Market Microstructure in the Digital Age

Theme I: Digital Market Design

Research Question: How does high-frequency trading affect price discovery, liquidity, and market quality?

PhD Seminar in Digital Finance

The Kyle (1985) Model: Foundation

Setup

- Asset with value $v \sim N(\bar{v}, \Sigma_0)$
- Informed trader observes v
- Noise traders: $u \sim N(0, \sigma_u^2)$
- Market maker sets price p

Market Maker's Problem

$$p = \mathbb{E}[v|x + u]$$

where x is informed order flow.

Linear Equilibrium

Informed trader: $x = \beta(v - \bar{v})$

Price: $p = \bar{v} + \lambda(x + u)$

Proposition (Kyle's Lambda)

In equilibrium:

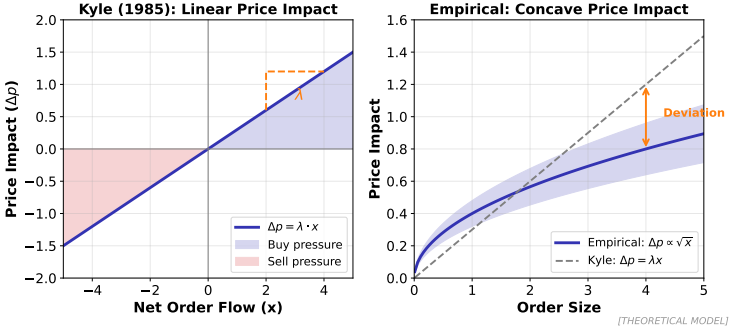
$$\lambda = \frac{\sqrt{\Sigma_0}}{2\sigma_u}$$

Interpretation

- λ = price impact per unit order
- Decreases with noise trading
- Increases with information asymmetry

Kyle (1985), "Continuous Auctions and Insider Trading," Econometrica

Kyle's Lambda: Price Impact Visualization



Price impact increases with order size; informed traders optimally conceal information.

Glosten-Milgrom (1985): Bid-Ask Spreads

Model Setup

- $v \in \{V_L, V_H\}$ with $\Pr(V_H) = \mu$
- Informed traders: fraction π
- Uninformed: buy/sell with prob 0.5

Equilibrium Spreads

$$A = \mathbb{E}[v|\text{buy}] = \frac{\mu(1-\pi)+\pi}{\mu(1-\pi)+(1-\mu)(1-\pi)+\pi} \cdot V_H + \ldots$$

Adverse Selection Component

Spread $S = A - B$ satisfies:

$$S \approx \pi(V_H - V_L)$$

Key Insight

Spread compensates for adverse selection:

- More informed trading $\rightarrow$ wider spread
- More volatile asset $\rightarrow$ wider spread
- Zero-profit condition for market makers

Glosten & Milgrom (1985), "Bid, Ask and Transaction Prices," JFE

High-Frequency Trading: The Speed Arms Race

HFT Characteristics

- Latency: < 1 microsecond
- Holding period: seconds to minutes
- Market share: 50%+ of equity volume
- Strategies: market-making, arbitrage

Technology Evolution

Year	Latency
2005	20 ms
2010	1 ms
2015	50 μs
2024	< 1 μs

Arms Race Model (Budish et al., 2015)
Speed investment s_i yields:

$$\pi_i(s_i, s_{-i}) = f(s_i - s_{-i}) - c(s_i)$$

Proposition

In equilibrium, total speed investment:

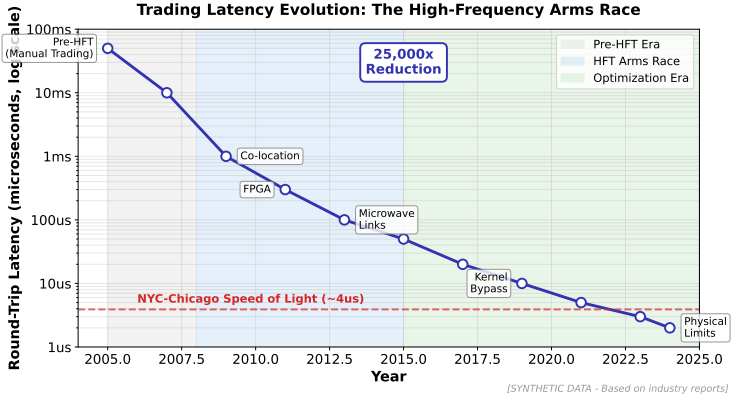
$$S^* = N \cdot \arg\max_s [f'(0)/N - c'(s)]$$

is socially wasteful.

Welfare Loss
Private value $>$ Social value of speed.

Budish, Cramton & Shim (2015), "The High-Frequency Trading Arms Race," QJE

The Speed Arms Race: Latency Evolution



Trading latency has declined from milliseconds to microseconds, driving billions in infrastructure investment.

Empirical Evidence: HFT Effects on Market Quality

Natural Experiments

Study	Setting	Effect
Hendershott+ (2011)	NYSE Autoquote	Spread ↓
Brogaard+ (2014)	NASDAQ HFT	Vol ↓
Menkveld (2013)	Chi-X entry	Spread ↓
Aquilina+ (2022)	UK speed bump	Mixed

Tse (2024) Meta-Analysis

- 47 papers, 1,200+ estimates
- HFT reduces spreads: -15% median
- But increases adverse selection for slow traders

Key Findings

Benefits:

- Tighter spreads (15-25%)
- Faster price discovery
- Lower explicit costs

Costs:

- Arms race waste ($billions)
- Flash crashes (May 2010)
- Increased complexity

Net Effect

Ambiguous; depends on trader type.

