

# Day 10: Real-World Assets — Pricing, Liquidity, and Market Design

Tokenization Risk Premia, Illiquidity Models, and Auction Theory

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PhD Seminar Series: Digital Finance Research

2026

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# A \$500K Apartment Sold in 1,000 Pieces

## Paris Apartment Tokenization (2024)

- Luxury apartment: \$500,000 market value
- 1,000 tokens at \$500 each
- 200+ investors from 15 countries
- Monthly rental income: \$8.33/token
- 24/7 secondary market trading
- No investor ever visited the apartment

## The Broader Market (2025)

- Tokenized Treasuries: \$9B+
- Tokenized private credit: \$12B
- Tokenized real estate: \$3B
- **Total RWA on-chain: \$30B+**
- BlackRock BUIDL: \$2.3B single fund

*"If this works, it is not just digital finance. It is the future of all finance."*

# Research Questions for Today

- 1 How do we price tokenized assets that carry **additional risk premia** (smart contract, liquidity, regulatory) beyond traditional factors?
- 2 What is the **term structure of tokenization risk premia**, and how does it evolve as the market matures?
- 3 How should illiquidity in tokenized markets be **measured and modeled** (Amihud-type for thin markets)?
- 4 What is the **optimal auction design** for tokenized bond issuance (Vickrey with participation costs)?
- 5 CLOB vs. AMM for **illiquid tokenized assets**: which market structure is theoretically superior?

## Key References

Amihud (2002), Vickrey (1961), Myerson (1981), Kyle (1985), OECD (2020), Gorton & Zhang (2023)  
[1–3]

# The Tokenization Stack: A Formal Framework

**Layer 5: Marketplace** — CLOB, AMM, OTC; secondary trading & price discovery

↑ trade/settle

**Layer 4: Token** — Digital representation of ownership (ERC-20/3643)

↑ mint/burn

**Layer 3: Smart Contract** — Token logic, compliance, distributions

↑ encoded rules

**Layer 2: Legal Wrapper** — SPV, trust, securities registration (ERC-3643)

↑ legal rights

**Layer 1: Physical Asset** — Real estate, bonds, commodities, art

# Today's Roadmap

- 1 RWA Pricing Theory
- 2 Liquidity Models for Tokenized Markets
- 3 Optimal Auction Design for Tokenized Issuance
- 4 Secondary Market Design: CLOB vs. AMM
- 5 Research Frontiers and Seminar Synthesis

# Outline

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# Tokenized Asset Pricing: Extended DCF Model

## Definition 1 (Tokenized Asset Value)

The value of a tokenized asset with cash flows  $\{CF_t\}_{t=1}^T$  is:

$$V = \sum_{t=1}^T \frac{CF_t}{(1 + r_f + \lambda_{\text{credit}} + \lambda_{\text{sc}} + \lambda_{\text{liq}} + \lambda_{\text{reg}})^t}$$

where the discount rate decomposes as:

- $r_f$ : risk-free rate
- $\lambda_{\text{credit}}$ : credit risk premium (traditional)
- $\lambda_{\text{sc}}$ : smart contract risk premium (exploit/bug risk)
- $\lambda_{\text{liq}}$ : liquidity risk premium (thin secondary market)
- $\lambda_{\text{reg}}$ : regulatory risk premium (legal uncertainty)

The **tokenization discount**  $\Delta V = V_{\text{trad}} - V_{\text{token}} > 0$  arises from  $\lambda_{\text{sc}} + \lambda_{\text{liq}} + \lambda_{\text{reg}} > 0$ .

# Calibrating $\lambda_{sc}$ : Smart Contract Risk Premium

## Empirical Approach

Define the annualized smart contract loss rate:

$$\lambda_{sc} = \frac{\text{Total exploited value (year)}}{\text{Total TVL (average)}}$$

## Historical Data (DeFi, 2020–2025)

Protocol Category	Total Exploits / TVL	Implied $\lambda_{sc}$
Blue-chip (Aave, Compound, MakerDAO)	<0.1% cumulative	0.02%/yr
Mid-tier (audited, >1 yr track record)	~0.5% cumulative	0.10%/yr
New protocols (<6 months)	2–5% cumulative	1–3%/yr
Unaudited / forked protocols	10–20% cumulative	5–10%/yr

$\lambda_{sc}$  is **decreasing** in protocol age and audit count — consistent with a learning/survival model.

