Day 2: DeFi Protocol Design

The Engineering of Automated Finance

Day 2 of 5

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Recap Day 1 & Today's Roadmap

Day 1 Recap: Pricing

- Log-normal model fails for crypto
- GARCH: time-varying vol, MLE estimation
- Merton: jumps via Poisson process
- Implied vol smile reveals model error

Day 2 Roadmap

1. Uniswap V2 recap: $x \cdot y = k$
2. **V3 concentrated liquidity** (derivation)
3. **Impermanent loss** (derivation)
4. Lending protocol economics
5. Composability and risks

Today: from pricing to protocol design. How do DeFi protocols automate financial functions (exchange, lending) using mathematical rules instead of intermediaries?

Hands-On

Build a V3 LP simulator in Python: compare range widths, fees, and IL.

$1.8 Billion in 24 Hours: The Uniswap V3 Launch

May 5, 2021: Uniswap V3 launched on Ethereum mainnet.

- $1.8B total value locked within 24 hours
- Promised **4,000**× capital efficiency over V2
- Introduced **concentrated liquidity**: LPs choose a price range
- Immediately became #1 DEX by volume

Key innovation:

- V2: liquidity spread uniformly over $(0, \infty)$
- V3: liquidity concentrated in $[p_a, p_b]$
- Same capital, dramatically more depth at the current price

V3 by the numbers (2021–2025):

Metric	Value
Cumulative volume	$> \$2T$
Peak daily volume	$10B
Deployed chains	15+
LP positions (NFTs)	5M+
Fee tiers	4

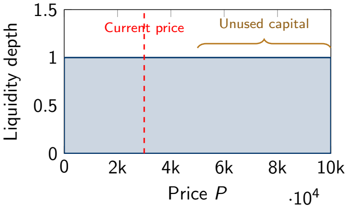
The catch

Higher returns but also **higher risk**: impermanent loss is amplified when price leaves your range.

V2's Capital Efficiency Problem

Uniswap V2: $x \cdot y = k$ [4]

- Liquidity is spread **uniformly** across all prices from 0 to ∞
- If ETH trades at \$3,000, liquidity exists at \$1, \$10, \$100,000
- 99% of capital sits at prices that will **never be reached**



Numerical example:

- LP deposits \$100,000 in ETH/USDC pool
- ETH price: \$3,000
- V2 spreads this across all prices
- Only ~\$500 is "active" near \$3,000
- **Capital utilization: ~0.5%**

V2 recap — worked example:

- Pool: $x = 10$ ETH, $y = 30{,}000$ USDC
- $k = 10 \times 30{,}000 = 300{,}000$
- Price $P = y/x = 3{,}000$ USDC/ETH
- Trader buys 1 ETH: new $x = 9$, $y = 300{,}000/9 = 33{,}333$

V3 Solution: Choose Your Price Range

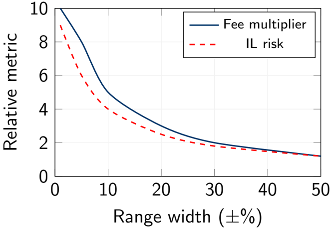
V3 idea: LPs concentrate liquidity in a chosen range $[p_a, p_b]$.

- All your capital is deployed **only** where it matters
- If price is in your range: you earn fees
- If price leaves your range: you earn nothing (but IL continues)

Capital efficiency gain:

- V2: spread over $(0, \infty)$
- V3 in $\pm 5\%$: liquidity concentrated in 10% of price space
- Effective depth: $\sim$4,000$\times$ more for the same capital

Tradeoff visualized:



Key insight

Narrow range = higher return AND higher risk. The LP must actively manage their position.

The Risk Side: Amplified Impermanent Loss

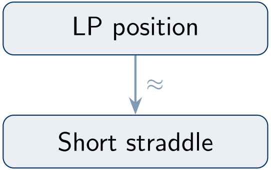
V2 impermanent loss:

- Price doubles ($r = 2$): IL $= -5.7\%$
- Annoying but manageable
- Fees often compensate

V3 concentrated IL:

- Same price move, $\pm 5\%$ range
- IL can be -50% or worse
- If price leaves range entirely: position becomes 100% of the losing token

Analogy from traditional finance:



Positive θ: earn fees
Negative γ: lose on big moves
Breakeven = fees > IL

Today: derive exactly how much IL you face, and when fees compensate.

Today's Structure

1. V3 Concentrated Liquidity
2. Impermanent Loss Derivation
3. Lending Protocol Economics
4. Composability and Risks
5. Hands-On: Uniswap V3 LP Simulator

Math level: we will *derive* the V3 liquidity formula and IL formula step by step. You should be able to reproduce these on paper.

