Day 6: DeFi Derivatives

Perpetual Futures, Power Perpetuals, and Structured Products

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The Week So Far

Days 1–3

- Jump-diffusion & stochastic vol pricing
- CFMM geometry, impermanent loss
- MEV, mechanism design, fee markets

Days 4–5

- ML microstructure, deep LOB forecasting
- Network risk, EVT, copulas
- CBDC, regulation, systemic fragility

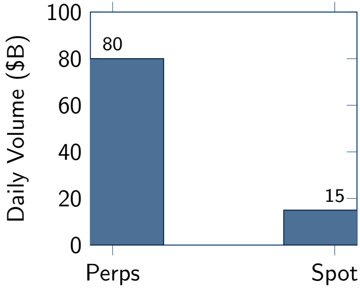
Today: The \$80B/day derivatives market that traditional finance never built.

The $80 Billion–per–Day Market You Have Never Heard Of

Key Facts

- Perpetual futures: **$80B daily volume** (vs. $15B spot)
- **No expiry, no delivery** — only a funding rate
- Invented by crypto (BitMEX, 2016; concept: Shiller, 1993)
- Dominate price discovery across all crypto assets

"Why would a derivative with no expiry and no physical delivery dominate the entire market?"



Perp volume $> 5\times$ spot across major venues (2024–2025).

Today's Roadmap

Perpetual Futures

Funding Rates · No-Arbitrage · Cost-of-Carry

What Is a Perpetual Future?

Definition 1

A **perpetual future** (perp) is a derivative contract tracking an underlying asset S with *no expiry date*. Price convergence to spot is enforced by a periodic **funding rate** $F(t)$ exchanged between long and short holders.

Key Properties

- Originated in theory: [6]
- First implementation: BitMEX (2016)
- Settlement: every 8 h (CEX) or continuous (DEX)
- Leverage: up to $125\times$ on major CEXs
- No physical delivery — purely cash-settled

Traditional Futures vs. Perps

	Trad.	**Perp**
Expiry	Yes	No
Delivery	Yes	No
Convergence	At T	Funding
Roll cost	Spread	0

No-Arbitrage Funding: Cost-of-Carry Derivation

Proposition 2 (Funding Rate from Cost-of-Carry)

Under no-arbitrage, the forward price satisfies $F_T = S_0\, e^{(r-y)T}$*, where* r *is the risk-free rate and* y *the convenience yield. For a perpetual (*$T \to 0$ *with continuous settlement):*

$$\boxed{f(t) = r - y(t)} \tag{1}$$

where $f(t)$ *is the instantaneous funding rate per unit time.*

Sketch.

1. Standard forward: $F_T = S_0 e^{(r-y)T}$. Log-basis: $\ln(F_T/S_0) = (r-y)T$.
2. A perp is the limit $T \to \Delta t$ with funding each Δt:

$$P_{\text{perp}} - S = S\left(e^{(r-y)\Delta t} - 1\right) \approx S(r-y)\Delta t.$$

3. Funding payment: $f \cdot \Delta t = (r-y)\Delta t$, hence $f = r - y$.

□

Perpetual Pricing Under Stochastic Convenience Yield

Model [ ]

Let the spot price and convenience yield follow:

$$\frac{\mathrm{d}S_t}{S_t} = \mu\,\mathrm{d}t + \sigma_S\,\mathrm{d}W_t^S, \tag{2}$$

$$\mathrm{d}y_t = \kappa(\bar{y} - y_t)\,\mathrm{d}t + \sigma_y\,\mathrm{d}W_t^y, \qquad \langle \mathrm{d}W^S, \mathrm{d}W^y \rangle = \rho\,\mathrm{d}t. \tag{3}$$

Instantaneous Funding

Under $\mathbb{Q}$, the fair funding rate is:

$$f(t) = r - y_t \tag{4}$$

Expected cumulative funding over $[0, T]$:

$$\mathbb{E}^{\mathbb{Q}}\left[\int_0^T f(t)\,\mathrm{d}t\right] = (r - \bar{y})T + \frac{y_0 - \bar{y}}{\kappa}\left(1 - e^{-\kappa T}\right)$$

Empirical Implications

- Persistent positive funding $\Rightarrow y < r$ (demand for leverage exceeds hedging)
- Mean-reversion speed κ: typical half-life 3–7 days
- Correlation $\rho < 0$: funding spikes when price rises
- Variance of funding $\propto \sigma_y^2$

Basis Dynamics: Ornstein–Uhlenbeck Model

Definition 3

The **basis** $b_t := P_{\mathrm{perp},t} - S_t$ measures the premium/discount of the perp relative to spot.

OU Model for Basis

$$\mathrm{d}b_t = -\alpha\, b_t\, \mathrm{d}t + \sigma_b\, \mathrm{d}W_t^b \tag{6}$$

where $\alpha > 0$ is the mean-reversion speed driven by funding.