# Term Structure of Tokenization Risk Premia

## Hypothesis

Each risk premium has a distinct **term structure**:

$$\lambda_i(t) = \lambda_i^{(0)} \cdot e^{-\kappa_i t} + \lambda_i^{(\infty)}$$

where  $\lambda_i^{(0)}$  is the initial premium,  $\kappa_i$  is the decay rate, and  $\lambda_i^{(\infty)}$  is the long-run floor.

## Estimated Parameters

Risk Premium	$\lambda^{(0)}$ (bps)	$\kappa$ (yr <sup>-1</sup> )	$\lambda^{(\infty)}$ (bps)
Smart contract $\lambda_{sc}$	100	0.5	5
Liquidity $\lambda_{liq}$	150	0.3	20
Regulatory $\lambda_{reg}$	75	0.7	5

Implication: the **tokenization discount narrows over time**. Early investors capture the compression as a risk premium

# Worked Example: Tokenized Corporate Bond Pricing

## Setup

3-year corporate bond, 5% annual coupon, face value \$1,000.

### Traditional Bond

Required return:  $r_f + \lambda_{\text{credit}} = 2\% + 3\% = 5\%$

$$V_{\text{trad}} = \frac{50}{1.05} + \frac{50}{1.05^2} + \frac{1050}{1.05^3} = \$1,000$$

(par, since coupon = required return)

### Tokenized Bond

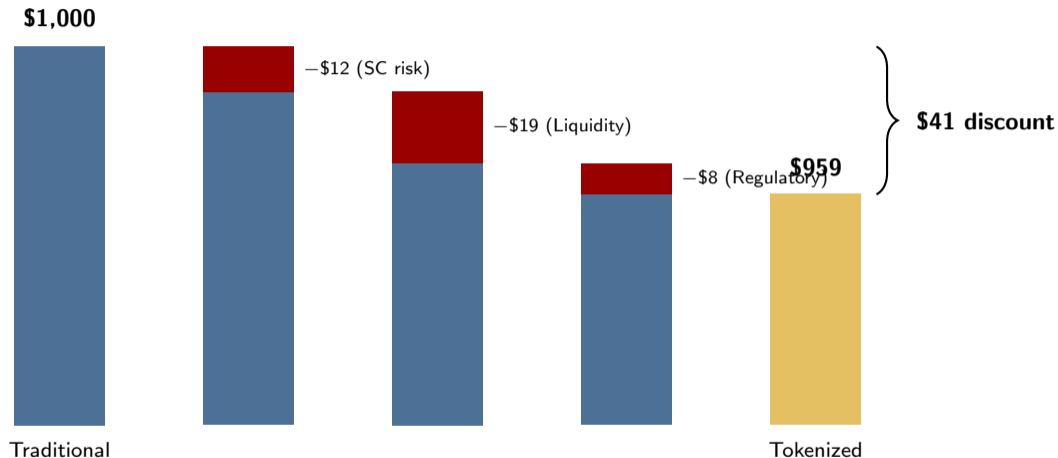
Additional premia:  $\lambda_{\text{sc}} = 0.50\%$ ,  $\lambda_{\text{liq}} = 0.75\%$ ,  
 $\lambda_{\text{reg}} = 0.25\%$ .

Total: 6.50%.

$$V_{\text{token}} = \frac{50}{1.065} + \frac{50}{1.065^2} + \frac{1050}{1.065^3} = \$958.74$$

**Tokenization discount:**  $\$1,000 - \$958.74 = \$41.26$  (4.1% below par)

# Tokenization Discount: Waterfall Decomposition



**Investment thesis:** As each risk premium compresses ( $\lambda_i \downarrow$ ), the tokenized bond reprices upward. Early holders capture the convergence.

# Tokenization Convenience Yield: The Offsetting Factor

## Definition

The **tokenization convenience yield**  $\gamma$  captures benefits *unique* to the tokenized version:

$$V_{\text{token}} = \sum_{t=1}^T \frac{CF_t}{(1 + r_f + \lambda_{\text{credit}} + \lambda_{\text{sc}} + \lambda_{\text{liq}} + \lambda_{\text{reg}} - \gamma)^t}$$

## Sources of Convenience Yield

- 1 **DeFi composability:** Use as collateral in Aave, MakerDAO (borrow against tokenized bonds at 80% LTV)
- 2 **Settlement speed:** T+0 vs. T+1/T+2  $\Rightarrow$  reduced counterparty risk ( $\gamma_{\text{settle}} \approx 2\text{--}5$  bps)
- 3 **24/7 trading:** Access during non-market hours
- 4 **Fractional ownership:** Lower minimum investment (\$100 vs. \$100K)
- 5 **Programmable distributions:** Automated coupon payments, no custodian

If  $\gamma > \lambda_{\text{sc}} + \lambda_{\text{liq}} + \lambda_{\text{reg}}$ : tokenized version trades at a **premium**.