V2 Review: The Constant Product Formula [ ]

Uniswap V2 invariant

$$x \cdot y = k$$

where x = reserve of token A, y = reserve of token B, k = constant.

Key properties:

- Marginal price: $P = \frac{y}{x} = \frac{dy}{dx}$ (slope of the hyperbola at current point)
- Adding liquidity: increase k while keeping price constant
- Swap: move along the curve $x \cdot y = k$
- Price impact: larger trades $\Rightarrow$ more slippage (curved, not linear)

Define liquidity: $L = \sqrt{k} = \sqrt{x \cdot y}$

This L is the key quantity. Note: $x = L/\sqrt{P}$ and $y = L \cdot \sqrt{P}$.

Verification

$x \cdot y = \frac{L}{\sqrt{P}} \cdot L\sqrt{P} = L^2 = k$. ✓

V3 Key Insight: Provide Liquidity Only in $[p_a, p_b]$

Uniswap V3: instead of providing liquidity across $(0, \infty)$, an LP chooses bounds $[p_a, p_b]$.

How it works:

- The LP's capital acts *as if* it were a V2 position, but only within $[p_a, p_b]$
- Outside the range: the position is dormant (100% in one token)
- This is implemented via "virtual reserves" — the protocol pretends the LP has more tokens than they actually deposited

The trick: virtual reserves.

- V2 formula $x \cdot y = k$ works with reserves from 0 to ∞
- V3 shifts the curve so the LP's real tokens cover exactly $[p_a, p_b]$
- The "virtual" reserves represent tokens the LP doesn't actually have

V3 modified invariant

$$(x + x_{\text{virtual}}) \cdot (y + y_{\text{virtual}}) = L^2$$

The real reserves x, y deplete to zero at the range boundaries.

Virtual Reserves and Liquidity Depth

Rewrite in terms of $\sqrt{P}$:

Since $x = L/\sqrt{P}$ and $y = L\sqrt{P}$ in V2, the V3 position with range $[p_a, p_b]$ uses:

$$x_{\text{real}} = L\left(\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{p_b}}\right), \qquad y_{\text{real}} = L(\sqrt{P} - \sqrt{p_a})$$

Boundary conditions:

- At $P = p_b$: $x_{\text{real}} = 0$ (all converted to token B)
- At $P = p_a$: $y_{\text{real}} = 0$ (all converted to token A)
- At $P = p_a = 0, p_b = \infty$: reduces to V2

Liquidity L from reserves

Given a deposit of Δx token A and Δy token B at current price $P \in [p_a, p_b]$:

$$L = \min\left(\frac{\Delta x}{\frac{1}{\sqrt{P}} - \frac{1}{\sqrt{p_b}}}, \frac{\Delta y}{\sqrt{P} - \sqrt{p_a}}\right)$$

Derivation: $L = \frac{\Delta y}{\sqrt{p_b} - \sqrt{p_a}}$ [ ]

Goal: how much token B (Δy) is needed to provide liquidity L in range $[p_a, p_b]$?

Step 1: In V2, the token B reserve at price P is $y = L\sqrt{P}$.
Step 2: For a V3 position, the LP only provides the *difference* in y between the range boundaries:

$$\Delta y = y(p_b) - y(p_a) = L\sqrt{p_b} - L\sqrt{p_a} = L\left(\sqrt{p_b} - \sqrt{p_a}\right)$$

Step 3: Solve for L:

$$\boxed{L = \frac{\Delta y}{\sqrt{p_b} - \sqrt{p_a}}}$$

Analogously for token A:

$$\Delta x = \frac{L}{\sqrt{p_a}} - \frac{L}{\sqrt{p_b}} = L\left(\frac{1}{\sqrt{p_a}} - \frac{1}{\sqrt{p_b}}\right) \implies L = \frac{\Delta x}{\frac{1}{\sqrt{p_a}} - \frac{1}{\sqrt{p_b}}}$$

Intuition

L measures how much *depth* the LP provides at each price tick within the range. Higher L

What Liquidity L Actually Means

L is the depth of the order book at each price.

- For a small price change $\Delta\sqrt{P}$:

$$\Delta y = L \cdot \Delta\sqrt{P}$$

- Higher $L \Rightarrow$ less price impact per dollar traded
- L is constant within a tick range
- Total L at a price $=$ sum of all LP positions covering that price

Units: L has units of $\sqrt{\text{token A} \cdot \text{token B}}$.

Comparison:

	V2	**V3** ($\pm 5\%$)
Capital	\$100k	\$100k
Range	$(0, \infty)$	$[2850, 3150]$
L at price	L_0	$\sim 40 \cdot L_0$
Eff. multiplier	$1\times$	$\sim$4,000$\times$

Why 4,000×?

