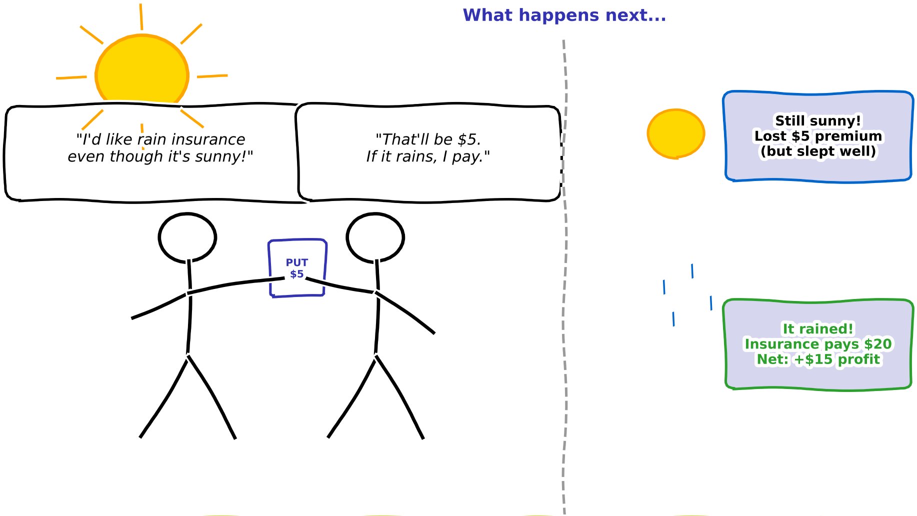
Lesson 4.2: Derivatives, Options, and Risk Transfer

Module 4: The Risk Problem

Prof. Dr. Joerg Osterrieder

Digital Finance — BSc Course

Buying Umbrella Insurance on a Sunny Day



Options = insurance: pay a small premium now for protection later

Learning Objectives

After completing this lesson, you will be able to:

1. **Explain** the difference between forwards, futures, swaps, and options [Understand]
2. **Draw** the payoff diagram for a call option, a put option, and basic combinations [Apply]
3. **Use** the Black–Scholes–Merton model as a calculator with 5 inputs [Apply]
4. **Interpret** the Greeks (Delta, Gamma, Vega, Theta) as sensitivity measures [Analyze]
5. **Evaluate** hedging strategies for FX exposure and interest-rate risk [Analyze]

Bloom's levels covered: Understand, Apply, Analyze

Objectives follow Bloom's taxonomy: Understand → Apply → Analyze.

Bridge: From Measuring Risk to Transferring Risk

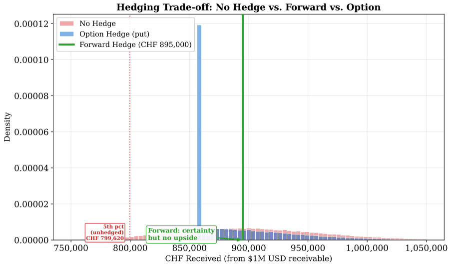
Previous lesson:

- We learned to **measure** risk — VaR, ES, volatility
- We can now quantify how much we might lose

The next question:

- What if we do not *want* to bear that risk?
- Can we pay someone else to carry it for us?
- **Yes — that is exactly what derivatives do.**

This lesson: How derivatives let you **transfer** risk to someone willing to bear it — and at what price.



Hedging reduces downside exposure but costs a premium.

We can measure risk. Now: how do we transfer it?

What Is a Derivative?

Definition: Derivative

A **derivative** is a financial contract whose value is *derived from* an underlying asset, rate, or index. The derivative itself is not the asset — it is a **side bet** on where the asset's price will go.

Common underlying assets:

- Stocks and stock indices
- Interest rates and bonds
- Foreign exchange rates
- Commodities (oil, gold, wheat)
- Credit risk (credit default swaps)
- Cryptocurrency

Why do derivatives exist?