Properties:

- Stationary distribution: $b \sim \mathcal{N}\left(0, \frac{\sigma_b^2}{2\alpha}\right)$
- Half-life: $t_{1/2} = \frac{\ln 2}{\alpha}$
- Empirical $\alpha \approx 0.1$–0.3 per 8 h $\Rightarrow$ half-life $\sim$2–7 settlement periods

Does Funding Eliminate the Basis?

No. Persistent premium exists because:

1. Risk premium for leverage provision
2. Demand asymmetry (net long bias)
3. Discrete settlement (not continuous)
4. Clamping truncates extreme adjustments

Funding Rate Mechanism: CEX vs. DEX

CEX Implementation (Binance)

$$F = \text{clamp}\left(\frac{P_{\text{perp}} - P_{\text{spot}}}{P_{\text{spot}}}, -c, c\right) + r_0 \Delta t \quad (7)$$

- Discrete: every 8 h
- Clamp: $c = 0.75\%$
- Base rate: $r_0 = 0.01\%$ per period
- TWAP-based price sampling

DEX Implementation (dYdX / Drift)

$$f(t) = \frac{P_{\text{TWAP}} - P_{\text{oracle}}}{P_{\text{oracle}} \cdot \tau} \quad (8)$$

- Continuous accrual (per-second)
- No clamp (protocol-specific caps)
- Oracle-based: Pyth / Chainlink
- Period τ: 1 h or 8 h normalization

Key Difference

CEX funding is *market-driven* (order book premium); DEX funding is *oracle-driven* (external price feed). This creates distinct arbitrage dynamics and manipulation surfaces.

Worked Example: Funding Rate Calculation

Setup

$P_{\text{perp}} = \$61{,}000$, $S = \$60{,}000$, $\text{Premium} = 1.67\%$, $c = 0.75\%$, $r_0 = 0.01\%$.

Step 1: Clamp premium

$$\min(1.67\%, 0.75\%) = 0.75\%$$

Step 2: Total funding rate

$$F = 0.75\% + 0.01\% = 0.76\% \text{ per } 8\,\text{h}$$

Step 3: Payment on \$100K notional long

$$\$100{,}000 \times 0.76\% = \$760 \text{ per } 8\,\text{h}$$

Step 4: Daily cost

$$3 \times \$760 = \$2{,}280/\text{day}$$

Step 5: Annualize

$$0.76\% \times 3 \times 365 = \mathbf{832.8\%}$$

Self-Correcting

Unsustainable cost forces longs to close ⇒ premium shrinks ⇒ funding normalizes within hours.

Cash-and-Carry Arbitrage: Formal Framework

Definition 4

Cash-and-carry: buy spot, short perp. Delta-neutral position that collects funding when $F > 0$.

Expected Return

$$\mathbb{E}[R] = \int_0^T F(t)\,\mathrm{d}t - c_{\mathrm{tx}} - c_{\mathrm{margin}} - \lambda_{\mathrm{exch}} \cdot L_{\mathrm{GD}} \quad (9)$$

where λ_{exch} is the exchange failure intensity and L_{GD} is the loss-given-default.

Risk decomposition:

- **Funding reversal:** $F < 0$ episodes
- **Liquidation gap:** short liquidated before spot sold
- **Exchange insolvency:** FTX-type event
- **Basis blowout:** margin calls during dislocations

Typical P&L (2024–2025)

Component	Annualized
Funding income	$+18\%$
Transaction cost	-0.5%
Margin cost	-1.0%
Exchange risk	-1.5%

Cash-and-Carry: Numerical Example

Setup

Capital: \$100K. $S = \$60{,}000$. $F = 0.03\%$ per 8 h (> 0: longs pay shorts).

1. **Establish:** Buy 0.833 BTC spot (\$49,980). Short 0.833 BTC perp (\$50,250 notional). Net $\Delta = 0$.
2. **Daily funding:** $0.833 \times \$60{,}300 \times 0.03\% \times 3 = \$45.19/\text{day}$.
3. **Annual:** $\$45.19 \times 365 = \$16{,}494 \Rightarrow$ **16.5% return**.
4. **Risk event:** BTC $+20\%$: short PnL $= -\$10\text{K}$, spot PnL $= +\$10\text{K}$. Net ≈ 0, *but* exchange may liquidate short before spot gain realized.

Sharpe Ratio

With $\sigma_{\text{funding}} \approx 8\%$:

$$\text{SR} = \frac{16.5\% - 5\%}{8\%} \approx 1.4$$

Competitive, but not risk-free.