- V2 spreads L over infinite range
- V3 concentrates same L in 10% band
- Effective $L_{V3} \approx L_{V2} \cdot \frac{1}{\sqrt{p_b}/\sqrt{p_a} - 1}$

Capital Efficiency: V3 in $\pm 5\%$ Range $\approx$ 4,000$\times$ V2

Setup: ETH/USDC pool, current price $P = 3{,}000$. LP provides \$10,000.

V2 position:

- Deposit: 1.667 ETH (\$5,000) + 5,000 USDC
- $k = 1.667 \times 5{,}000 = 8{,}333$
- $L = \sqrt{k} = 91.3$
- This L is spread over $(0, \infty)$

V3 position ($\pm 5\%$, range $[2{,}850, 3{,}150]$):

- $\sqrt{p_a} = \sqrt{2850} = 53.39$, $\sqrt{p_b} = \sqrt{3150} = 56.12$
- $\Delta\sqrt{P} = 56.12 - 53.39 = 2.73$
- $L = \Delta y / \Delta\sqrt{P}$; with same \$5,000 in USDC: $L = 5000/2.73 = 1{,}832$
- **Effective L:** $1{,}832/91.3 \approx 20\times$ **per dollar**
- But we also need less token A, so total capital efficiency: $\approx$ **4,000**$\times$

Interpretation

A \$10,000 V3 position in $\pm 5\%$ provides the same depth as a \$40M V2 position.

Checkpoint

Quick Question

If you narrow your LP range from $\pm 10\%$ to $\pm 5\%$, does your L increase or decrease for the same deposit amount?

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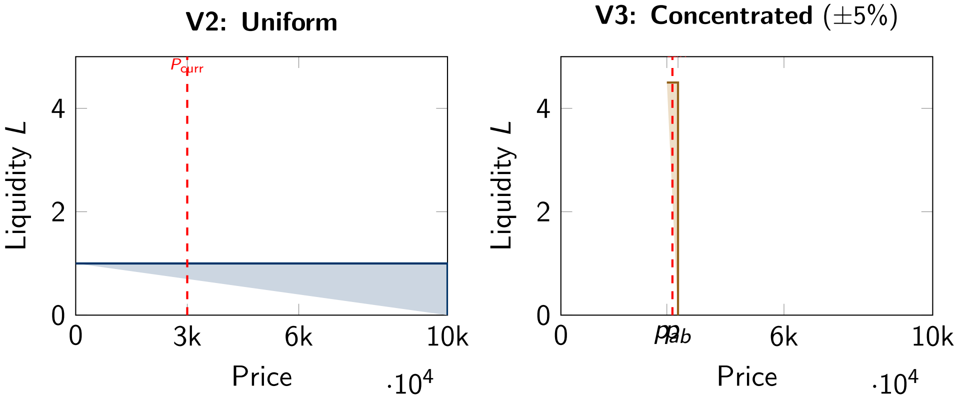
Answer

L **increases**. Since $L = \Delta y / (\sqrt{p_b} - \sqrt{p_a})$, a narrower range means a smaller denominator, so the same capital produces higher liquidity depth. This is the core of V3 capital efficiency—but it also amplifies impermanent loss if price leaves the range.

V2 vs. V3 Liquidity Distribution



Same capital, dramatically different depth. V3 LPs earn more fees per dollar but must actively manage their range as prices move.

Fee Mechanics in V3

Fee tiers: V3 offers four fee levels for different pair types.

Fee tier	Rate	Typical use
0.01%	$\gamma = 0.9999$	Stablecoin pairs (USDC/USDT)
0.05%	$\gamma = 0.9995$	Correlated pairs (WETH/stETH)
0.30%	$\gamma = 0.9970$	Standard pairs (ETH/USDC)
1.00%	$\gamma = 0.9900$	Exotic / low-liquidity pairs

How fees work:

- Trader submits swap of Δ tokens
- Protocol takes fee: effective input $= \gamma \cdot \Delta$ where $\gamma = 1 - \text{fee}$
- Fee is distributed pro-rata to in-range LPs based on their L
- LP's fee share: fee $\times \frac{L_{\text{LP}}}{\sum L_{\text{in-range}}}$

Concentrated LP advantage

An LP with 10$\times$ higher L (from concentration) earns 10$\times$ more fees per dollar of capital.

Tick Math: Discretizing the Price Space

V3 uses discrete "ticks" to represent prices:

$$P(i) = 1.0001^{i}$$

Each tick i represents a price. The tick spacing depends on the fee tier.

