DeFi: Automated Market Makers and Lending

Theme III: Decentralized Finance

Research Question: How do decentralized exchanges and lending protocols work, and what are their inherent risks?

PhD Seminar in Digital Finance

Constant Product AMMs: Theory

Uniswap v2 Mechanism
Invariant: $x \cdot y = k$
Trade Δx for Δy:

$$(x + \Delta x)(y - \Delta y) = k$$

Implied Price

$$p = \frac{y}{x}$$

After trade:

$$p' = \frac{y - \Delta y}{x + \Delta x}$$

Properties
Marginal Price

$$\frac{dy}{dx} = -\frac{y}{x} = -p$$

Price Impact
For trade size Δx:

$$\text{Impact} = \frac{\Delta y}{\Delta x} - p = -\frac{p \cdot \Delta x}{x + \Delta x}$$

Slippage
Large trades move price significantly.

Adams et al. (2021), "Uniswap v3 Core" – concentrated liquidity extension

Impermanent Loss: The LP's Dilemma

Definition

LP provides at price p_0, withdraws at p_1.
Value if held: $V_{hold} = x_0 \cdot p_1 + y_0$
Value in pool: $V_{pool} = 2\sqrt{k \cdot p_1}$

Impermanent Loss

$$IL = \frac{V_{pool}}{V_{hold}} - 1 = \frac{2\sqrt{r}}{1+r} - 1$$

where $r = p_1/p_0$.

IL by Price Change

Price Change	IL
1.25x	-0.6%
1.50x	-2.0%
2x	-5.7%
3x	-13.4%
5x	-25.5%

Break-Even

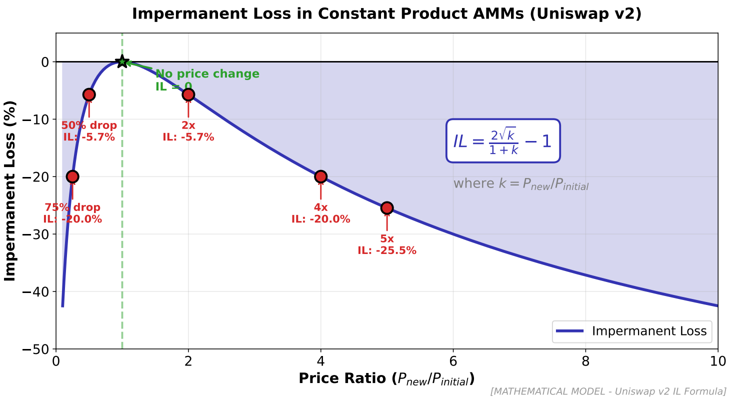
LP profitable if:

$$\text{Fees earned} > IL$$

High volume, stable pairs preferred.

IL is “impermanent” only if price returns; otherwise realized loss

Impermanent Loss: Mathematical Visualization



IL increases non-linearly with price deviation; 50% price change yields 5.7% loss.

AMM Design Space

Curve: StableSwap
For stablecoins, use:

$$x + y + \frac{A \cdot xy}{(x+y)/2} = D + \frac{A \cdot D}{2}$$

Low slippage near peg.
Uniswap v3: Concentrated Liquidity
LPs choose price range $[p_a, p_b]$:

$$\text{Virtual liquidity} = L = \frac{\Delta y}{\sqrt{p_b} - \sqrt{p_a}}$$

Higher capital efficiency but:

- Active management required
- Higher IL in range

Optimal AMM Design (2024)
Curry, Fan & Parkes (2024):
"Differentiable economics" approach.
Key Insight
Optimal bonding curve depends on:

- Expected trading patterns
- Arbitrageur behavior
- LP risk preferences

No Universal Optimum
Trade-off: Execution quality vs. LP returns.

Curry, Fan & Parkes (Feb 2024), "Optimal AMMs: Differentiable Economics," arXiv

DeFi Lending: Overcollateralized Loans

Mechanism (Aave, Compound)

1. Deposit collateral (e.g., ETH)
2. Borrow up to LTV ratio (e.g., 75%)
3. Interest accrues algorithmically
4. Liquidation if undercollateralized

Interest Rate Model

$$r_{borrow} = r_0 + \frac{U}{U_{optimal}} \cdot r_{slope1}$$

where U = utilization ratio.

Key Parameters (Aave v3)

Parameter	Value
Max LTV (ETH)	80%
Liquidation threshold	82.5%
Liquidation bonus	5%
Reserve factor	10%

Capital Efficiency

Overcollateralization limits leverage. "Capital-inefficient" vs. TradFi.

Aave v3 Documentation (2022) – current market leader by TVL

Liquidation Cascades and Systemic Risk

Cascade Mechanism

1. Price drops $\rightarrow$ loans undercollateralized
2. Liquidators seize and sell collateral
3. Selling pressure $\rightarrow$ further price drop
4. More loans undercollateralized
5. Repeat...

Moallemi & Patange (2024)
"Fixed-Spread Liquidation Lending"
Proposes gradual liquidation to reduce cascades.