Post-FTX Lesson

Counterparty risk is *not* negligible.
$\lambda_{\text{exch}} \approx 2\%/\text{year} \Rightarrow$ expected loss $\approx 1\%/\text{year}$ on full capital at risk.

Basis Convergence Under Stochastic Funding

Model

Basis $b_t = P_t^{\perp} - S_t$ follows:

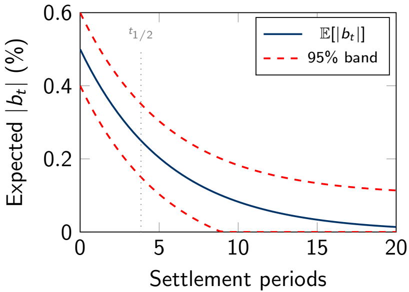
$$\mathrm{d}b_t = -\alpha b_t\,\mathrm{d}t + \sigma_b\,\mathrm{d}W_t \tag{10}$$

Stationary variance:

$$\mathrm{Var}(b_\infty) = \frac{\sigma_b^2}{2\alpha} \tag{11}$$

Calibration (BTC, 2024):

- $\alpha \approx 0.18$ per 8 h
- $\sigma_b \approx 0.0032$
- Half-life ≈ 3.9 settlement periods (≈ 31 h)



Basis decays exponentially but never reaches zero due to structural demand premium.

DEX Perp Mechanics: Virtual AMM (vAMM)

Definition 5

A **virtual AMM (vAMM)** uses a CFMM pricing formula (e.g., $x \cdot y = k$) with no real reserves. The pool is "virtual": it determines the execution price but collateral is held in a separate vault.

Perpetual Protocol v1 Model

Virtual reserves (x_v, y_v) with $x_v y_v = k$:

$$P_{\text{mark}} = \frac{y_v}{x_v} \tag{12}$$

Trade of size Δx:

$$P_{\text{exec}} = \frac{y_v}{x_v + \Delta x} \tag{13}$$

Slippage $= |P_{\text{exec}} - P_{\text{mark}}|$.

Oracle-Based Models (GMX, Hyperliquid)

- Execution at oracle price (zero slippage for small trades)
- Liquidity from GLP/HLP pool (LPs are counterparty)
- Open interest caps limit directional exposure
- Funding rate = function of long/short imbalance

Trade-off: vAMM has endogenous price discovery but higher slippage; oracle model has lower slippage but oracle dependency risk

vAMM Price Impact and Depth

Proposition 6 (Price Impact in vAMM)

For a constant-product vAMM with invariant $k = x_v y_v$, the **marginal price impact** *of a trade of size q (in base asset units) is:*

$$\Delta P(q) = P_0 \left(\frac{1}{1 + q/x_v} - 1 \right) \approx -\frac{P_0\, q}{x_v} \quad \text{for } q \ll x_v \tag{14}$$

The **depth parameter** *$\lambda = P_0/x_v$ measures Kyle-style price impact per unit trade.*

Implications for Perp DEXs

- k is a tunable parameter (protocol governance sets virtual depth)
- Higher $k \Rightarrow$ lower slippage but higher LP exposure
- Unlike real AMMs, no impermanent loss (virtual reserves)
- Risk: protocol must manage insurance fund for shortfalls

Connection to Kyle λ: $\lambda_{\text{vAMM}} = 1/x_v$ vs. $\lambda_{\text{Kyle}} = \sigma_v/(2\sigma_u)$ [5].

Power Perpetuals

Constant Gamma · Squeeth · Variance Swaps

Power Perpetuals: Definition

Definition 7 ([ ])

A **power perpetual** of order p is a derivative whose mark price tracks S^p (the p-th power of the underlying), with no expiry. Funding is paid to maintain the peg:

$$F(t) = \frac{p(p-1)}{2}\sigma^2(t)\,\Delta t \tag{15}$$

Special Cases

p	Name	Funding
1	Linear perp	0
2	Squeeth	$\sigma^2 \Delta t$
3	Cubed perp	$3\sigma^2 \Delta t$
0.5	Sqrt perp	$-\frac{1}{8}\sigma^2 \Delta t$

Key Insight

- $p > 1$: **long convexity** $\Rightarrow$ pay funding
- $p < 1$: **short convexity** $\Rightarrow$ receive funding
- $p = 1$: linear, no convexity cost
- Funding = replication cost of convexity

Derivation: Funding Rate from Black–Scholes Replication

Theorem 8 (Power Perp Replication Cost)

Under GBM $\mathrm{d}S = \mu S\,\mathrm{d}t + \sigma S\,\mathrm{d}W$, *the value* $V = S^p$ *satisfies:*

$$\mathrm{d}V = pS^{p-1}\,\mathrm{d}S + \tfrac{1}{2}p(p-1)S^{p-2}(\mathrm{d}S)^2 = pV\frac{\mathrm{d}S}{S} + \tfrac{1}{2}p(p-1)\sigma^2 V\,\mathrm{d}t \tag{16}$$

Proof.