Fee tier	Tick spacing	Price increment
0.01%	1	0.01%
0.05%	10	0.10%
0.30%	60	0.60%
1.00%	200	2.02%

Why ticks?

- Gas efficiency: each LP position snaps to tick boundaries
- $\sqrt{P(i)} = 1.0001^{i/2}$: precomputed, no square roots on-chain
- Total active liquidity changes only at tick boundaries
- Within a tick: constant L, simple swap math

Impermanent Loss: Setup

Setup: at time $t = 0$, LP deposits into a V2 pool:

- x_0 units of token A (e.g., ETH)
- y_0 units of token B (e.g., USDC)
- Initial price: $P_0 = y_0 / x_0$
- Pool constant: $k = x_0 \cdot y_0$

At time t: price changes to $P = r \cdot P_0$ where $r = P/P_0$ is the price ratio.

Two strategies to compare:

1. **LP:** keep tokens in the AMM pool (rebalanced by arbitrageurs)
2. **HODL:** hold the initial amounts x_0, y_0 in a wallet

Question: how does V_{LP} compare to V_{HODL} as P changes?

Impermanent loss:

$$\text{IL}(r) = \frac{V_{\text{LP}}}{V_{\text{HODL}}} - 1$$

(Always ≤ 0: the LP always underperforms HODL, ignoring fees.)

V2 Impermanent Loss: Derivation

Step 1: LP portfolio value at price P. From $x \cdot y = k$ and $P = y/x$:

$$x = \sqrt{k/P}, \qquad y = \sqrt{kP}$$

$$V_{\text{LP}} = x \cdot P + y = \sqrt{k/P} \cdot P + \sqrt{kP} = 2\sqrt{kP}$$

Step 2: HODL portfolio value:

$$V_{\text{HODL}} = x_0 \cdot P + y_0 = x_0 P + x_0 P_0 = x_0(P + P_0)$$

Since $k = x_0 y_0 = x_0^2 P_0$, we have $\sqrt{k} = x_0\sqrt{P_0}$.

$$V_{\text{LP}} = 2x_0\sqrt{P_0 P}$$

Step 3: take the ratio with $r = P/P_0$:

$$\frac{V_{\text{LP}}}{V_{\text{HODL}}} = \frac{2x_0\sqrt{P_0 P}}{x_0(P + P_0)} = \frac{2\sqrt{P_0 \cdot rP_0}}{rP_0 + P_0} = \frac{2\sqrt{r}}{1 + r}$$

V2 Impermanent Loss Formula

IL Normalized: $\text{IL}(r) = \frac{2\sqrt{r}}{1+r} - 1$

Properties:

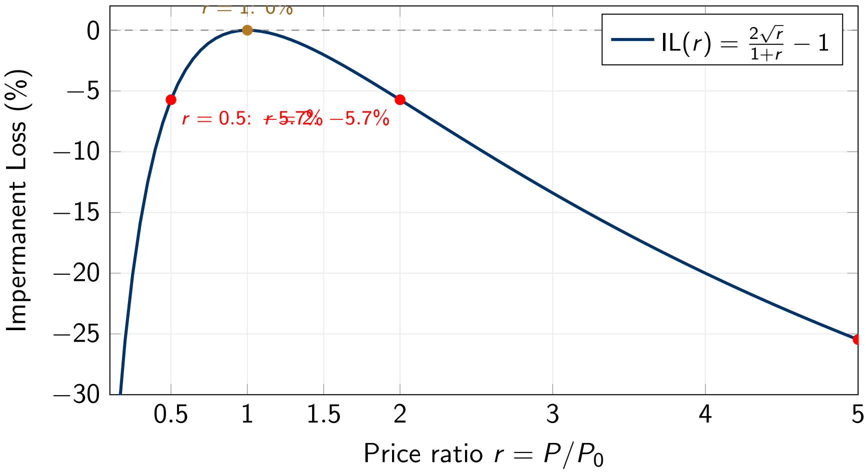
- $\text{IL}(1) = 0$: no loss if price unchanged
- $\text{IL}(r) < 0$ for all $r \neq 1$: LP always underperforms HODL
- Symmetric in log-space: $\text{IL}(r) = \text{IL}(1/r)$
- "Impermanent" because loss disappears if price returns to P_0

Key values:

$r = P/P_0$	IL	Meaning
0.50	−5.72%	Price halved
0.75	−1.03%	−25% move
1.00	0%	No change
1.50	−2.02%	+50% move
2.00	−5.72%	Price doubled
5.00	−25.46%	5× increase

Note the symmetry: $r = 0.5$ and $r = 2$ give the same IL

Impermanent Loss Curve



V3 Amplified Impermanent Loss

Concentrated liquidity amplifies IL. The narrower the range, the more IL.

