Lesson 4.1 Exercises: Measuring Market Risk

Module 4: The Risk Problem

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Digital Finance — BSc Course

Exercise 1: VaR from Sorted Returns

Scenario: You manage a CHF 500,000 equity portfolio. Over the past 200 trading days, you recorded daily returns. The 10 worst daily returns (sorted) are:

Rank (worst)	Daily Return
1st	-4.8%
2nd	-3.9%
3rd	-3.4%
4th	-3.1%
5th	-2.7%
6th	-2.4%
7th	-2.2%
8th	-2.0%
9th	-1.8%
10th	-1.6%

Tasks:

a. What is the 95% daily VaR as a percentage? (Hint: 5% of 200 = ?)
b. Convert this to a CHF amount for the CHF 500,000 portfolio.
c. What is the 95% Expected Shortfall (ES) as a percentage?
d. In one sentence, explain what the ES number tells you that VaR does not.

Difficulty: Introductory — tests the "sort and count" intuition.

Exercise 2: Monte Carlo — The Coin-Flip Risk Game

Scenario: You play a coin-flip game 10 times. Each flip:

- Heads (50%): you **win CHF 120**
- Tails (50%): you **lose CHF 100**

Tasks:

a. What is the **expected profit** from playing 10 rounds? Show the calculation.
b. A friend says "positive expected value means I cannot lose money." Explain why this is wrong.
c. You simulate 8 players, each playing 10 rounds. Their total profits/losses are: +CHF 340, −CHF 80, +CHF 120, −CHF 420, +CHF 560, −CHF 200, +CHF 240, −CHF 300. Sort these from worst to best and estimate the "87.5% VaR" of this game. (Hint: 1 out of 8 = 12.5%.)
d. Why would 10,000 simulated players give a more reliable VaR estimate than 8 players?

Difficulty: Introductory–Intermediate — builds Monte Carlo intuition from scratch.

Exercise 3: Three Roads to VaR

Scenario: A portfolio has the following characteristics based on 500 days of historical data:

- Mean daily return: $+0.03\%$
- Daily standard deviation: 1.4%
- The 25th worst historical return: -3.1%

Tasks:

a. Calculate the 95% VaR using the **variance-covariance method**. (Use: $\text{VaR} \approx -(mean - 1.65 \times \sigma)$.)
b. What is the 95% VaR using **historical simulation**? (Hint: 5% of 500 = ?)
c. The two answers differ. Give one reason **why** the methods produce different results.
d. Which method would you trust more if the return data has fat tails? Justify your answer.

Difficulty: Intermediate — requires applying and comparing two methods.

Exercise 4: Diversification in Action

Scenario: You consider two assets for a portfolio:

	Stock A	**Stock B**
Expected annual return	8%	12%
Annual volatility	15%	25%

The correlation between A and B is $\rho = 0.2$.

Tasks:

a. Calculate the portfolio expected return if you invest 60% in A and 40% in B.
b. Calculate the portfolio volatility using $\sigma_p = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \rho \sigma_A \sigma_B}$.
c. Compare the portfolio volatility to the weighted average of the two volatilities ($0.6 \times 15\% + 0.4 \times 25\%$). By how many percentage points did diversification reduce risk?
d. Recalculate the portfolio volatility if $\rho = 1.0$. What happens to the diversification benefit?

Difficulty: Intermediate — requires the portfolio variance formula.

Exercise 5: Why the Bell Curve Lies

Scenario: Under a normal distribution, a return of -3σ or worse should occur with probability 0.13% (about once in 740 trading days, or roughly once every 3 years).

You examine 20 years of daily stock index data (approximately 5,040 trading days) and find **28 days** with returns worse than -3σ.

Tasks:

a. Under the normal distribution, how many -3σ events would you **expect** in 5,040 days?
b. How many times more frequently did extreme losses actually occur compared to the normal prediction?
c. Explain in 2–3 sentences what "fat tails" means and why this matters for risk management.
d. A colleague says: "We should just ignore outliers — they are rare anomalies." Write a one-paragraph response explaining why this is dangerous.

Difficulty: Intermediate — requires reasoning about distributional assumptions.