Apply Itô's lemma to $V(S) = S^p$:

$$V_S = pS^{p-1}, \qquad V_{SS} = p(p-1)S^{p-2} \tag{17}$$

$$\mathrm{d}V = V_S\,\mathrm{d}S + \tfrac{1}{2}V_{SS}\sigma^2 S^2\,\mathrm{d}t = pS^{p-1}\,\mathrm{d}S + \tfrac{1}{2}p(p-1)\sigma^2 S^p\,\mathrm{d}t \tag{18}$$

A portfolio of p units of $S^{p-1}\cdot S = pV$ delta-hedges the first term. The residual $\frac{1}{2}p(p-1)\sigma^2 V\,\mathrm{d}t$ is the **replication cost** = funding rate per unit time. □

$$\boxed{\ \cdots\ \frac{p(p-1)}{\cdots}\ \cdots^{2}} \tag{19}$$

Constant Gamma: The Key Innovation

Proposition 9 (Constant Dollar Gamma)

For $V = S^p$, the **dollar gamma** *is:*

$$\Gamma_\$ = S^2 \cdot V_{SS} = p(p-1)\,S^p = p(p-1)\,V \tag{20}$$

This is proportional to the position value — constant as a fraction of notional.

Comparison with Vanilla Options

	Option	**Power Perp**
Gamma	Varies	Constant
Expiry	Yes	No
Roll cost	High	Zero
Greeks mgmt	Complex	Simple

For Squeeth ($p = 2$):

$$V_{SS} = 2 \tag{21}$$

$$\Gamma_\$ = 2S^2 = 2V \tag{22}$$

$$\text{“Gamma per dollar”} = \frac{\Gamma_\$}{V} = 2 \tag{23}$$

The gamma-to-value ratio is *constant and strike-independent*. This eliminates the need for strike selection and rolling.

Squeeth (S^2): Implementation

Opyn Squeeth (2022)

- Tracks ETH^2 (normalized by a scaling factor η)
- Mark price: $\eta \cdot (\text{ETH}/\text{USD})^2$
- Index price: on-chain TWAP of $(\text{ETH}/\text{USD})^2$
- Funding: continuously accrued, realized as **in-kind reduction** (long Squeeth position size shrinks over time)

Basis: Mark vs. Index

$$b_t = P_t^{\text{mark}} - P_t^{\text{index}} \tag{24}$$

- Positive basis: market prices convexity above fair value
- Negative basis: discount (rare, indicates forced selling)

In-Kind Funding

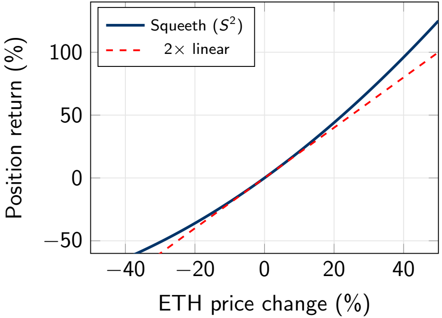
Position normalization factor η_t:

$$\eta_{t+\Delta t} = \eta_t \cdot e^{-\sigma^2 \Delta t} \tag{25}$$

After 1 year at $\sigma = 80\%$:

$$\eta_T = \eta_0 \cdot e^{-0.64} \approx 0.527\, \eta_0$$

Squeeth Payoff: Convexity in Action



Numerical Comparison

ETH move	Squeeth	2× linear
+50%	+125%	+100%
+10%	+21%	+20%
−10%	−19%	−20%
−50%	−75%	−100%

Key Property

- Upside: **more** than 2× levered
- Downside: **less** than 2× levered
- This asymmetry = **convexity**
- Cost: funding $\approx \sigma^2/\text{year}$

Power Perps and the Variance Risk Premium

Theorem 10 (Squeeth $\equiv$ Variance Swap [ ])

A long Squeeth position is economically equivalent to a **perpetual variance swap***:*

$$\textit{Squeeth funding} = \sigma^2_{\textit{implied}} \cdot \Delta t \qquad \textit{vs.} \qquad \textit{Realized P\&L contribution} = \sigma^2_{\textit{realized}} \cdot \Delta t \tag{26}$$

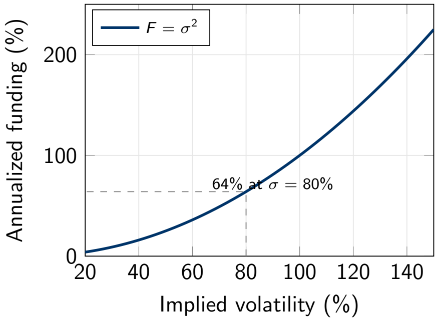
Variance Risk Premium (VRP)

$$\text{VRP} = \mathbb{E}[\sigma^2_{\text{implied}}] - \mathbb{E}[\sigma^2_{\text{realized}}] > 0 \tag{27}$$

- Squeeth **longs buy variance**: profitable when $\sigma_{\text{realized}} > \sigma_{\text{implied}}$
- Squeeth **shorts sell variance**: harvest VRP (analogous to selling options)
- Empirical VRP in ETH: ∼10–20% (annualized σ^2 terms)

Connection to traditional finance: VIX futures, variance swaps on equity indices. Squeeth provides the same exposure *permissionlessly on-chain*.