Concentration leverage factor:

A V3 position in $[p_a, p_b]$ behaves like a V2 position with

$$\text{leverage} = \frac{1}{\sqrt{p_b/p_a} - 1} \cdot \frac{\sqrt{p_b/p_a}}{1}$$

Simplified for symmetric range $P_0 \cdot (1 \pm \delta)$:

- Range $\pm 50\%$: leverage $\approx 2.4\times$ V2 IL
- Range $\pm 10\%$: leverage $\approx 10\times$ V2 IL
- Range $\pm 1\%$: leverage $\approx 100\times$ V2 IL

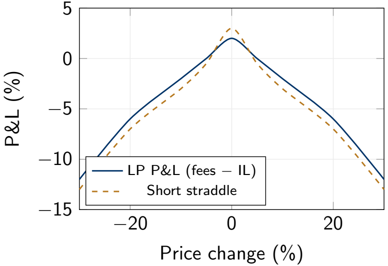
When price leaves the range:

- $P > p_b$: position is 100% token B (e.g., all USDC, no ETH)
- $P < p_a$: position is 100% token A (e.g., all ETH, no USDC)
- LP has been "adversely selected": holds the depreciating token

Key risk

LP ≈ Short Straddle: The Options Analogy

Payoff comparison:



Greeks analogy:

Greek	Option	LP
$\Theta > 0$	Time decay	Fee income
$\Gamma < 0$	Convexity loss	IL
Δ	Hedge ratio	Rebalancing
Vega	Vol exposure	IL risk

Key insight:

- LP earns "theta" (fees) in exchange for "gamma" (IL)
- More volatile markets ⇒ more IL but also more fees
- Breakeven depends on the ratio

Breakeven: When Do Fees Compensate IL?

Breakeven condition: fee APY $\geq$ expected IL loss rate.

Approximate formula: For a V2 position with daily volatility σ_d and fee rate f on volume V:

$$\text{Fee APY} \approx \frac{f \cdot V \cdot 365}{2 \cdot \text{TVL}}, \qquad \text{Expected IL rate} \approx \frac{\sigma_d^2}{2} \cdot 365$$

Breakeven when Fee APY $>$ IL rate.

Typical numbers (ETH/USDC V3 $\pm$10%):

- Fee tier: 0.30%
- Daily volume / TVL: $\sim$15%
- Fee APY: $\sim$16%
- ETH $\sigma_d \approx$ 4%, IL rate $\approx$ 29%
- **Net:** $-$13% (IL wins!)

When LPs profit:

- Low volatility periods (IL small)
- High volume / TVL ratio (fees high)
- Stablecoin pairs (tiny IL)
- Range matches price action

Cartoon placeholder: "Impermanent Loss: The Unwanted Rebalancing" — an LP character watching arbitrageurs take their

Lending Protocols: Aave and Compound [ ]

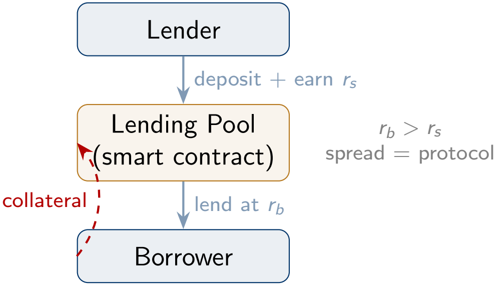
How it works:

1. **Deposit** collateral (e.g., ETH)
2. **Borrow** another asset (e.g., USDC) up to a loan-to-value limit
3. **Pay interest** on borrowed amount
4. **Earn interest** on deposited collateral

No credit checks: all loans are *over-collateralized*. Borrow \$1,000 $\Rightarrow$ must deposit $\geq$\$1,500.

Key difference from TradFi:

- No identity, no credit score
- Algorithmic interest rates
- Instant liquidation via smart contracts



By the numbers (2025):

- Aave TVL: ~\$15B
- Compound TVL: ~\$3B
- Combined annual interest: ~\$500M

Interest Rate Kink Model

Utilization rate: $U = \frac{\text{Total Borrowed}}{\text{Total Supplied}} \in [0, 1]$