- **Hedging:** Transfer unwanted risk to someone willing to bear it
- **Speculation:** Take a position on future price movements with leverage
- **Price discovery:** Futures prices reveal market expectations

Global derivatives market notional: over $600 trillion — far larger than the global stock market.

The Four Building Blocks of Derivatives

Type	Obligation	Analogy	Cost
Forward	Both must trade	"Handshake deal" to buy a car in 6 months at an agreed price	Zero upfront
Future	Both must trade	Same as forward, but standardized and exchange-traded	Margin deposit
Swap	Both must exchange	"I'll pay your mortgage, you pay mine"	Zero upfront
Option	Buyer chooses	"Insurance policy" — pay premium, exercise if needed	Premium

Key distinction:

- Forwards, futures, and swaps are **obligations** — both sides must perform
- Options are **rights** — the buyer can walk away if the contract is not profitable
- This asymmetry is why options require an **upfront premium**

Forwards/futures/swaps = obligations (symmetric risk). Options = rights (asymmetric risk, hence the premium).

Forward Contracts: The Simplest Derivative

Definition: Forward Contract

A **forward** is an agreement to buy or sell an asset at a **pre-agreed price** (the forward price) on a **specific future date**. No money changes hands until maturity.

Example: A European exporter expects to receive $1 million in 90 days.

- Today's EUR/USD rate: 1.10 (so $1M = €909,091)
- Risk: If EUR strengthens to 1.15, the $1M is only worth €869,565 — a €39,526 loss
- **Hedge:** Enter a 90-day forward to sell $1M at the forward rate of 1.105
- Result: The exporter locks in €905,000 regardless of where the rate moves

Trade-off: If EUR weakens to 1.05, the exporter *still* gets €905,000 — missing the upside of €952,381. Forwards lock in **both** sides.

Forwards remove uncertainty by fixing a price today — but you give up upside to eliminate downside.

Futures vs. Forwards: Exchange-Traded vs. OTC

Feature	Forward	Future
Trading venue	Over-the-counter (OTC)	Exchange (CME, Eurex)
Contract terms	Customizable	Standardized
Counterparty risk	Direct (bilateral)	Clearinghouse guarantees
Settlement	At maturity	Daily mark-to-market
Margin required	Negotiable	Mandatory (initial + variation)
Liquidity	Lower	Higher
Typical users	Corporates, banks	Speculators, hedgers, funds

Daily mark-to-market (futures):

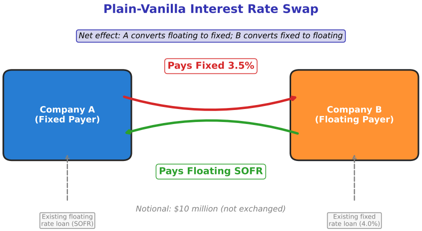
- Every day, gains and losses are settled in cash via the margin account
- If your margin drops below the **maintenance margin**, you receive a **margin call**
- This eliminates the risk that the loser cannot pay at maturity

Futures reduce counterparty risk via the clearinghouse, but require daily cash management (margin calls).

Interest Rate Swaps: Exchanging Cash Flows

Definition: Interest Rate Swap

An **interest rate swap (IRS)** is an agreement where two parties exchange interest payments on the same notional amount (the face value of a derivative contract): one pays a **fixed rate**, the other pays a **floating rate** (e.g., Secured Overnight Financing Rate or SOFR).



Why use a swap?

- A company with a floating-rate loan wants certainty
- It enters a swap: pays fixed 3.5%, receives floating SOFR
- Net effect: the floating-rate loan becomes **effectively fixed**

Market size: Interest rate swaps are the **largest** derivative market by notional value (over $400 trillion).

Swaps let you convert a floating-rate exposure to fixed (or vice versa) without refinancing.

Options as Insurance Contracts

Definition: Option

An **option** gives the buyer the **right, but not the obligation**, to buy (call) or sell (put) an underlying asset at a pre-agreed **strike price** on or before a specified **expiration date**. The buyer pays a **premium** upfront.