Funding Cost as a Function of Volatility



Quadratic Cost Structure

$$F_{\text{annual}} = \sigma^2 \tag{28}$$

- At $\sigma = 50\%$: funding $= 25\%/\text{yr}$
- At $\sigma = 80\%$: funding $= 64\%/\text{yr}$
- At $\sigma = 120\%$: funding $= 144\%/\text{yr}$

Practical Consequence

Long Squeeth is only profitable if realized vol exceeds implied vol used to set funding. This is the variance risk premium trade.

Greeks of Power Perpetuals

Complete Greeks for $V = S^p$

$$\Delta = V_S = p\,S^{p-1} \tag{29}$$

$$\Gamma = V_{SS} = p(p-1)\,S^{p-2} \tag{30}$$

$$\Gamma_\$ = S^2\Gamma = p(p-1)\,S^p = p(p-1)\,V \tag{31}$$

$$\mathcal{V} = \frac{\partial V}{\partial \sigma} = p(p-1)\,\sigma\,V\,\Delta t \quad \text{(vega, via funding sensitivity)} \tag{32}$$

$$\Theta = -\tfrac{1}{2}p(p-1)\,\sigma^2\,V \quad \text{(time decay = funding outflow)} \tag{33}$$

Put–Call Parity Analog

Long S^2 (Squeeth) + short $2S$ (delta hedge) = long variance exposure:

$$\mathrm{d}V - 2S\,\mathrm{d}S = \sigma^2 S^2\,\mathrm{d}t = \sigma^2 V\,\mathrm{d}t \tag{34}$$

The delta-hedged Squeeth has P&L proportional to σ^2

Replicating and Hedging Squeeth

Static Replication

From the payoff S^2:

$$S^2 = 2\int_0^{\infty} (S-K)^+ \,\mathrm{d}K + S^2\big|_{S=0} \qquad (35)$$

Squeeth $\equiv$ portfolio of all-strike calls (weighted uniformly).

Dynamic Hedge

Delta-hedge with $\Delta = 2S$ units of spot:

$$\mathrm{d}\Pi = \mathrm{d}(S^2) - 2S\,\mathrm{d}S = \sigma^2 S^2\,\mathrm{d}t \qquad (36)$$

Hedging P&L = realized variance $\times$ notional.

Practical Hedging Considerations

- Continuous rebalancing impossible $\Rightarrow$ discrete hedging error
- Hedging error $\sim \mathcal{O}(\sigma^2\sqrt{\Delta t})$ per period
- Gas costs on Ethereum: \$2–10 per hedge rebalance
- Optimal rebalance frequency: balance gas cost vs. tracking error

Hedge Frequency Trade-off

Frequency	Error	Gas/yr
1 min	0.1%	\$5M
1 h	1.2%	\$88K
1 day	6.5%	\$3.7K

Generalizations: Higher-Order Power Perps

p-th Power Perp Funding and Use Cases

p	Instrument	Exposure	Ann. Funding
2	Squeeth	Variance	σ^2
3	Cubed perp	Skewness	$3\sigma^2$
-1	Inverse perp	Short convexity	σ^2 (received)
0.5	Sqrt perp	Dampened upside	$\frac{1}{8}\sigma^2$ (received)

Portfolio Construction

Any smooth payoff $g(S)$ can be approximated via Taylor expansion:

$$g(S) \approx \sum_{k=0}^{n} \frac{g^{(k)}(S_0)}{k!}(S - S_0)^k \tag{37}$$

Power perps of orders $1, 2, 3, \ldots$ span the payoff

Open Research Questions

- Optimal p for tail-risk hedging?
- Multi-asset power perps: $S_1^{p_1} S_2^{p_2}$?
- Stochastic vol extensions: $f = \frac{p(p-1)}{2}\sigma_t^2$?
- Discrete funding bias and its correction?

Option Vaults & Structured Products

DOVs · Greeks Management · DeFi CPPI

Decentralized Option Vaults (DOVs)

Definition 11

A **DOV** is a smart contract that automates option-selling strategies. Depositors provide collateral; the vault sells covered calls or cash-secured puts each epoch via on-chain auction, distributing premium to depositors.