Piecewise linear interest rate

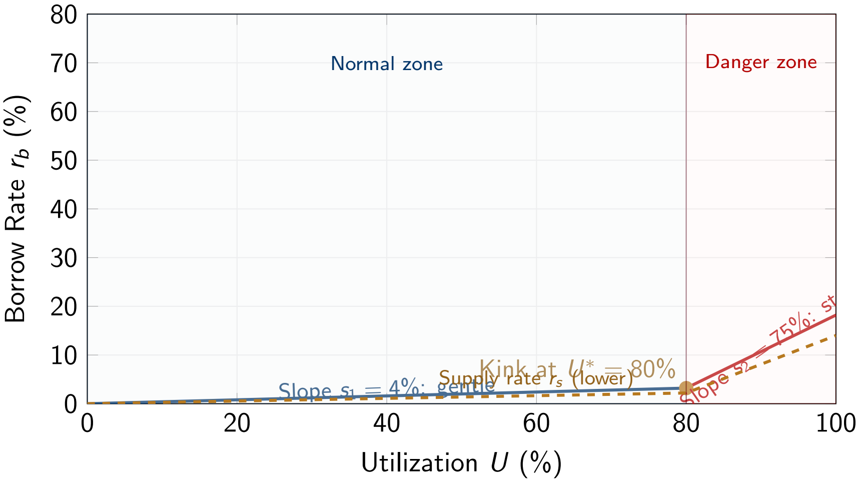
$$r_b(U) = \begin{cases} r_0 + s_1 \cdot U & \text{if } U \leq U^* \\ r_0 + s_1 \cdot U^* + s_2 \cdot (U - U^*) & \text{if } U > U^* \end{cases}$$

where $s_2 \gg s_1$ (steep slope above kink U^*).

Typical parameters (USDC on Aave):

Parameter	Value	Meaning
r_0	0%	Base rate
s_1	4%	Slope below kink
U^*	80%	Optimal utilization
s_2	75%	Slope above kink

Interest Rate Kink: Visual



Health Factor and Liquidation

Health factor

$$H = \frac{\text{Collateral Value} \times \text{LTV}_{\text{max}}}{\text{Outstanding Debt}}$$

Liquidation triggered when $H < 1$.

Example:

- Deposit 10 ETH at \$3,000 = \$30,000 collateral
- LTV_{max} for ETH on Aave: 80%
- Borrow \$20,000 USDC
- $H = \frac{30{,}000 \times 0.80}{20{,}000} = 1.20 \quad \text{(safe)}$

If ETH drops to \$2,500:

- Collateral: $10 \times 2{,}500 = \$25{,}000$
- $H = \frac{25{,}000 \times 0.80}{20{,}000} = 1.00 \quad \text{(at liquidation threshold!)}$
- Any further drop: liquidation bot repays part of debt and claims collateral at a discount (typically 5–10% bonus)

Flash Loans: Atomic Borrowing

Definition

A flash loan lets you borrow **any amount** with **zero collateral**, provided you repay within the **same transaction**.

How is this possible?

- Ethereum transactions are **atomic**: all steps succeed or all revert
- Smart contract checks at the end: was the loan repaid?
- If not $\Rightarrow$ entire transaction reverts (as if it never happened)
- No credit risk for the lender (mathematically impossible to default)

Legitimate uses:

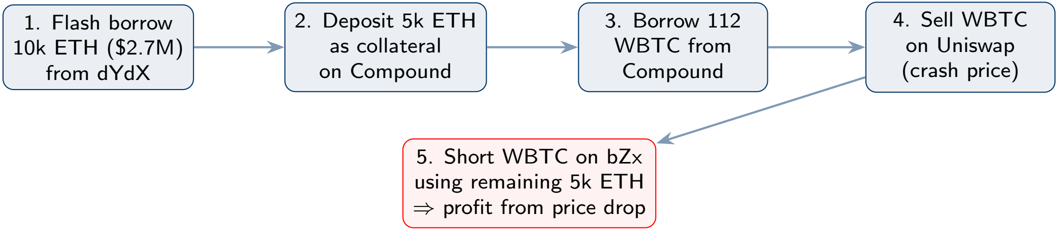
1. **Arbitrage:** borrow \$10M, trade across DEXes, repay + keep profit
2. **Collateral swap:** switch from ETH to WBTC collateral without closing position
3. **Self-liquidation:** avoid liquidation penalty by repaying yourself

Fee: Aave charges 0.09% per flash loan (flat, regardless of amount or duration).

“Money Legos”: Protocol Composability [ ]

DeFi protocols can be combined like building blocks.