Tse (June 2024), "HFT, Asset Pricing, and Market Microstructure," SSRN

Market Fragmentation and Best Execution

Fragmentation Trend
US equity trading venues:

- 1990: 2 exchanges (NYSE, NASDAQ)
- 2024: 16 exchanges + 40 ATSs
- Off-exchange: 45% of volume

Theoretical Trade-Off
Consolidation: Better price discovery
Fragmentation: More competition

$$W = \alpha \cdot \text{PriceDisc} + (1 - \alpha) \cdot \text{Competition}$$

Best Execution Problem
Router chooses venue j to minimize:

$$C_j = \text{spread}_j + \text{impact}_j + \text{delay}_j$$

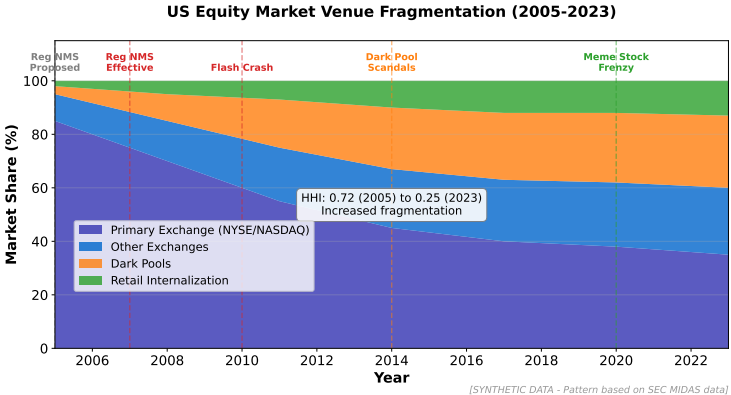
Smart Order Routing

- NBBO: National Best Bid/Offer
- Reg NMS: Trade-through protection
- Payment for order flow: Conflicts

Research Finding
Fragmentation benefits HFTs disproportionately (arbitrage opportunities).

O'Hara & Ye (2011), "Is Market Fragmentation Harming Market Quality?" JFE

Market Fragmentation: Trading Venue Evolution



Market share has shifted from traditional exchanges to alternative venues and dark pools.

Market Design: Continuous vs. Frequent Batch Auctions

Continuous Limit Order Book

- Price-time priority
- Continuous matching
- Speed advantage valuable

Problem: Encourages wasteful speed competition.

Frequent Batch Auctions (FBA)

- Discrete time intervals (e.g., 100ms)
- Pro-rata or random allocation
- Eliminates speed advantage

BCS Proposal

Budish, Cramton & Shim argue for:

FBA interval $\approx$ 100ms

Trade-Offs

	CLOB	FBA
Speed value	High	Zero
Spread	Lower	Higher
Arms race	Yes	No
Complexity	Lower	Higher

IEX Speed Bump

350μs delay: Partial solution.

IEX market share grew to 3% but continuous markets still dominate

Open Research Questions

Publishable Research Directions

1. **Optimal Market Design Under Heterogeneous Traders**
   - RQ: What auction format maximizes welfare when traders differ in speed?
   - Method: Mechanism design with asymmetric information about speed
   - Gap: Existing models assume homogeneous traders
2. **Cross-Asset HFT Spillovers**
   - RQ: How does HFT in one market affect liquidity in related markets?
   - Method: Exploit regulatory changes (MiFID II) as natural experiment
   - Gap: Most studies focus on single-market effects
3. **Retail Order Flow and Market Quality**
   - RQ: Does payment for order flow harm retail investors?
   - Method: Compare execution quality across routing regimes
   - Gap: SEC debate lacks causal evidence

Market microstructure offers clean identification opportunities

Exercises

Mathematical
In the Kyle model, show that:

$$\text{Var}(p_T - v) = \frac{\Sigma_0}{2}$$

Interpret: How much information is revealed by trading?
Hint: Use $p_T = \bar{v} + \lambda(x + u)$ and equilibrium β, λ.

Empirical
Using TAQ data:

1. Compute effective spreads for 10 stocks
2. Compare pre/post decimalization
3. Decompose into adverse selection vs realized spread

Data: Wharton TAQ

Research Proposal
Draft 1-page proposal:

- "HFT and Flash Crashes"
- Identify episode for study
- Specify testable hypothesis
- Data requirements

Due: Week 3 – Mathematical exercise recommended for theory-focused students

Kyle model derivation is a classic prelim question

Required Readings

Core Papers (Read Before Class)

1. **Kyle** (1985). "Continuous Auctions and Insider Trading." *Econometrica*, 53(6), 1315-1335.
   - Focus: Model setup, Proposition 1, equilibrium derivation
2. **Budish, Cramton & Shim** (2015). "The High-Frequency Trading Arms Race." *QJE*, 130(4), 1547-1621.
   - Focus: Sections 1-3, welfare analysis

Supplementary

- Glosten & Milgrom (1985): Bid-ask spread foundations
- Tse (2024): Recent HFT meta-analysis – SSRN
- O'Hara (2015): "High Frequency Market Microstructure" – JFE survey

Kyle (1985) is foundational; expect detailed knowledge for discussion