Mechanism

1. Depositors lock ETH (or USDC) in vault
2. Each Friday: vault sells weekly options
3. Strike selected algorithmically (e.g., 10Δ call)
4. Premium distributed pro-rata
5. At expiry: settlement vs. strike

Major Protocols

Protocol	Strategy	Peak TVL
Ribbon	Covered call	\$300M
Friktion	Volt strategies	\$100M
StakeDAO	Multi-chain	\$50M
Thetanuts	Exotic	\$80M

DOV Greeks Management

Vault-Level Greeks (Short Covered Call)

$$\Delta_{\text{vault}} = 1 - \Delta_{\text{call}} \approx 1 - N(d_1) \tag{38}$$

$$\Gamma_{\text{vault}} = -\Gamma_{\text{call}} = -\frac{n(d_1)}{S\sigma\sqrt{\tau}} < 0 \tag{39}$$

$$\mathcal{V}_{\text{vault}} = -\mathcal{V}_{\text{call}} < 0 \quad \text{(short vega: benefits from vol decline)} \tag{40}$$

$$\Theta_{\text{vault}} = -\Theta_{\text{call}} > 0 \quad \text{(positive theta: earns time decay)} \tag{41}$$

The Gamma Trap

As $S \to K$ near expiry ($\tau \to 0$):

$$|\Gamma_{\text{vault}}| = \frac{n(d_1)}{S\sigma\sqrt{\tau}} \to \infty \tag{42}$$

Near-strike, near-expiry: extreme negative gamma creates **adverse delta swings**. DOV rebalancing exacerbates price moves (pro-cyclical hedging).

DOV Profitability: Implied vs. Realized Volatility

Vault P&L Decomposition

The vault profits when:

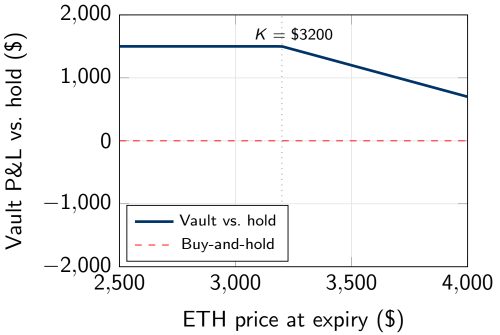
$$\sigma_{\text{implied}} > \sigma_{\text{realized}} \tag{43}$$

Premium collected $\propto \sigma_{\text{implied}}\sqrt{\tau}$.
Losses occur when S exceeds K:

$$\text{Loss} = \max(S_T - K, 0) - \text{Premium} \tag{44}$$

Empirical (ETH 2022–2024):

- IV $>$ RV in 68% of weekly epochs
- Average premium yield: 20–30% ann.
- Drawdown in Nov 2022: -35%



Vault outperforms below breakeven, underperforms in strong rallies.

DOV Worked Example: Covered Call Vault

Setup

Vault holds 10 ETH at $S = \$3{,}000$. Sells 10 weekly $K = \$3{,}200$ calls. Premium: 0.5 ETH ($\approx \$1{,}500$).

$S_T = \$3{,}100$

Options expire worthless.
Vault: $10 \times \$3100 + \$1500$
$= \$32{,}500$.
Return: $+5\%$ for the week.

$S_T = \$3{,}500$

Exercised at K.
Vault: $10 \times \$3200 + \$1500$
$= \$33{,}500$.
Hold: $10 \times \$3500 = \$35{,}000$.
Under by \$1,500.

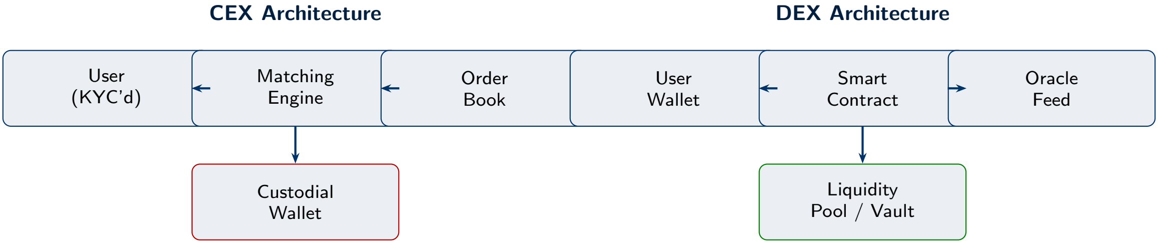
$S_T = \$2{,}500$

Options expire worthless.
Vault: $10 \times \$2500 + \$1500$
$= \$26{,}500$.
Hold: \$25,000.
Cushion: $+\$1{,}500$.

Key Insight

DOV is a **systematic short-volatility strategy**. It harvests the variance risk premium but sacrifices tail upside. Equivalent to selling the upper tail of the return distribution.