# SDF Approach to Tokenized Asset Pricing

## No-Arbitrage Pricing

Under the risk-neutral measure  $\mathbb{Q}$ :

$$V_0 = \mathbb{E}^{\mathbb{Q}} \left[ \sum_{t=1}^T \frac{CF_t \cdot \mathbb{1}_{\{\text{no exploit by } t\}}}{(1 + r_f)^t} \right]$$

where  $\mathbb{1}_{\{\text{no exploit by } t\}}$  is the survival indicator.

## With Poisson Exploit Arrivals

Let exploits arrive as a Poisson process with intensity  $\mu$  (exploits/year). Conditional on no exploit, full cash flow. On exploit, recovery  $R \in [0, 1]$ :

$$V_0 = \sum_{t=1}^T \frac{CF_t \cdot e^{-\mu t} + CF_t \cdot R \cdot (1 - e^{-\mu t})}{(1 + r_f + \lambda_{\text{credit}})^t}$$

Simplifying with  $R = 0$  (total loss on exploit):

# Multi-Factor Model for Tokenized Assets

## Factor Model

Expected return of tokenized asset  $i$ :

$$\mathbb{E}[R_i] - r_f = \beta_i^{\text{mkt}} \lambda_{\text{mkt}} + \beta_i^{\text{credit}} \lambda_{\text{credit}} + \beta_i^{\text{DeFi}} \lambda_{\text{DeFi}} + \beta_i^{\text{liq}} \lambda_{\text{liq}}$$

where the **DeFi factor** captures systematic smart contract and protocol risk across the tokenized asset ecosystem.

## Empirical Identification

- $\lambda_{\text{mkt}}$ : Traditional equity market premium ( $\sim 5\%$ )
- $\lambda_{\text{credit}}$ : Corporate bond spread indices
- $\lambda_{\text{DeFi}}$ : Composite index of DeFi exploit losses + TVL drawdowns
- $\lambda_{\text{liq}}$ : Amihud illiquidity measure for tokenized assets

Open question: Is  $\lambda_{\text{DeFi}}$  priced? Does it diversify traditional factor exposure?

# The Tokenization Basis: Empirical Evidence

## Definition 2 (Tokenization Basis)

$$\text{Basis}_t = Y_t^{\text{token}} - Y_t^{\text{trad}}$$

where  $Y^{\text{token}}$  is the yield on the tokenized instrument and  $Y^{\text{trad}}$  is the yield on the identical traditional instrument.

## Observed Basis (2024–2025)

Asset Class	Basis (bps)	Direction
Tokenized US Treasuries (BUIDL)	15–25	Token yields more
Tokenized corporate bonds	50–100	Token yields more
Tokenized real estate	200–400	Token yields more
Tokenized money markets	5–15	Token yields more

Basis narrows as TVL grows: BlackRock BUIDL basis compressed from 30 bps to 15 bps over 12 months as AUM grew from \$500M to \$2.2B

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# Amihud (2002) Illiquidity Ratio [ ]

## Definition 3

The **Amihud illiquidity ratio** for asset  $i$  over period  $[1, D]$ :

$$\text{ILLIQ}_i = \frac{1}{D} \sum_{d=1}^D \frac{|R_{i,d}|}{V_{i,d}}$$

where  $R_{i,d}$  is the daily return and  $V_{i,d}$  is the daily dollar volume.

## Interpretation

- Measures the **price impact per dollar traded**
- High ILLIQ  $\Rightarrow$  each dollar of volume moves the price significantly
- Can be interpreted as Kyle's  $\lambda$ :  $\text{ILLIQ} \approx \lambda / \bar{V}$

## Application to Tokenized Assets

Tokenized real estate:  $\text{ILLIQ} \approx 10^{-2}$  (highly illiquid).

# Modified Amihud for Thin Tokenized Markets

## Problem

Standard Amihud assumes daily trading. Many tokenized assets trade <5 times/week. Zero-volume days dominate.

## Definition 4 (Adjusted Illiquidity Measure)

$$\text{ILLIQ}_i^{\text{adj}} = \frac{1}{D_{\text{active}}} \sum_{d: V_{i,d} > 0} \frac{|R_{i,d}|}{V_{i,d}} + \underbrace{\frac{D_{\text{zero}}}{D} \cdot \kappa}_{\text{zero-volume penalty}}$$

where  $D_{\text{active}}$  is the number of trading days,  $D_{\text{zero}} = D - D_{\text{active}}$ , and  $\kappa$  is a calibrated penalty for non-trading periods.