Case study: bZx Flash Loan Attack (Feb 2020)



Profit: $350k (in one transaction, zero capital at risk)

The attack exploited:

- Uniswap as a price oracle (manipulable with large trades)
- Composability: 5 protocols in one atomic transaction
- Flash loans: no capital needed

DeFi Risks: A Taxonomy

Smart contract risk:

- Code bugs ⇒ loss of funds
- $3.8B stolen in 2022 alone
- Formal verification helps but is incomplete
- Audits reduce but don't eliminate risk

Oracle risk:

- Protocols rely on price feeds
- Manipulated oracles ⇒ incorrect liquidations
- Chainlink reduces this but adds centralization

Governance risk:

Composability risk ("dependency chains"):

- Protocol A depends on Protocol B's output
- If B fails, A fails too
- Example: stETH depeg affects all protocols using stETH as collateral

Economic risk:

- Impermanent loss (as derived)
- Liquidation cascades
- Bank-run dynamics on lending pools

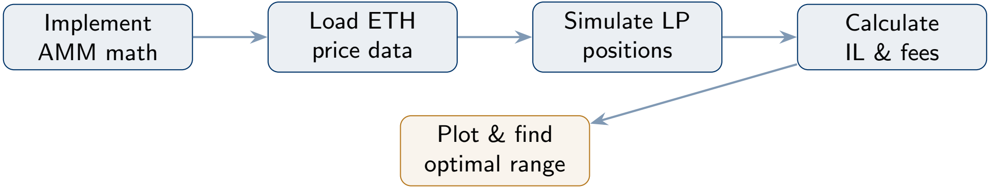
The core tension

Hands-On

Uniswap V3 LP Simulator

V2 vs. V3 · Range Width · IL vs. Fees

Exercise Overview: Compare V2 vs. V3 LP Positions



Tools

- Python 3.10+
- `numpy`: AMM math
- `pandas`: price data handling
- `matplotlib`: visualization

Learning Objectives

- Implement V2 and V3 AMM formulas
- Quantify IL for different range widths
- Understand the fee/IL tradeoff
- Find optimal LP range given volatility

Step 1: Implement V2 and V3 AMM Math

V2 functions

```
def v2_value(x0, y0, P0, P):
    """LP value under V2 at price P."""
    k = x0 * y0
    return 2 * np.sqrt(k * P)

def v2_il(r):
    """IL as a fraction, given price ratio r = P/P0."""
    return 2 * np.sqrt(r) / (1 + r) - 1
```

V3 functions

```
def v3_liquidity(dy, pa, pb):
    """Compute L from token B deposit and range."""
    return dy / (np.sqrt(pb) - np.sqrt(pa))

def v3_value(L, P, pa, pb):
    """V3 LP position value at price P."""
    P_clamped = np.clip(P, pa, pb)
    x = L * (1/np.sqrt(P_clamped) - 1/np.sqrt(pb))
```

Step 2: Load ETH/USDC Historical Prices

Code

```
import yfinance as yf
import pandas as pd

eth = yf.download('ETH-USD', start='2023-01-01', end='2025-12-31')
prices = eth['Close'].dropna()

print(f"Price range: $prices.min():.0f -- $prices.max():.0f")
print(f"Current: $prices.iloc[-1]:.0f")
print(f"Daily vol: prices.pct_change().std()*100:.1f%")
```

Expected output (2023–2025):

- Price range: ~\$1,000 to ~\$4,800
- Daily volatility: ~3–5%
- Several regime changes (bull/bear/sideways)

Key setup:

- Choose a starting date and set P_0 = price on that day
- Assume LP deposits \$10,000 (split 50/50 at P_0)

Step 3: Simulate LP Positions with Different Range Widths

Code structure

```
P0 = prices.iloc[0]
capital = 10000
ranges = [0.05, 0.10, 0.20, 1.0]   # +/- 5%, 10%, 20%, full

for delta in ranges:
    pa = P0 * (1 - delta) if delta < 1 else 0.01
    pb = P0 * (1 + delta) if delta < 1 else 1e8
    # Compute L for this range
    dy = capital / 2   # half in USDC
    L = v3_liquidity(dy, pa, pb)
    # Track daily value
    for P in prices:
        value = v3_value(L, P, pa, pb)
        hodl = capital * (P/P0 + 1) / 2
```

Compare four strategies:

Strategy	Range	Expected behavior

Step 4: Calculate IL, Fees, Net Return for Each

IL calculation

```
for each day t:
    r = prices[t] / P0
    il_v2[t] = v2_il(r)
    il_v3[t] = v3_value(L, prices[t], pa, pb) / hodl_value[t] - 1
```

Fee estimation (simplified):