CEX vs. DEX Derivatives: Architecture Comparison



CEX Advantages

- Latency: ~5 ms
- Depth: $50B daily volume
- Insurance fund: $1B+ (Binance)

DEX Advantages

- Self-custody (no FTX risk)
- Transparent on-chain execution
- Permissionless, 24/7/365

DEX Perp Microstructure: Information and MEV

Information Asymmetry

- **CEX:** Order flow hidden; exchange has info advantage
- **DEX:** Pending orders visible in mempool $\Rightarrow$ front-running
- MEV on perp DEXs: sandwich attacks on large position opens/closes

DEX Volume Growth Post-FTX

- 2022: 1.5% of total perp volume
- 2025: $\sim$5% of total perp volume
- Hyperliquid: fastest-growing perp DEX

Formal Model: MEV in Perp DEXs

Let trader submit order of size q at oracle price P. MEV searcher observes in mempool and front-runs with q_f:

$$P_1 = P + \lambda q_f \quad \text{(front-run)} \tag{45}$$
$$P_2 = P_1 + \lambda q \quad \text{(victim)} \tag{46}$$
$$P_3 = P_2 - \lambda q_f \quad \text{(back-run)} \tag{47}$$

Searcher profit:

$$\pi_{\text{MEV}} = q_f \cdot \lambda q - 2c_{\text{gas}} \tag{48}$$

where λ is Kyle-style price impact [5].

Structured Products: Principal-Protected Notes in DeFi

Definition 12

A **DeFi principal-protected note** splits capital into a safe tranche (lending) and a risky tranche (options), guaranteeing approximate return of principal.

Construction

Capital C, lending rate r_L, option cost c_{opt}:

$$C_{\text{safe}} = \frac{C}{1 + r_L T} \approx C(1 - r_L T) \tag{49}$$

$$C_{\text{risky}} = C - C_{\text{safe}} = C \cdot r_L T \tag{50}$$

$$\text{Guarantee:} \quad C_{\text{safe}}(1 + r_L T) \geq C \tag{51}$$

The risky tranche buys ATM calls:

$$n_{\text{calls}} = \frac{C_{\text{risky}}}{c_{\text{opt}}}$$

Numerical Example

$C = \$10{,}000$, $r_L = 5\%$, $T = 1\text{yr}$.

- Safe: \$9,524 → \$10,000 at maturity
- Risky: \$476 buys ETH calls
- If ETH +50%: total ≈ \$12,475
- If ETH flat/down: total ≈ \$10,000

Additional Risk

Smart contract failure in lending protocol ⇒ principal is *not* protected.

Formal Framework: CPPI in DeFi

Constant Proportion Portfolio Insurance [ ]

$$\text{Cushion:} \quad C_t = V_t - G \cdot e^{-r(T-t)} \tag{52}$$

$$\text{Risky allocation:} \quad E_t = m \cdot C_t \tag{53}$$

$$\text{Safe allocation:} \quad B_t = V_t - E_t \tag{54}$$

where V_t is portfolio value, G is the guarantee level, and m is the multiplier.

DeFi Implementation

1. B_t in Aave/Compound lending
2. E_t in Squeeth or ETH calls
3. Rebalance on-chain (gas cost!)
4. Multiplier m governs upside participation

Jump-to-Default Risk

Standard CPPI assumes continuous trading. In DeFi:

- Block times ⇒ discrete rebalancing
- Smart contract exploit ⇒ jump-to-zero on safe tranche

Composability: Risk Stacking in DeFi Structured Products

Composability Chain

A single DeFi structured product may compose:

1. Staking (Lido stETH)
2. Lending (Aave: collateralize stETH)
3. Borrow USDC against stETH
4. Buy Squeeth with borrowed USDC
5. Earn staking yield + Squeeth convexity

Each layer adds a **multiplicative** risk factor.

Risk Decomposition

Total survival probability:

$$\mathbb{P}[\text{no loss}] = \prod_{i=1}^{n}(1 - p_i) \qquad (55)$$

where p_i is the failure probability of layer i.

5-Layer Stack

Layer	p_i (annual)
Lido	0.5%
Aave	1.0%
Borrow	2.0%
Squeeth	1.5%
Bridge	1.0%

DeFi vs. TradFi Structured Products

Feature	**TradFi**	**DeFi**
Transparency	Opaque term sheet	Open-source contract
Fees	2–5% embedded	Gas + protocol fee
Counterparty	Bank balance sheet	Smart contract
Settlement	T+2	Instant (1 block)
Accessibility	Accredited only	Permissionless
Customization	Fixed products	Composable modules
Regulation	Full oversight	Minimal
Risk: insolvency	Yes (bank)	No
Risk: code bug	No	Yes

Research Implication

The **smart contract risk premium** is the DeFi-specific risk factor absent in TradFi. Pricing it requires actuarial models for exploit frequency and severity.