The insurance analogy:

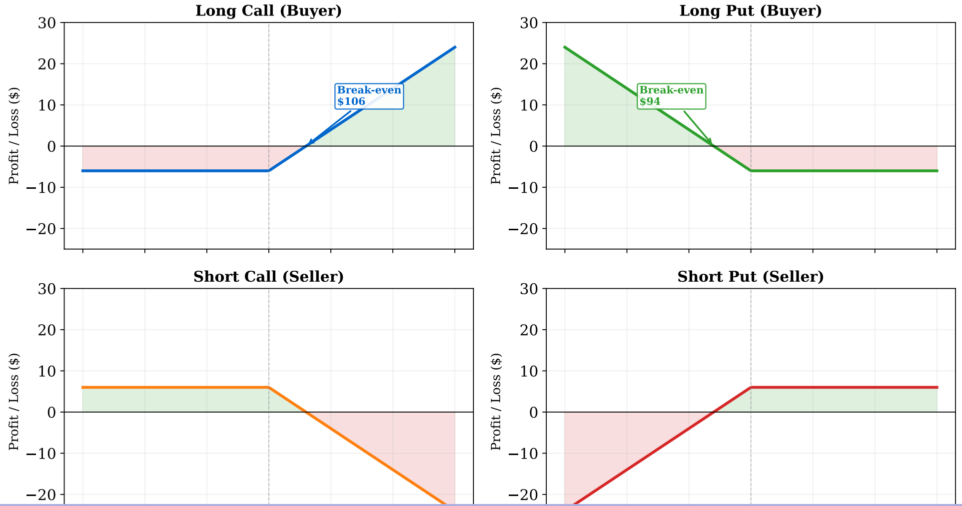
Insurance Concept	**Option Equivalent**	**Example**
Premium	Option premium	Pay \$5 today
Coverage amount	Strike price	Protected at \$100
Claim event	Stock falls below strike	Stock drops to \$80
Payout	Intrinsic value	Receive \$100 − \$80 = \$20
No claim	Option expires worthless	Stock stays above \$100

Key insight: Like insurance, you pay a small amount (premium) now for large protection later. If nothing bad happens, you lose only the premium.

An option = insurance contract: premium now for protection later. Maximum loss is the premium paid.

Call and Put Options: Payoff Diagrams

Option Payoff Diagrams (Strike = $100, Premium = 6$)



Strike Price and Moneyness

Definition: Strike Price

The **strike price** (exercise price) is the pre-agreed price at which the option holder can buy (call) or sell (put) the underlying asset.

Moneyness — where is the stock relative to the strike?

Term	Call Option	Put Option
In-the-money (ITM)	Stock $>$ Strike	Stock $<$ Strike
At-the-money (ATM)	Stock $\approx$ Strike	Stock $\approx$ Strike
Out-of-the-money (OTM)	Stock $<$ Strike	Stock $>$ Strike

Why moneyness matters:

- ITM options are more expensive (higher intrinsic value)
- OTM options are cheap but unlikely to pay off — like cheap insurance with a high deductible
- ATM options have the most time value and the highest Vega (sensitivity to volatility)

Moneyness tells you how likely the option is to pay off at expiration, which drives its price.

No-Arbitrage: The Fundamental Pricing Principle

No-Arbitrage Principle

In an efficient market, it is impossible to earn a **risk-free profit** with zero investment. If two portfolios have the same future payoffs, they must have the same price **today**. Otherwise, traders would arbitrage the difference away instantly.

Put–Call Parity (with continuous dividend yield q):

$$C - P = S_0 e^{-qT} - Ke^{-rT}$$

For a non-dividend-paying stock ($q = 0$) this reduces to $C - P = S_0 - Ke^{-rT}$. **In plain English:**

- Owning a call and selling a put (same strike, same expiry) produces the *same payoff* as owning the stock (net of dividends) and borrowing the present value of the strike
- If these two portfolios had different prices, you could buy the cheap one and sell the expensive one for a guaranteed profit
- **This cannot persist** — arbitrageurs eliminate the gap

No-arbitrage is the bedrock of derivative pricing: same future payoff $\Rightarrow$ same price today.