## Calibration

Set  $\kappa$  such that a hypothetical minimum-size trade on a zero-volume day would have price impact equal to the full bid-ask spread:

$$\kappa = \frac{s}{v}$$

# Cross-Asset Liquidity Comparison

Asset	Avg Daily Vol	Bid-Ask	ILLIQ	Depth
SPY (S&P 500 ETF)	\$30B	0.01%	$10^{-12}$	Deep
AAPL	\$10B	0.02%	$10^{-11}$	Deep
BTC/USD	\$15B	0.05%	$10^{-11}$	Deep
Token. Treasury (BUIDL)	\$50M	0.10%	$10^{-8}$	Moderate
Token. Corp. Bond	\$1M	1–2%	$10^{-5}$	Thin
Token. Real Estate	\$2.5K	5–10%	$10^{-2}$	Minimal

## Key Observation

**Four orders of magnitude** separate tokenized Treasuries from tokenized real estate. Tokenization enables trading but does not create liquidity — liquidity requires market depth, which requires participants.

# Endogenous Liquidity: The Chicken-and-Egg Problem

## Model

Let  $L_t$  denote liquidity (daily volume) and  $N_t$  the number of participants. Dynamics:

$$\frac{dN_t}{dt} = \alpha \cdot L_t - \delta \cdot N_t \quad (\text{participants attract when liquid}) \quad (1)$$

$$\frac{dL_t}{dt} = \beta \cdot N_t - \gamma \cdot L_t \quad (\text{liquidity grows with participants}) \quad (2)$$

## Equilibria

- **Absorbing state:**  $N^* = L^* = 0$  — no participants, no liquidity (death spiral)
- **Interior equilibrium:**  $N^* = \frac{\alpha\beta}{\delta\gamma}$ ,  $L^* = \frac{\beta N^*}{\gamma}$  — positive feedback sustains market
- **Critical mass:**  $N_0 > N_c = \frac{\delta\gamma}{\alpha\beta}$  required to escape the death spiral

Implication: Tokenized asset issuers must **subsidize** initial liquidity to reach  $N_c$ . Market maker incentive programs serve this function.

# Calibrating $\lambda_{liq}$ from Market Data

## Approach: Matched-Pair Regression

For asset  $i$  with both traditional ( $Y_i^{trad}$ ) and tokenized ( $Y_i^{token}$ ) versions:

$$Y_i^{token} - Y_i^{trad} = \alpha + \beta_1 \cdot ILLIQ_i^{adj} + \beta_2 \cdot SC\_Score_i + \beta_3 \cdot Reg\_Index + \varepsilon_i$$

$\beta_1$  identifies the marginal liquidity premium;  $\beta_2$  the smart contract premium.

## Illustrative Results

Variable	Coefficient	Interpretation
ILLIQ <sup>adj</sup>	0.85 (0.12)	1 unit ILLIQ → 85 bps premium
SC_Score (1–10)	8.2 (2.1)	1 point worse → 8.2 bps premium
Reg_Index (0–1)	22.5 (5.8)	Full uncertainty → 22.5 bps

$R^2 \approx 0.62$  — the three factors explain most of the tokenization basis.

# Market Impact in Thin Tokenized Markets

## Kyle (1985) Adapted for Tokenized Assets [ ]

In Kyle's model, price impact of order  $Q$ :

$$\Delta p = \lambda \cdot Q, \quad \lambda = \frac{\sigma_v}{2\sigma_u}$$

For tokenized assets:  $\sigma_u$  is small (few noise traders)  $\Rightarrow \lambda$  is large.

## Numerical Example: Selling 100 Tokenized RE Tokens

- Fair value: \$500/token. Order book depth: 30 tokens at \$480 (best bid)
- Selling 100 tokens: first 30 fill at \$480, next 70 must lower price
- Estimated average execution: \$470/token
- **Slippage:**  $(\$500 - \$470)/\$500 = 6\%$
- For comparison: selling \$50K of AAPL slippage  $< 0.01\%$

This market impact **is** the liquidity premium that investors demand ex ante.