Liquidation Mechanics: Perp DEXs

Liquidation Condition

A position is liquidated when:

$$\text{Margin Ratio} = \frac{\text{Collateral} + \text{Unrealized PnL}}{\text{Position Notional}} < m_{\text{min}} \quad (56)$$

Typical $m_{\text{min}} = 2.5\%–6.25\%$ (leverage dependent).

CEX Liquidation

- Internal liquidation engine
- Insurance fund absorbs shortfall
- Auto-deleveraging (ADL) as last resort
- Binance insurance: $> \$1\text{B}$

DEX Liquidation

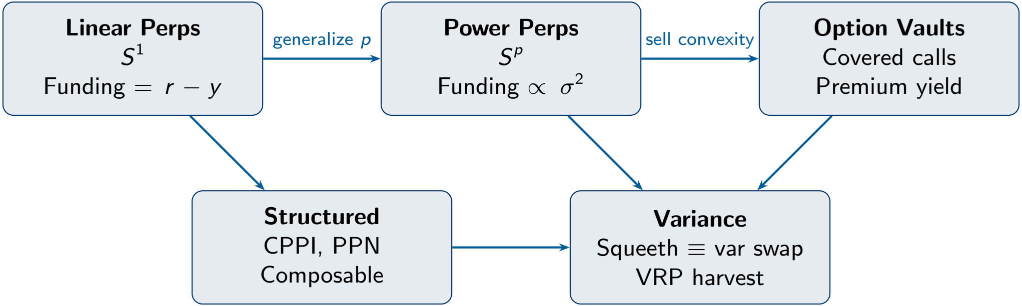
- Permissionless: anyone can liquidate
- Liquidator earns penalty fee
- On-chain: gas cost + block delay
- Bad debt $\Rightarrow$ socialized across LPs

DEX liquidation cascades during high-vol events: multiple positions liquidated in sequence $\Rightarrow$ price impact $\Rightarrow$ more liquidations [4].

Synthesis & Research Frontiers

Open Problems · Research Directions

Unified Framework: DeFi Derivatives Landscape



Unifying Principle

All DeFi derivatives can be viewed through the lens of **convexity exposure** and its cost (funding/premium). Linear perps: zero convexity. Power perps: long convexity. DOVs: short convexity.

Open Research Questions

Pricing and Hedging

1. Optimal funding mechanism design under adverse selection
2. Power perps with stochastic volatility: $f(t) = \frac{p(p-1)}{2}\sigma_t^2$
3. Discrete funding bias quantification
4. Cross-chain arbitrage dynamics

Market Microstructure

1. vAMM depth calibration and welfare
2. Oracle manipulation incentives in perp DEXs
3. Liquidation cascade modeling
4. MEV extraction on perp positions

Structured Products

1. Smart contract risk premium estimation
2. DOV Greeks management across protocols
3. Composability risk: correlated SC failures
4. CPPI under block-time constraints

PhD Project Ideas

- Empirical: basis dynamics across CEX/DEX with OU estimation
- Theoretical: optimal power p for tail hedging
- Applied: DOV strategy backtesting with real on-chain data

Key Equations: Day 6 Summary

Perpetual Futures

$$f(t) = r - y(t) \quad \text{(funding = cost-of-carry)} \quad (57)$$

$$\mathrm{d}b_t = -\alpha b_t\,\mathrm{d}t + \sigma_b\,\mathrm{d}W_t \quad \text{(basis mean-reversion)} \quad (58)$$

Power Perpetuals

$$f = \frac{p(p-1)}{2}\,\sigma^2 \quad \text{(replication cost)} \quad (59)$$

$$\Gamma_\$ = p(p-1)\,V \quad \text{(constant dollar gamma)} \quad (60)$$

$$\eta_{t+\Delta t} = \eta_t \cdot e^{-\sigma^2 \Delta t} \quad \text{(in-kind funding decay)} \quad (61)$$

Structured Products

$$E_t = m \cdot (V_t - G\,e^{-r(T-t)}) \quad \text{(CPPI risky allocation)} \quad (62)$$

Looking Ahead: Day 7

Tomorrow: AI Agents, LLMs, and Autonomous Finance

- AI agents as POMDPs: formalize perception, decision, action
- Multi-agent game theory in DeFi: Nash equilibria among bots
- LLM oracles: calibration, adversarial manipulation, incentives
- Account abstraction: session keys and safe delegation
- Information aggregation with autonomous agents

*"If perpetual futures were the first crypto-native derivative,
AI agents may be the first crypto-native market participant."*

Thank You

Questions, Discussion, Research Ideas

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