Lesson 4.2 Exercises: Derivatives, Options, and Risk Transfer

Module 4: The Risk Problem

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Digital Finance — BSc Course

Exercise 1: Option Payoff Calculations

Scenario (hypothetical exercise): You consider the following four option positions:

Position	Type	Strike	Premium Paid
A	Long call	\$100	\$6
B	Long put	\$100	\$4
C	Long call	\$110	\$2
D	Long put	\$90	\$3

Tasks:

a. For each position, calculate the **payoff at expiration** and **net profit** if the stock closes at \$85, \$95, \$100, \$105, and \$115.

b. For each position, determine the **break-even price** (where net profit = 0).

c. Draw the payoff diagram for positions A and B on the same chart. At which stock price does the call profit exactly equal the put profit?

d. A friend says: "I can never lose more than the premium on an option." Is this true for a buyer? For a seller? Explain.

Difficulty: Introductory — tests arithmetic and payoff logic.

Exercise 2: Using the BSM Calculator

Scenario: Use the BSM model as a calculator. You are given a European call option with:

Input	**Value**
Stock price (S)	\$100
Strike price (K)	\$100 (ATM)
Time to expiration (T)	0.25 years (3 months)
Risk-free rate (r)	5% per year
Volatility (σ)	20% per year

Tasks: (Use an online BSM calculator or the formula provided by your instructor.)

a. Compute the BSM call price. (Expected answer: approximately \$4.62.)

b. If volatility doubles to 40%, what is the new call price? By how much (in dollars and percent) did it increase?

c. If time to expiration increases to 1 year (all else equal), what happens to the price? Why?

d. Which single input has the **largest impact** on the option price for this ATM option? Justify your answer by varying each input by 10%.

Difficulty: Intermediate — uses BSM as a tool, not a derivation.

Exercise 3: Interpreting the Greeks

Scenario: Your broker provides the following Greeks for a call option you own (all values synthetic):

Greek	Value
Delta (Δ)	0.55
Gamma (Γ)	0.04
Vega ($\mathcal{V}$)	0.18
Theta (Θ)	-0.06

Tasks:

a. The stock rises by \$3. Using Delta only, estimate the option price change.

b. Now use both Delta and Gamma for a more accurate estimate of the same \$3 move. (Hint: the Gamma correction is $\frac{1}{2} \times \Gamma \times (\Delta S)^2$.)

c. Implied volatility increases by 2 percentage points (e.g., from 20% to 22%). How much does the option gain?

d. Five trading days pass with no stock movement and no volatility change. How much value does the option lose?

e. A friend says: "I want an option with high Delta and low Theta." Explain why this is difficult to achieve.

Difficulty: Intermediate — tests Greek interpretation and arithmetic.

Exercise 4: Delta Hedging Step-by-Step

Scenario: You have sold 10 call option contracts (each covering 100 shares) on stock XYZ. The current Delta of each option is 0.50.

Tasks:

a. How many shares of XYZ must you hold to be **Delta-neutral** right now? Show the calculation.

b. The stock rises by \$2 and the option's Delta increases to 0.60. How many *additional* shares must you buy to restore Delta neutrality?

c. The stock then drops by \$4 and Delta falls to 0.40. How many shares must you sell?

d. Over these two rebalancing steps, you bought shares at a higher price and sold at a lower price. Calculate the approximate dollar loss from rebalancing (assume initial stock price \$50).

e. Explain in 2–3 sentences why Delta hedging is often described as "buying high, selling low." What compensates the hedger for this cost?

Difficulty: Intermediate–Advanced — requires multi-step tracking.

Exercise 5: FX Hedging Decision

Scenario: A Swiss watchmaker expects to receive \$2 million from a US distributor in 6 months. Today's USD/CHF rate is 0.90 (i.e., \$1 = CHF 0.90).

Two hedging options are available:

Instrument	Details	Cost
6-month forward	Lock in USD/CHF = 0.895	Zero upfront
6-month put option	Strike USD/CHF = 0.89, protects below 0.89	CHF 30,000 premium

Tasks:

a. If the company uses the forward, how many CHF will it receive in 6 months?

b. If USD/CHF drops to 0.82 at expiry, what does the company receive under each strategy?

c. If USD/CHF rises to 0.98 at expiry, what does the company receive under each strategy?

d. Calculate the exchange rate at which both strategies yield the **same CHF amount**. Show your work.

e. Write a 3–4 sentence recommendation to the CFO. Which strategy do you recommend and why?

Difficulty: Advanced — requires comparison analysis and recommendation.

Exercise 6: Duration and Convexity

Scenario: A bond portfolio has the following characteristics:

Bond	Market Value	Duration	Convexity
Bond A (2-year)	\$2,000,000	1.9	5
Bond B (10-year)	\$3,000,000	8.2	85
Bond C (30-year)	\$1,000,000	19.5	420

Tasks:

a) Calculate the **portfolio-weighted duration**. (Weight by market value.)

b) Calculate the **portfolio-weighted convexity**.

c) Using duration alone, estimate the portfolio value change if rates rise by 1%.

d) Now add the convexity adjustment. By how much does convexity improve the estimate? (Convexity adjustment $= \frac{1}{2} \times C \times (\Delta y)^2 \times V$.)

e) If rates rise by 2% instead, recalculate both estimates. Is the convexity correction larger or smaller? Why?

Difficulty: Intermediate–Advanced — requires weighted calculations.

Exercise 7: Corporate Hedging Strategy

Scenario: An electronics manufacturer imports components priced in Japanese yen. Annual import bill: JPY 500 million. Current USD/JPY: 150 (so the bill is approximately $3.33 million).

Three strategies are under consideration:

1. **No hedge:** Accept the JPY exposure.
2. **Forward hedge (100%):** Lock in USD/JPY = 149 for the full amount.
3. **Option hedge (100%):** Buy a USD/JPY put at 148, premium = $80,000.

Tasks:

a) Calculate the USD cost under each strategy if USD/JPY is 140 at settlement (yen strengthened — adverse for the importer).

b) Calculate the USD cost under each strategy if USD/JPY is 160 at settlement (yen weakened — favorable for the importer).

c) Create a table comparing the three strategies across: cost if JPY strengthens, cost if JPY weakens, upfront cost, and maximum possible cost.

d) Under what circumstances would you recommend strategy 3 (options) over strategy 2 (forwards)?

Difficulty: Advanced — integrates forward, option, and no-hedge analysis.

Exercise 8: Comprehensive Case – Tech Startup Equity Risk

Scenario: A venture capital fund holds 500,000 shares of a pre-IPO tech company currently valued at $40/share ($20M position). The IPO is expected in 6 months. The fund manager wants to protect the downside while keeping upside exposure.

Available instruments:

- 6-month put options: Strike $35, premium $2.50/share; Strike $30, premium $1.00/share
- 6-month forward: Sell at $39.50/share

Tasks:

a) Calculate the total cost of each put option hedge for the full 500,000 shares.

b) For each strategy (forward, $35 put, $30 put, no hedge), calculate the portfolio value if the stock is at $25, $35, $40, $50, and $60 at expiry. Include hedge costs.

c) Which strategy has the highest maximum gain? The lowest maximum loss?

d) The fund manager says: "The $30 put is cheap insurance — let's buy that." Explain the risk of choosing the cheaper put. When would it fail to protect the portfolio?

e) Write a 4–5 sentence recommendation. Consider the fund's need to report performance to investors.

Difficulty: Advanced–Integrative — combines all lesson concepts in a realistic case.