# Optimal Liquidation: Almgren–Chriss for Tokenized Assets

## Setup (Almgren and Chriss, 2001)

Sell  $X$  tokens over interval  $[0, T]$ . Trading trajectory  $x_t$  ( $x_0 = X$ ,  $x_T = 0$ ). Objective:

$$\min_{x_t} \mathbb{E}[\text{implementation shortfall}] + \lambda_{\text{risk}} \cdot \text{Var}[\text{shortfall}]$$

## Optimal Strategy

With linear permanent impact  $g(v) = \gamma v$  and linear temporary impact  $h(v) = \eta v$ :

$$x_t^* = X \cdot \frac{\sinh(\kappa(T-t))}{\sinh(\kappa T)}, \quad \kappa = \sqrt{\frac{\lambda_{\text{risk}} \sigma^2}{\eta}}$$

## Thin Market Complication

For tokenized RE with  $\eta \gg \gamma$  and  $\sigma \gg \gamma$ : optimal  $T$  may be **weeks**, not minutes. Patient liquidation is critical but conflicts with stated “24/7 liquidity.”

# DeFi Composability as a Liquidity Source

## Using Tokenized Assets as DeFi Collateral

- 1 Deposit tokenized Treasury (BUIDL) into Aave/MakerDAO
- 2 Borrow stablecoins at 80% LTV
- 3 No need to *sell* the asset for liquidity  $\Rightarrow$  reduces selling pressure
- 4 Effective liquidity: borrowing against collateral substitutes for trading

## Impact on Liquidity Premium

If fraction  $\phi$  of token holders can use DeFi borrowing instead of selling:

$$\lambda_{\text{liq}}^{\text{eff}} = (1 - \phi) \cdot \lambda_{\text{liq}}^{\text{raw}} + \phi \cdot \underbrace{(r_{\text{borrow}} - Y_{\text{asset}})^+}_{\text{borrowing cost if any}}$$

For BUIDL at 5% yield, DeFi borrow rate 3%: borrowing is *profitable*  $\Rightarrow \lambda_{\text{liq}}^{\text{eff}} < \lambda_{\text{liq}}^{\text{raw}}$ .

DeFi composability directly compresses the liquidity premium — a unique tokenization advantage.

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# Primary Market Design: Why Auctions?

## The Issuance Problem

An issuer tokenizes a \$10M bond. How to allocate tokens?

- **Fixed price:** Issuer sets price — risks under/overpricing
- **Book building:** Traditional underwriter process — expensive, slow
- **Auction:** Transparent, price discovery via competitive bidding
- On-chain auctions: **programmable**, atomic settlement, verifiable

## Auction Theory Application

- **Independent Private Values (IPV):** Each bidder has a private valuation  $v_i \sim F$  (e.g., different required yields)
- **Common Value (CV):** Asset has objective value  $v$  but bidders receive noisy signals  $s_i = v + \varepsilon_i$  (winner's curse)
- Tokenized bonds: **hybrid** — common credit quality but private liquidity needs and DeFi composability valuations

# Vickrey (1961) Auction: Truth-Telling Optimality

## Definition 5 (Vickrey (Second-Price) Auction)

$N$  bidders submit sealed bids  $b_1, \dots, b_N$ . Highest bidder wins and pays the **second-highest bid**:  $p = b_{(2)}$ .

## Theorem 6 (Dominant Strategy Incentive Compatibility)

In a Vickrey auction with IPV, bidding  $b_i = v_i$  (true value) is a **weakly dominant strategy**.

## Proof.

Fix bidder  $i$  with value  $v_i$ . Let  $m = \max_{j \neq i} b_j$ .

**Case 1:**  $v_i > m$ . Bidding  $v_i$  wins; pays  $m$ ; profit  $v_i - m > 0$ . Any  $b_i > m$  also wins with same payment. Any  $b_i < m$  loses. Truthful is weakly optimal.

**Case 2:**  $v_i < m$ . Bidding  $v_i$  loses (profit 0). Bidding  $b_i > m$  wins but pays  $m > v_i$  (negative profit). Truthful is strictly better.

**Case 3:**  $v_i = m$ . Indifferent. Profit = 0 either way. □ □

# Multi-Unit VCG Auction for Token Issuance

## Definition 7 (Multi-Unit Vickrey Auction)

Issuer sells  $K$  identical tokens. Bidder  $i$  submits demand schedule  $(q_i, b_i)$  (quantity, per-unit bid). Allocation: top  $K$  units by bid price. Each winner pays the  $(K + 1)$ -th highest bid (uniform price VCG).

## Example: Tokenized Bond Auction

- $K = 10,000$  tokens. 50 bidders.
- Bids sorted by price: \$1,015, \$1,012, \$1,010, ..., \$1,001, \$999, \$995, ...
- 10,000th highest bid: \$1,001. 10,001st: \$999.
- All winners pay \$999/token (the “market-clearing” price)
- Revenue:  $10,000 \times \$999 = \$9,990,000$

On-chain implementation: smart contract collects sealed bids (commit-reveal scheme), computes allocation, settles atomically.

