In-Class Activity: TBM1 — Liquidation Math — SOLUTIONS

Digital Finance – BSc Course — Prof. Dr. Joerg Osterrieder

Trustless Business Models | 15–18 min | Groups of 3–4

Context: In DeFi lending (Aave, Compound), overcollateralization replaces credit scoring. A borrower posts more collateral than they borrow; smart contracts auto-liquidate if the collateral ratio falls below a threshold. **Starting position:** 5 ETH at \$3,200 collateral; borrowed \$8,000 USDC; liquidation at ratio = 125 %; liquidation penalty 8 %; gas per tx \$18.

Exercise 1 (completed): Ratio = (5 × price) / 8,000.

Scenario	ETH Price	Collateral Value	Ratio (%)	SAFE / LIQUIDATED
A	\$3,200	\$16,000	200.0 %	**SAFE**
B	\$2,600	\$13,000	162.5 %	**SAFE**
C	\$2,200	\$11,000	137.5 %	**SAFE (close)**
D	\$1,900	\$9,500	118.8 %	**LIQUIDATED**
E	\$1,600	\$8,000	100.0 %	**LIQUIDATED**

Exercise 2 (completed): At scenario C (ETH = \$2,200), applying an extra −15 % shock gives ETH = \$1,870.

Option	Action	Ratio @ \$2,200	Ratio @ \$1,870
(i)	Do nothing	137.5 %	116.9 % **LIQ**
(ii)	Add 0.5 ETH (5.5 ETH total)	151.3 %	128.6 % **safe (barely)**
(iii)	Repay \$2,000 (debt \$6,000)	183.3 %	155.8 % **SAFE**

Winner: **(iii) Repay \$2,000 USDC**. Reducing debt is more capital-efficient than adding a small amount of collateral when the collateral asset itself is the volatile side.

Exercise 3 (completed): Stated APR = 5.0 %. Monthly borrow interest on \$8,000 = 8000 × 0.05 / 12 ≈ \$33.33. Gas = 4 × \$18 = \$72. No-liquidation 30-day cost = \$33.33 + \$72 = \$105.33 on \$8,000 for 30 days. Annualised effective = (105.33 / 8000) × 12 ≈ **15.8 %**. With a liquidation event the 8 % penalty adds \$640, so 30-day cost = \$745.33 and annualised effective ≈ **111.8 %**.

Debrief: Stated 5 % vs realised 15.8 % (no liquidation) already doubles or triples a typical bank rate. A single liquidation pushes it past 100 %. DeFi is competitive only for short-duration, large-size, on-chain-native use cases where the alternative is *no* credit access at all.

Answer Key **E1:** A 200%, B 162.5%, C 137.5%, D 118.8% **LIQ**, E 100% **LIQ**. **E2:** After another −15 %: (i) 116.9% LIQ, (ii) 128.6%, (iii) 155.8%. Winner **(iii) repay debt**. **E3:** No-liq 30-day cost \$105.33 → effective APR ≈ **15.8 %**. With liq \$745.33 → ≈ **111.8 %**. Core lesson: gas + penalty risk make DeFi lending **much more expensive than the headline APR**.