BSM Model: What It Is

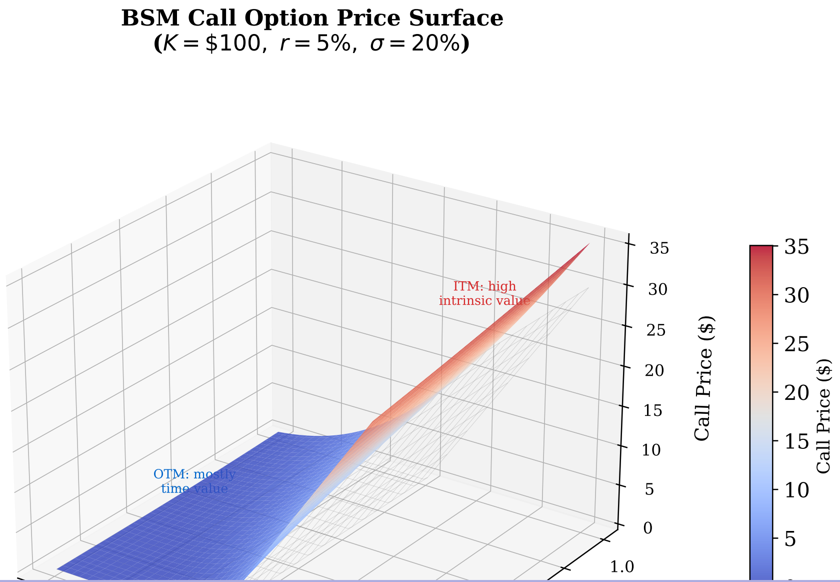
The **Black–Scholes–Merton (BSM)** model is a **calculator** that takes 5 inputs and outputs a fair option price. Think of it as a pricing formula, like a mortgage calculator — you do not need to derive the formula to use it.

The 5 inputs:

#	Input	Meaning
1	S — Stock price	Where the stock is now
2	K — Strike price	Your "insurance deductible"
3	T — Time to expiry	How long the insurance lasts
4	r — Risk-free rate	Time value of money
5	σ — Volatility	How wildly the stock swings

What BSM is NOT: It is not a prediction of where the stock will go. It is a *fair price* assuming volatility stays constant. The real world is messier.

BSM = calculator with 5 inputs. You do not need to derive it — you need to understand what each input means.


BSM Call Option Price Surface
($K = \$100,\ r = 5\%,\ \sigma = 20\%$)
ITM: high
intrinsic value
OTM: mostly
time value
Call Price ($)
35
30
25
20
15
10
5
0
1.0
Call Price ($)
35
30
25
20
15
10
5

The Greeks: Sensitivity Dashboard

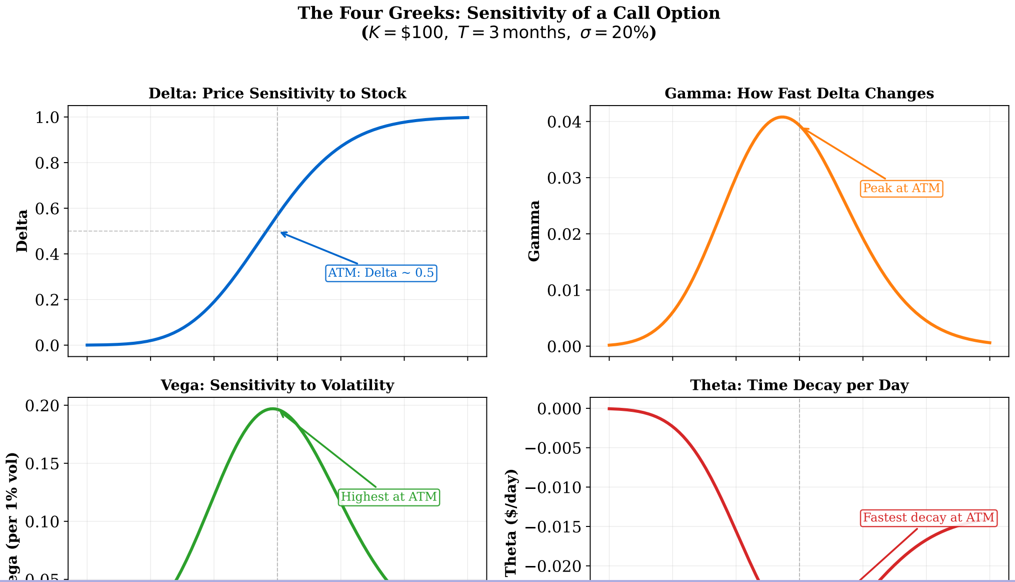
What Are the Greeks?