# Vickrey Auction with Participation Costs

## Model (Samuelson, 1985; Stegeman, 1996)

Each bidder incurs cost  $c > 0$  to participate (due diligence, KYC, gas fees). Entry decision: participate iff  $\mathbb{E}[\text{surplus}] \geq c$ .

## Equilibrium Entry

With  $N$  potential bidders, each with  $v_i \sim U[0, 1]$ :

- Bidder enters iff  $v_i \geq v^*$ , where  $v^*$  solves:

$$\mathbb{E}[\max(v_i - p, 0) \mid v_i = v^*] = c$$

- Expected number of entrants:  $n^* = N \cdot (1 - F(v^*))$
- Higher  $c \Rightarrow$  fewer entrants  $\Rightarrow$  lower revenue for issuer

## Tokenization Impact on $c$

- Traditional issuance:  $c = \text{legal review} + \text{underwriter fees} \sim \$50\text{K} - \$500\text{K}$

# Myerson (1981) Optimal Auction Design

## Theorem 8 (Revenue Equivalence)

Any auction mechanism in which (i) the bidder with the highest value wins and (ii) the lowest-value bidder gets zero surplus, yields the same expected revenue:

$$\mathbb{E}[\text{Revenue}] = \mathbb{E} \left[ v_{(1)} - \frac{1 - F(v_{(1)})}{f(v_{(1)})} \right]$$

where  $v_{(1)}$  is the highest order statistic and  $\frac{1-F(v)}{f(v)}$  is the inverse hazard rate.

## Optimal Reserve Price

The revenue-maximizing reserve price  $r^*$  satisfies:

$$r^* = r^* + \frac{1 - F(r^*)}{f(r^*)} = 0 \quad \Rightarrow \quad r^* \text{ such that virtual value} = 0$$

For  $F = U[0, 1]$ :  $r^* = 1/2$ . Excludes low-value bidders to extract more surplus.

# On-Chain Auction: Smart Contract Design

## Commit–Reveal Protocol

- 1 **Commit phase** (blocks 1 to  $B_1$ ): Bidders submit  $h_i = \text{Hash}(b_i \parallel \text{nonce}_i)$  and lock deposit
- 2 **Reveal phase** (blocks  $B_1$  to  $B_2$ ): Bidders reveal  $(b_i, \text{nonce}_i)$ . Contract verifies  $\text{Hash}(b_i \parallel \text{nonce}_i) = h_i$
- 3 **Settlement** (block  $B_2 + 1$ ): Contract computes allocation and price, transfers tokens to winners, refunds losers

## Advantages Over Traditional Book Building

Property	Traditional	On-Chain Auction
Transparency	Opaque	Fully verifiable
Settlement	T+2	Atomic (same block)
Intermediary	Underwriter (2–5% fee)	Smart contract (gas only)
Access	Institutional only	Permissionless (with KYC)
Manipulation	Possible (allocation bias)	Cryptographic fairness

# Winner's Curse in Tokenized Asset Auctions

## Common Value Setting

True asset value  $v$  is unknown. Bidder  $i$  receives signal  $s_i = v + \varepsilon_i$ ,  $\varepsilon_i \sim \mathcal{N}(0, \sigma^2)$ .

## Winner's Curse

The winner has the *highest signal*  $\Rightarrow$  likely overestimates  $v$ :

$$\mathbb{E}[v|\text{win}] = \mathbb{E}[v|s_i = \max_j s_j] < s_i$$

Rational bidders shade their bids:  $b_i = s_i - \beta(N, \sigma)$  where  $\beta$  increases with  $N$  (more competitors) and  $\sigma$  (more noise).

## Tokenization-Specific Winner's Curse

- Tokenized bonds: common credit quality  $v$  + private DeFi composability value
- Issuers can mitigate by publishing on-chain audit data, reserve reports
- More information  $\Rightarrow$  smaller  $\sigma \Rightarrow$  less bid shading  $\Rightarrow$  higher revenue

# Dutch Auction: Alternative for Token Issuance

## Mechanism

Price starts high and decreases over time (or blocks). Each bidder buys at the current price when they decide to act. All tokens sold at the final clearing price.

## GDA: Gradual Dutch Auction (Paradigm, 2022)

On-chain variant with continuous price decay:

$$p(t) = p_0 \cdot e^{-\alpha \cdot (t-t_0)} + p_{\text{floor}}$$

where  $\alpha$  controls the decay rate and  $p_{\text{floor}}$  is the reserve.