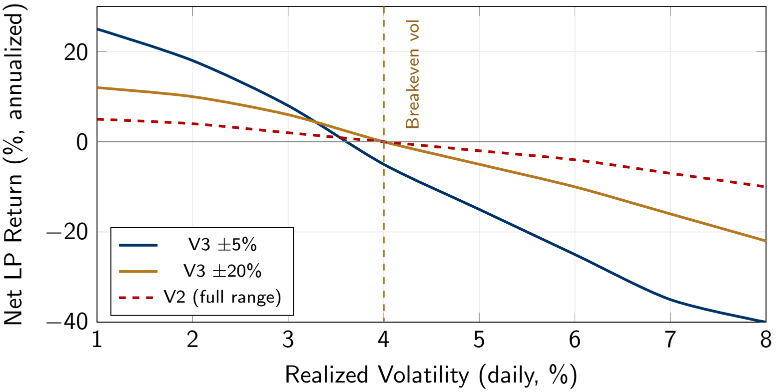
- Assume daily volume = 15% of TVL (typical for ETH/USDC)
- Fee per day = $0.003 \times 0.15 \times \text{capital} \times \frac{L_{\text{you}}}{L_{\text{total}}}$
- For simplicity, assume your share of L is proportional to concentration advantage

Fee simulation (approximate)

```
fee_rate = 0.003   # 0.30% tier
daily_volume_ratio = 0.15
concentration_mult = 1 / (2 * delta) if delta < 1 else 1
daily_fee = fee_rate * daily_volume_ratio * capital * concentration_mult / 365
```

Step 5: Plot Net LP Return vs. Price Volatility

The key tradeoff chart:



Interpretation: narrow ranges are highly profitable in low-vol environments but disastrous when volatility spikes. The breakeven volatility depends on the range width.

Step 6: Find Optimal Range Width for Given Volatility

Optimization

```
from scipy.optimize import minimize_scalar

def net_return(delta, sigma_daily, fee_rate=0.003):
    """Expected net return for range width delta."""
    fee_income = fee_rate * 0.15 * 365 / (2 * delta)
    expected_il = sigma_daily**2 / (2 * delta) * 365   # approximate
    return -(fee_income - expected_il)   # negative for minimization

result = minimize_scalar(net_return, bounds=(0.01, 1.0),
                        method='bounded', args=(0.04,))
print(f"Optimal range: +/- result.x*100:.1f%")
```

Typical results:

Daily volatility	Optimal range (±)
2% (low vol)	±8–12%
4% (normal)	±15–25%
6% (high vol)	±30–50% or full range

Discussion

Active LP management:

- Should LPs rebalance ranges when price moves?
- Rebalancing = close position + open new one
- Costs: gas fees + realized IL
- Benefit: stay in range, keep earning fees
- Many "LP management" protocols automate this (Arrakis, Gamma)

Research questions:

- Is LP-ing on V3 a positive-sum game?
- Who profits: LPs, traders, or arbitrageurs?

Flash loans: innovation or exploit?

- **Pro:** democratize access to capital, enable efficient arbitrage, reduce market inefficiencies
- **Con:** enable oracle manipulation, governance attacks, and protocol exploits
- Key insight: flash loans don't create risk—they *reveal* existing vulnerabilities

Open question

If protocols were perfectly designed, flash loans would only do useful things (arbitrage, liquidation). The attacks exploit design

Day 2: Summary and Key Takeaways

What we learned:

1. V3 concentrated liquidity: $L = \frac{\Delta y}{\sqrt{p_b} - \sqrt{p_a}}$
2. Capital efficiency: $\sim$4,000$\times$ in $\pm$5% range
3. V2 IL: $\text{IL}(r) = \frac{2\sqrt{r}}{1+r} - 1$
4. V3 amplifies IL proportional to concentration
5. LP $\approx$ short straddle (fees vs. IL)
6. Lending: kink model keeps utilization stable
7. Flash loans: atomic, zero-collateral borrowing

Key formulas:

- V2: $x \cdot y = k$, $P = y/x$
- V3: $L = \Delta y / (\sqrt{p_b} - \sqrt{p_a})$
- IL: $\frac{2\sqrt{r}}{1+r} - 1$
- Health: $H = \frac{\text{Coll} \times \text{LTV}}{\text{Debt}}$

Day 3 Preview

Blockchain Economics:

- Consensus mechanisms & game theory
- MEV: the hidden tax on DeFi users
- Fee market design (EIP-1559)

Reading: [1] (V3 whitepaper), [4] (AMM economics).

References I

[1] Hayden Adams et al. *Uniswap V3 Core*. 2021.

[2] Campbell R. Harvey, Ashwin Ramachandran, and Joey Santoro. *DeFi and the Future of Finance*. Wiley, 2021.

[3] Lioba Heimbach and Luying Huang. *DeFi Leverage*. BIS Working Paper 1171. Bank for International Settlements, 2024.

[4] Alfred Lehar and Christine A. Parlour. "Decentralized Exchange: The Uniswap Automated Market Maker". In: *Journal of Finance* 80.1 (2025), pp. 321–374.