The **Greeks** are numbers that tell you how much the option price changes when one input changes by a small amount. Think of them as the **speedometer, fuel gauge, and temperature gauge** of your option position.

Greek	What It Measures	Plain English
Delta (Δ)	Price change per \$1 stock move	"How many shares does my option act like?"
Gamma (Γ)	How fast Delta changes	"Is my Delta accelerating?"
Vega ($\mathcal{V}$)	Price change per 1% vol move	"How sensitive to fear/uncertainty?"
Theta (Θ)	Price change per day passing	"How much do I lose each day?"

Key point: You do not need calculus to use the Greeks. They are **read from a table** or **computed by software**. Your job is to *interpret* them.

The Greeks are sensitivities: Delta = stock exposure, Gamma = convexity, Vega = volatility exposure, Theta = time decay.


The Four Greeks: Sensitivity of a Call Option
($K = \$100$, $T = 3$ months, $\sigma = 20\%$)
Delta: Price Sensitivity to Stock
Delta
1.0
0.8
0.6
0.4
0.2
0.0
ATM: Delta ~ 0.5
Gamma: How Fast Delta Changes
Gamma
0.04
0.03
0.02
0.01
0.00
Peak at ATM
Vega: Sensitivity to Volatility
0.20
0.15
0.10
Highest at ATM
Theta: Time Decay per Day
Theta ($/day)
0.000
−0.005
−0.010
−0.015
−0.020
Fastest decay at ATM

Delta: How Much Does My Option Move?

Delta (Δ) tells you how much the option price changes when the stock moves \$1:

Delta Value	Interpretation
$\Delta = 0.50$	Option moves \$0.50 for every \$1 stock move (ATM call)
$\Delta = 0.90$	Option moves almost 1-for-1 with the stock (deep ITM call)
$\Delta = 0.10$	Option barely moves with the stock (deep OTM call)
$\Delta = -0.50$	Put option moves \$0.50 *opposite* to stock

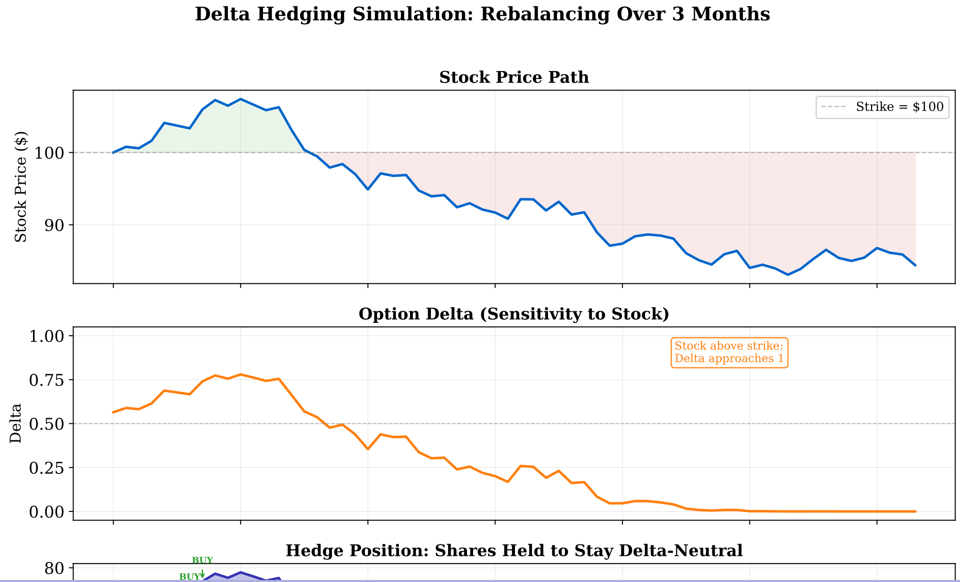
Delta as a hedge ratio:

- If you own 100 shares and buy a put with $\Delta = -0.50$, you need **2 puts** to hedge
- Because each put offsets \$0.50 of stock movement, so 2 puts offset \$1.00
- This is the essence of **Delta hedging**

Shortcut: Delta $\approx$ probability the option expires in-the-money (roughly).

Delta is the most important Greek: it tells you your effective stock exposure at any moment.

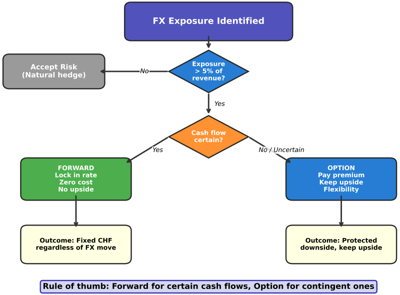
Delta Hedging in Practice



FX Hedging: Should a Company Hedge Currency Risk?

The decision framework:

1. **Identify exposure:** Does the company have revenue or costs in a foreign currency?
2. **Quantify risk:** How much could the exchange rate move? (Use VaR from Lesson 4.1)
3. **Choose instrument:**
   - Forward → lock in rate (no upside)
   - Option → pay premium, keep upside
4. **Evaluate cost vs. benefit:** Is the premium worth the protection?



Rule of thumb:

- Hedge if the exposure is **material** (¿5% of revenue)
- Use forwards for **certain** cash flows
- Use options for **uncertain** cash flows (e.g., bids)

The choice between forward and option depends on whether the cash flow is certain or contingent.

The Yield Curve: Why Shape Matters

Definition: Yield Curve

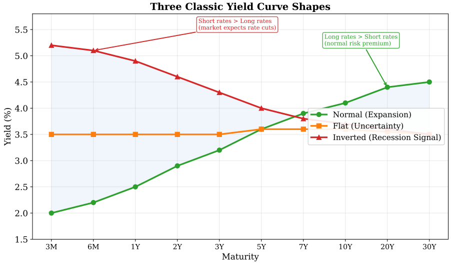
The **yield curve** plots interest rates (yields) of bonds with the same credit quality but different maturities. It shows the **term structure of interest rates**.

Three classic shapes:

- **Normal (upward):** Long rates > short rates — economy growing
- **Flat:** All rates similar — uncertainty, transition
- **Inverted:** Short rates > long rates — recession signal

Why it matters for risk:

- Bond prices are driven by interest rates
- The yield curve tells you *which* rates affect *which* bonds



An inverted yield curve has preceded every US recession in the last 50 years — it is the most watched signal in finance.

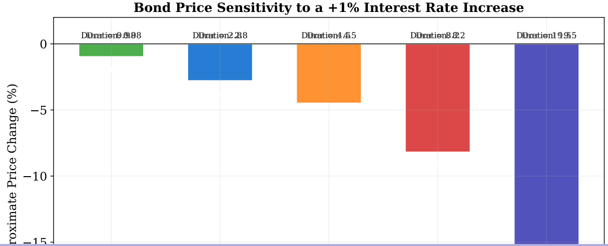
Duration: How Sensitive Is My Bond?

Definition: Duration

Duration measures how much a bond's price changes when interest rates change by 1%. A bond with duration 5 loses approximately 5% of its value if rates rise by 1%.

Intuition: Duration is like "interest rate Delta."