## Comparison for Tokenized Bond Issuance

Property	Sealed-Bid (VCG)	Dutch	LMSR/AMM
Price discovery	Excellent	Good	Continuous
Bid privacy	Yes (commit-reveal)	No (public)	N/A

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# Secondary Market Structure: CLOB vs. AMM for Illiquid RWAs

## The Design Question

Given a tokenized asset with  $ILLIQ \approx 10^{-5}$  and daily volume  $\sim \$1M$ : should the secondary market use a CLOB or an AMM?

Criterion	CLOB	AMM (CPMM/LMSR)
Liquidity source	Active market makers	Passive LPs / protocol
Spread	Endogenous (inventory risk)	Exogenous (pool size)
Zero-volume periods	No liquidity	Always available
Price discovery	Excellent	Delayed (oracle dependent)
Capital efficiency	High (concentrated)	Low (distributed)
Manipulation risk	Flash crash (pull orders)	Pool drain (sandwich)
Best for	High-volume assets	Low-volume, nascent assets

# AMM Design for Illiquid Tokenized Assets

## Concentrated Liquidity (Uniswap V3 Style)

For a tokenized bond trading near par (\$1,000):

- Concentrate liquidity in [\$950, \$1,050] (5% band)
- Capital efficiency:  $\sim 20\times$  vs. full-range AMM
- Effective depth equivalent to much larger pool

## Custom Bonding Curve for RWAs

Instead of  $xy = k$ , use a curve that reflects the asset's fundamentals:

$$p(x) = V_{\text{NAV}} + \eta \cdot \frac{\Delta x}{L}$$

where  $V_{\text{NAV}}$  is the oracle-fed net asset value,  $\eta$  is a spread parameter,  $\Delta x$  is the trade size, and  $L$  is pool depth. Advantages:

- Price anchored to NAV (no drift from fundamental value)
- Spread scales linearly with trade size (incentivizes small trades)

# Theoretical Comparison: Expected Execution Cost

## CLOB Expected Cost (Glosten–Milgrom Framework)

For an order of size  $Q$  with adverse selection probability  $\alpha$ :

$$C_{\text{CLOB}}(Q) = \underbrace{s_0 \cdot Q}_{\text{spread cost}} + \underbrace{\lambda \cdot Q^2}_{\text{price impact}} + \underbrace{\mathbb{1}_{\{Q > D\}} \cdot \infty}_{\text{if exceeds depth}}$$

where  $s_0 = 2\alpha\sigma_v/(1 - 2\alpha)$  (Glosten–Milgrom spread) and  $D$  is available depth.

## AMM Expected Cost (CPMM)

For a constant-product AMM with reserves  $(x, y)$ :

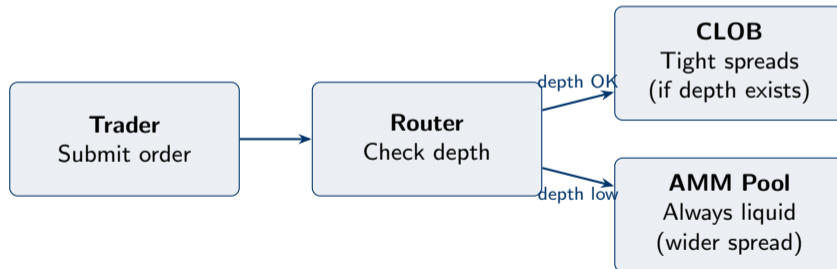
$$C_{\text{AMM}}(Q) = Q \cdot p - y \cdot \left(1 - \frac{x}{x + Q}\right) = \frac{p \cdot Q^2}{x + Q}$$

No depth limit — always executable but with increasing slippage.

# Hybrid Market Design: CLOB + AMM

## Architecture

- 1 **Primary liquidity:** CLOB for market makers and informed traders
- 2 **Backstop liquidity:** AMM pool that fills orders when CLOB is empty
- 3 Smart contract routes: check CLOB depth first; if insufficient, route to AMM



This design is used by Blur (NFTs) and emerging RWA exchanges. Combines CLOB efficiency with AMM availability.