Exercise 6: Rolling vs EWMA Volatility

Scenario: You observe the following 5 daily returns for a stock: $+0.5\%$, -0.3%, $+0.8\%$, -3.5% (shock!), $+0.2\%$.

Tasks:

a. Calculate the **simple standard deviation** of all 5 returns. (Treat the mean as 0 for simplicity.)
b. Now calculate the standard deviation using only the **last 3 returns** (rolling window of 3). How does it differ from (a)?
c. In qualitative terms (no calculation needed), would an EWMA estimate ($\lambda = 0.94$) after these 5 days be **closer to** the rolling-3 estimate or the full-sample estimate? Why?
d. Why do risk managers prefer EWMA or GARCH over simple rolling windows in practice?

Difficulty: Intermediate — combines computation with conceptual reasoning.

Exercise 7: When Diversification Fails

Scenario: A fund holds 50% stocks and 50% corporate bonds. In normal times, the correlation between stocks and corporate bonds is $\rho = 0.30$. During a financial crisis, the correlation jumps to $\rho = 0.85$.
Both assets have an annual volatility of 18%.

Tasks:

a. Calculate the portfolio volatility in normal times ($\rho = 0.30$).
b. Calculate the portfolio volatility during the crisis ($\rho = 0.85$).
c. By what percentage did portfolio risk increase from normal times to the crisis?
d. A risk model estimated the portfolio's VaR using normal-time correlations. Was this VaR estimate too high, too low, or correct during the crisis? Explain.
e. Name one asset class that historically maintained a **low or negative** correlation with stocks during crises. Why is this valuable?

Difficulty: Advanced — requires portfolio math and qualitative analysis.

Exercise 8: Comprehensive Case — Startup Risk Report

Scenario: You are a risk analyst at a fintech startup that manages CHF 10 million in client assets across a simple portfolio: 70% stocks (annual vol 16%), 30% bonds (annual vol 6%), correlation $= -0.10$.

Tasks:

a. Calculate the portfolio's annual volatility.
b. Convert to daily volatility (divide by $\sqrt{252}$).
c. Estimate the 95% daily VaR in CHF using the variance-covariance method (VaR $\approx 1.65 \times \sigma_{\text{daily}} \times$ portfolio value).
d. Your CTO asks: "What is the worst-case daily loss?" Explain why VaR does **not** answer this question and suggest a complementary measure.
e. Write a 3-sentence "risk summary" for the startup's board, using plain language (no jargon). Include the VaR number, its meaning, and one key caveat.

Difficulty: Advanced–Integrative — combines all lesson concepts into a realistic scenario.

Answer Key

Exercise 1:

(a) 5% of 200 = 10. The 10th worst return is the 95% VaR = -1.6%.

(b) CHF 500,000 $\times$ 1.6% = **CHF 8,000**.

(c) ES = average of the 10 worst: $(4.8 + 3.9 + 3.4 + 3.1 + 2.7 + 2.4 + 2.2 + 2.0 + 1.8 + 1.6)/10 = 27.9/10 = -2.79\%$.

(d) ES tells you the *average* severity of losses on the worst 5% of days, not just the boundary.

Exercise 2:

(a) Expected profit per flip $= 0.5 \times 120 + 0.5 \times (-100) = +10$. Over 10 rounds: **+CHF 100**.

(b) Expected value is the *average* over many repetitions. In any single sequence of 10 flips, you can get unlucky and lose money. Variance creates a range of outcomes.

(c) Sorted: -420, -300, -200, -80, $+120$, $+240$, $+340$, $+560$. The 1st worst out of 8 (12.5%) gives 87.5% VaR = $-$**CHF 420**. (With only 8 observations, this is imprecise.)

(d) More simulations reduce sampling error. 10,000 paths give a much smoother distribution and more reliable percentile estimates.

Exercise 3:

(a) VaR $= -(0.03\% - 1.65 \times 1.4\%) = -(-2.28\%) =$ **2.28%**.

(b) 5% of 500 = 25. The 25th worst = -3.1%, so historical VaR = 3.1%.

(c) The variance-covariance method assumes normally distributed returns. If the actual distribution has fatter tails, the 5th percentile will be more extreme than the normal approximation.

(d) Historical simulation, because it uses actual data without assuming a normal shape. Fat-tailed data will naturally produce a larger VaR.