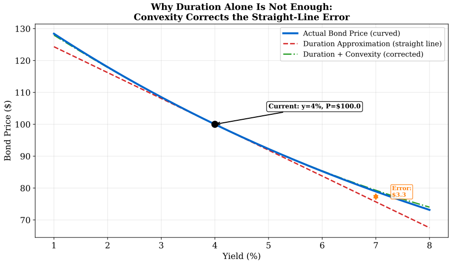
Bond Type	Duration	Interest Rate Sensitivity
1-year Treasury bill	≈ 1 year	Low — barely moves
10-year Treasury note	≈ 8 years	Moderate
30-year Treasury bond	≈ 20 years	High — moves a lot



Convexity: Why Duration Is Not Enough

Definition: Convexity

Convexity measures how much the duration itself changes as rates move. It captures the **curvature** of the price–yield relationship — the part that a simple duration estimate misses.



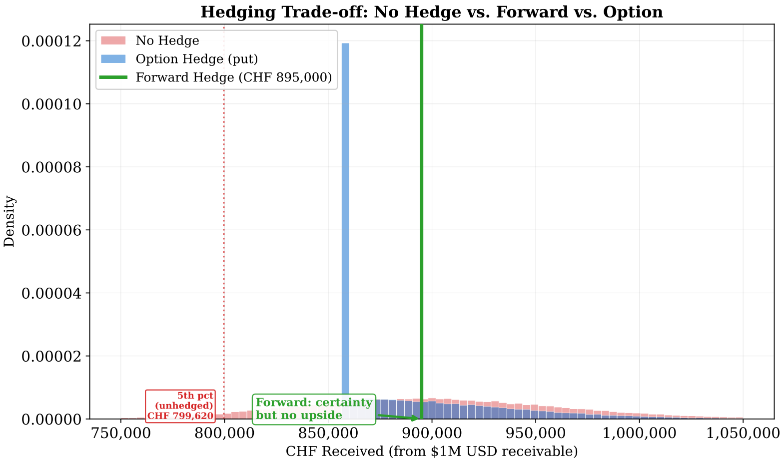
Why convexity matters:

- Duration gives a **straight-line approximation**
- For small rate changes ($<$0.5%), duration is enough
- For large rate changes ($>$1%), the straight line is wrong
- Convexity **corrects** the estimate

Analogy: Duration is like Delta (first-order). Convexity is like Gamma (second-order). Same idea, different market.

Convexity is the Gamma of the bond world: it measures how much your sensitivity changes as rates move.

Hedging in Practice: Cost vs. Benefit



The hedging paradox:

- **Perfect hedge:** Eliminates all risk, but also eliminates all upside and costs money
- **Partial hedge:** Reduces worst-case losses, preserves some upside

Real-World Example: Airline Fuel Hedging

Scenario: An airline expects to burn 10 million gallons of jet fuel next year.

Strategy	**Cost Today**	**If Fuel +30%**	**If Fuel −20%**
No hedge	$0	+$9M cost increase	$6M savings
Forward (lock $3/gal)	$0	Protected	Miss savings
Call option ($3 strike)	$2M premium	Protected above $3	Keep savings

Trade-offs:

- The forward is "free" but **locks in the price** — no benefit if fuel drops
- The option costs $2M but lets the airline **benefit from falling prices**
- This is the classic forward vs. option choice: **certainty vs. flexibility**

Real example: Several airlines that aggressively hedged fuel before 2020 were locked into paying high prices when demand (and fuel costs) collapsed during the pandemic.

Hedging protects against adverse moves, but real-world timing matters — even good hedges can look bad in hindsight.

Risk-Neutral Pricing: The Core Intuition

How does BSM actually calculate the price?

The key idea behind **risk-neutral pricing**:

1. Imagine a world where **everyone is risk-neutral** (nobody demands extra return for taking risk)
2. In this world, every asset grows at the **risk-free rate**
3. Calculate the **expected payoff** of the option in this imaginary world
4. **Discount it back** to today at the risk-free rate
5. That is the option's fair price *in the real world too*

Why does this work?

