**.

(b) Weighted convexity $= (2M \times 5 + 3M \times 85 + 1M \times 420)/6M = (10 + 255 + 420)/6 =$ **114.17**.

(c) Duration: $-7.98 \times 0.01 \times 6M =$ −**\$478,800**.

(d) Convexity adjustment: $\frac{1}{2} \times 114.17 \times 0.0001 \times 6M =$ +\$34,251. Corrected: −478,800 + 34,251 = −**\$444,549**.

(e) At +2%: Convexity adjustment $= \frac{1}{2} \times 114.17 \times 0.0004 \times 6M =$ +\$137,004. Correction is 4× larger because it scales with $(\Delta y)^2$.

# Answer Key (continued)

**Exercise 7:**

(a) At USD/JPY = 140: No hedge: 500M/140 = **\$3,571,429**. Forward: 500M/149 = **\$3,355,705**. Option: exercise put at 148, cost = 500M/148 + \$80,000 = **\$3,458,378**.

(b) At USD/JPY = 160: No hedge: 500M/160 = **\$3,125,000**. Forward: 500M/149 = **\$3,355,705** (locked in, now more expensive). Option: let put expire, cost = 500M/160 + \$80,000 = **\$3,205,000**.

(d) Options are preferred over forwards when: (i) the cash flow is uncertain (may not materialize), (ii) management wants to preserve upside from favorable FX moves, or (iii) the company has low tolerance for opportunity cost of being locked in.

**Exercise 8:**

(a) \$35 put: 500,000 × \$2.50 = **\$1,250,000**. \$30 put: 500,000 × \$1.00 = **\$500,000**.

(b) At \$25: Forward = \$19.75M. Put \$35 = 500K × 35 − 1.25M = \$16.25M. Put \$30 = 500K × 30 − 0.5M = \$14.5M. No hedge = \$12.5M.
At \$60: Forward = \$19.75M. Put \$35 = 500K × 60 − 1.25M = \$28.75M. Put \$30 = 500K × 60 − 0.5M = \$29.5M. No hedge = \$30M.

(c) Highest max gain: No hedge (unlimited). Lowest max loss: Forward (\$19.75M guaranteed).

(d) The \$30 put only protects below \$30 — if the stock drops to \$32 (a 20% decline), the put provides **zero protection**. The fund loses \$4M with no hedge benefit. The "gap" between \$30 and \$40 is unprotected.