Answer Key (continued)

Exercise 4:

(a) $E[R_p] = 0.6 \times 8\% + 0.4 \times 12\% = 4.8\% + 4.8\% =$ **9.6%**.

(b) $\sigma_p = \sqrt{0.6^2 \times 0.15^2 + 0.4^2 \times 0.25^2 + 2 \times 0.6 \times 0.4 \times 0.2 \times 0.15 \times 0.25} = \sqrt{0.0081 + 0.01 + 0.0036} = \sqrt{0.0217} =$ **14.73%**.

(c) Weighted average $= 0.6 \times 15\% + 0.4 \times 25\% = 19\%$. Diversification saved $19\% - 14.73\% =$ **4.27 percentage points**.

(d) If $\rho = 1.0$: $\sigma_p = \sqrt{0.0081 + 0.01 + 0.018} = \sqrt{0.0361} = 19.0\%$. No diversification benefit — portfolio vol equals weighted average.

Exercise 5:

(a) Expected $= 5{,}040 \times 0.0013 \approx$ **6.6 events**.

(b) Actual / expected $= 28/6.6 \approx$ **4.2 times more often**.

(c) Fat tails mean the probability of extreme returns is much higher than predicted by a normal distribution. This matters because risk models using normal assumptions systematically underestimate the frequency and severity of crashes.

Exercise 6:

(a) Returns: 0.5, −0.3, 0.8, −3.5, 0.2 (%). Assuming mean = 0: $\sigma = \sqrt{(0.25 + 0.09 + 0.64 + 12.25 + 0.04)/5} = \sqrt{2.654} =$ **1.63%**.

(b) Last 3: 0.8, −3.5, 0.2. $\sigma = \sqrt{(0.64 + 12.25 + 0.04)/3} = \sqrt{4.31} =$ **2.08%**. Higher because the shock is a larger proportion.

(c) Closer to rolling-3 because EWMA places more weight on the recent shock (−3.5%), amplifying its effect relative to the full-sample estimate.

Answer Key (continued)

Exercise 7:

(a) $\sigma_p = \sqrt{0.5^2 \times 0.18^2 + 0.5^2 \times 0.18^2 + 2 \times 0.5 \times 0.5 \times 0.30 \times 0.18 \times 0.18} = \sqrt{0.0081 + 0.0081 + 0.00486} = \sqrt{0.02106} = \mathbf{14.51\%}$.

(b) $\sigma_p = \sqrt{0.0081 + 0.0081 + 2 \times 0.25 \times 0.85 \times 0.0324} = \sqrt{0.0081 + 0.0081 + 0.01377} = \sqrt{0.02997} = \mathbf{17.31\%}$.

(c) Increase $= (17.31 - 14.51)/14.51 =$ **19.3%** increase in portfolio risk.

(d) Too low. The model assumed $\rho = 0.30$, giving lower portfolio vol. The actual crisis correlation of 0.85 meant less diversification, so true risk was higher than modeled.

(e) Government bonds (especially U.S. Treasuries) historically maintained negative or low correlation with stocks during crises (flight to safety). This makes them valuable as a hedge.

Exercise 8:

(a) $\sigma_p = \sqrt{0.7^2 \times 0.16^2 + 0.3^2 \times 0.06^2 + 2 \times 0.7 \times 0.3 \times (-0.10) \times 0.16 \times 0.06} = \sqrt{0.012544 + 0.000324 - 0.000403} = \sqrt{0.012465} = \mathbf{11.16\%}$.

(b) Daily vol $= 11.16\%/\sqrt{252} = 11.16\%/15.87 = \mathbf{0.703\%}$.

(c) VaR $= 1.65 \times 0.703\% \times 10{,}000{,}000 =$ **CHF 116,000** (approx.).

(d) VaR does not give the worst-case loss; it gives the loss exceeded only 5% of the time. Expected Shortfall measures the average loss on those worst 5% days, providing more information about tail severity.

(e) "On 95% of trading days, we expect client portfolios to lose no more than approximately CHF 116,000. This means on about 1 in 20 trading days, losses could exceed this amount. This estimate assumes recent market conditions continue — in a severe crisis, actual losses could be significantly larger."