- Because you can **replicate** the option payoff by continuously trading the stock
- The replication cost is the same regardless of risk preferences
- This is the **no-arbitrage argument**: replication cost = option price

You do not need to believe this world is real. It is a mathematical trick that gives the right answer because of no-arbitrage.

Risk-neutral pricing is the engine inside BSM: pretend everyone is risk-neutral, compute expected payoff, discount.

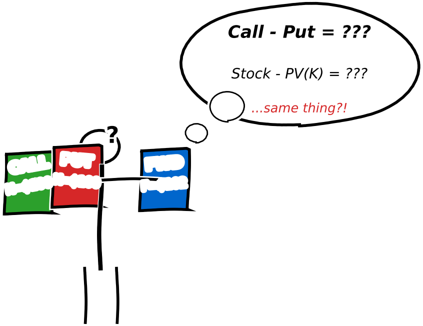
Derivatives and FinTech: What Is Changing?

Innovation	**Traditional**	**FinTech / DeFi**
Options trading	Broker + exchange	App-based (Robinhood, eToro)
Clearing	Central counterparty	Smart contract escrow
Hedging tools	Bank-designed, OTC	Parameterized DeFi protocols
Greeks computation	Bloomberg terminal	Open-source Python libraries
Pricing data	Expensive data feeds	On-chain oracles (Chainlink)

Key trends:

- **Democratization:** Retail investors now trade options on mobile apps
- **DeFi derivatives:** On-chain options (Opyn, Lyra) use smart contracts as the clearinghouse
- **Risk:** Easier access $\neq$ better understanding — retail losses on options have surged

Technology makes derivatives more accessible, but the underlying risk is unchanged — complexity is not reduced.



Put-call parity: Where you realize finance is just algebra with payoffs.

Sometimes the best way to remember a concept is to laugh about it.

Key Takeaways

1. **Derivatives** are contracts whose value derives from an underlying asset — the 4 building blocks are forwards, futures, swaps, and options
2. **Options = insurance:** Pay a premium now, receive protection later. Max loss = premium
3. **BSM** is a **calculator** with 5 inputs (S, K, T, r, σ) — you use it, you do not need to derive it
4. **The Greeks** are sensitivities: Delta (stock exposure), Gamma (curvature), Vega (volatility), Theta (time decay)
5. **Delta hedging** neutralizes stock exposure but requires continuous rebalancing
6. **Duration** and **convexity** are the bond world's equivalent of Delta and Gamma
7. **No-arbitrage** is the bedrock: same payoff $\Rightarrow$ same price today
8. Hedging is **not free** — it trades potential upside for certainty

Derivatives are tools for transferring risk. Understanding the trade-offs is more important than the math.

Summary / Next Lesson Preview

This lesson: We learned how derivatives let you **transfer risk** to someone willing to bear it — forwards lock prices, options provide insurance, swaps exchange cash flows.

Key vocabulary:

- Forward, future, swap, option
- Strike price, moneyness
- Call, put, premium
- Black–Scholes–Merton
- Delta, Gamma, Vega, Theta
- No-arbitrage, put–call parity
- Duration, convexity
- Yield curve

Next lesson (M4L3): *Institutional Risk Management* — How banks aggregate thousands of individual risks into a single capital requirement. We will cover Basel III/IV, Risk-Weighted Assets, stress testing, and why bank capital regulation exists.

Review: Can you name the 5 BSM inputs and explain what each Greek measures in one sentence?

Self-Check: Derivatives & Options

Attempt these before turning the page.

1. [Understand] State the five Black-Scholes inputs and name the Greek that measures sensitivity to each.
2. [Apply] Call option: $S = \$100$, $K = \$105$, $T = 0.25$ yr, $r = 5\%$, $\sigma = 20\%$. The option premium is \$2.00. If S jumps to \$102 instantaneously (no time passes), what is the *approximate* new premium using Delta $= 0.45$ and Gamma $= 0.035$?
3. [Analyze] "Higher volatility is bad for the writer of the option, good for the buyer." True or false? Explain with Vega.

Solutions hidden unless `\solutionstrue` **is set before compiling.**