Empirical Evidence
ScienceDirect (2024): Aave analysis

- 25K+ liquidation events studied
- Cascade probability: 15% during crashes
- Median cascade depth: 3 rounds

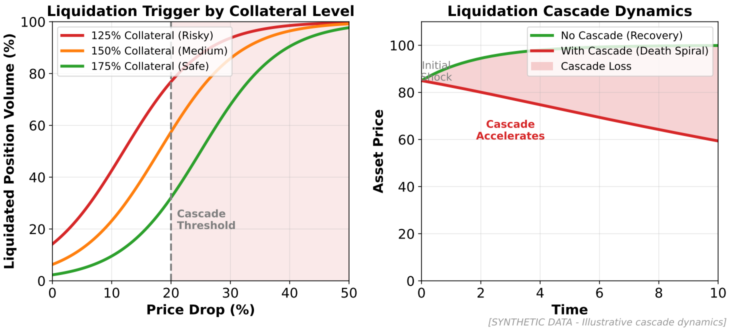
Key Finding
Liquidations discourage new lending:

$$\frac{\partial TVL}{\partial \text{Liquidations}} < 0$$

Design Implication
Smoother liquidation curves reduce systemic risk.

Moallemi & Patange (Mar 2024), "Fixed-Spread Liquidation Lending," SSRN/FC'24

Liquidation Cascades: Systemic Risk Simulation



Price shocks trigger cascading liquidations; severity depends on collateralization ratios and liquidity.

DeFi vs. Traditional Finance: Structural Comparison

Advantages of DeFi

- **Composability**: Lego-like building
- **Transparency**: All on-chain
- **Permissionless**: No gatekeepers
- **24/7**: No market hours
- **Programmable**: Smart contracts

Efficiency Gains

	DeFi	TradFi
Settlement	Minutes	T+2
Access	Global	Regional
Fees	Variable	Fixed

Limitations of DeFi

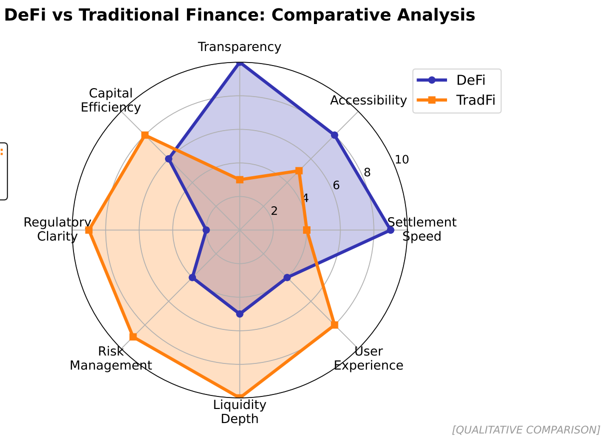
- **Overcollateralization**: Capital-inefficient
- **Volatility**: Gas fees, slippage
- **Smart contract risk**: Bugs, hacks
- **Scalability**: Throughput limits
- **Regulation**: Legal uncertainty

OECD (2023) Assessment

"DeFi liquidations create procyclicality comparable to traditional margin calls, but with less circuit breaker protection."

OECD (2023), "DeFi Liquidations: Volatility and Liquidity" – policy analysis

DeFi vs TradFi: Multi-Dimensional Comparison



TradFi Advantages:
- Regulation
- Risk Mgmt
- Liquidity

DeFi Advantages:
- Speed
- Transparency
- Accessibility

DeFi excels in transparency and speed but lags in capital efficiency and user protection.

Open Research Questions

Publishable Research Directions

1. **Optimal AMM Design**
   - RQ: What bonding curve maximizes total welfare (LPs + traders)?
   - Method: Mechanism design with heterogeneous agents
   - Gap: Existing designs heuristic; no provably optimal AMM
2. **DeFi Systemic Risk**
   - RQ: How interconnected are DeFi protocols, and what triggers cascades?
   - Method: Network analysis of on-chain data
   - Gap: Limited quantification of DeFi contagion channels
3. **LP Returns Decomposition**
   - RQ: What fraction of LP returns comes from fees vs. adverse selection?
   - Method: Transaction-level analysis, informed vs. uninformed flow
   - Gap: Aggregate IL known; flow composition understudied

DeFi offers rich on-chain data for empirical research

Exercises

Mathematical

Derive impermanent loss formula. Starting with:

- $x_0 y_0 = k$
- $p_0 = y_0 / x_0$
- Price changes to p_1

Show:

$$IL = \frac{2\sqrt{p_1/p_0}}{1 + p_1/p_0} - 1$$

Empirical

Using Dune Analytics:

1. Track Uniswap v3 LP positions
2. Compute realized IL
3. Compare to fee income
4. Identify profitable strategies

Data: Dune/Flipside

Research Proposal

Draft 1-page proposal:

- "DeFi Lending Cascade Risk"
- Define cascade metric
- Identify trigger events
- Policy implications

Due: Week 9 – IL derivation is elegant; understand the intuition

On-chain data is public and granular – excellent for empirical research

Required Readings

Core Papers (Read Before Class)

1. **Bartoletti et al.** (2021). "A Theory of Automated Market Makers in DeFi." *FC 2021*.
   - Focus: Sections 2-3, AMM properties
2. **Moallemi & Patange** (2024). "Fixed-Spread Liquidation Lending in DeFi." *SSRN/FC'24*.
   - Focus: Liquidation mechanism design

Supplementary

- Adams et al. (2021): Uniswap v3 whitepaper
- ScienceDirect (2024): "Do Liquidations Discourage Lending in DeFi?"
- Curry, Fan & Parkes (2024): Optimal AMM – arXiv

Bartoletti provides theoretical foundation; Moallemi addresses systemic risk