# RFQ Markets for Institutional RWA Trading

## Why RFQ for Tokenized Assets?

- Institutional investors prefer **block trades** (large size, minimal impact)
- CLOB and AMM both suffer from large-order slippage
- RFQ: trader requests quotes from designated market makers; best quote wins

## On-Chain RFQ Protocol

- 1 Trader broadcasts encrypted RFQ: "Buy 500 tokens of Bond-X"
- 2  $M$  market makers respond with quotes (signed, time-limited)
- 3 Smart contract selects best quote, executes atomically
- 4 All quotes visible post-execution (transparency)

## Theoretical Advantage

With  $M$  competing market makers, the expected spread:

2

# Comprehensive Market Structure Comparison

Property	CLOB	CPMM	Conc. Liq.	RFQ	Hybrid
Always liquid	No	Yes	Yes	Depends on <i>M</i>	Yes
Tight spread	Yes (if depth)	No	Moderate	Yes (if <i>M</i> high)	Best of both
Large orders	Good (if depth)	Slippage	Moderate	Excellent	Good
Price discovery	Excellent	Delayed	Moderate	None (taker)	Good
Capital req.	MM capital	LP deposits	LP deposits	MM capital	Mixed
Manipulation	Order spoofing	Sandwich	Sandwich	Collusion	Mitigated
Gas cost	Medium	Low	Medium	Low	Medium
<b>Best for</b>	Liquid	Illiquid	Mid-tier	Institutional	General

**Recommendation:** Tokenized Treasuries (liquid) → CLOB. Tokenized real estate (illiquid) → AMM or hybrid. Institutional blocks → RFQ.

# Legal Wrapper Efficiency Analysis

## Cost of Legal Infrastructure

Wrapper Type	Setup Cost	Ongoing Cost	Jurisdiction
US SPV (Reg D)	\$50–200K	1–2%/yr	Delaware
EU DLT Pilot	\$30–100K	0.5–1%/yr	Luxembourg
Swiss DLT Act	\$20–80K	0.3–0.8%/yr	Zug
Cayman SPV	\$15–50K	0.5–1.5%/yr	Cayman Islands

## Legal Wrapper Efficiency Ratio

$$\eta_{\text{legal}} = \frac{\text{Investor protection quality}}{\text{Annual cost as \% of AUM}}$$

- Swiss DLT Act: highest  $\eta$  (strong legal recognition, low cost)
- US Reg D: lowest  $\eta$  (fragmented regulation, high compliance cost)

# Global Regulatory Landscape for Tokenized Assets

Feature	US	EU (DLT Pilot)	Switzerland	Singapore
Legal recognition	Fragmented	DLT Pilot Regime	DLT Act (2021)	MAS framework
Token as security	Yes (Howey)	Yes (MiFID II)	Register security	Case-by-case
Max issuance	No limit (Reg D)	€6B per DLT-TSS	No limit	No limit
Retail access	Accredited only	With limits	Permitted	Accredited only
Settlement	T+0 possible	T+0 permitted	T+0 standard	T+0 possible
DeFi composability	Unclear	Limited	Permitted	Case-by-case
<b>Readiness score</b>	5/10	7/10	9/10	6/10

Switzerland leads due to the DLT Act's explicit recognition of blockchain-based register securities — no separate legal wrapper needed for certain asset types.

# Outline

- 1 RWA Pricing Theory
- 2 Liquidity Models for Tokenized Markets
- 3 Optimal Auction Design for Tokenized Issuance
- 4 Secondary Market Design: CLOB vs. AMM
- 5 Research Frontiers and Seminar Synthesis

# The \$600 Trillion Question

## Global Asset Market

Asset Class	Estimated Value
Real estate	\$330T
Fixed income (bonds)	\$130T
Equities	\$110T
Alternatives (PE, VC, commodities)	\$30T
<b>Total addressable market</b>	<b>~\$600T</b>
Currently tokenized (2025)	\$30B
<b>Penetration rate</b>	<b>0.005%</b>

## McKinsey / Boston Consulting Group Projections

- 2030 estimate: \$4–16T tokenized (optimistic: 2.7% penetration)
- If tokenization captures 5% of global assets: \$20T market

# Open Research Problems

- 1 **Cross-chain liquidity aggregation:** Same RWA token on Ethereum, Polygon, Solana — how to unify liquidity across chains without fragmenting the order flow?
- 2 **Dynamic risk premium models:** Bayesian updating of  $\lambda_{sc}$  as protocol-specific exploit data accrues. Conjugate prior models.
- 3 **Optimal legal wrapper design:** Mechanism design for SPV structures that minimize cost while maximizing enforceability (joint law + CS problem)
- 4 **Tokenized derivatives:** Options and futures on tokenized RWAs — adapting Black–Scholes when underlying is illiquid and discrete
- 5 **Systemic risk in tokenized markets:** If tokenized Treasuries become major DeFi collateral, a smart contract exploit creates Treasury market contagion

# Seminar Synthesis: Ten Days of Digital Finance Research

Day	Topic	Key Model	Open Question
1	Crypto Derivatives	Heston, Bates, Fourier	Crypto-specific vol surface
2	DeFi Mathematics	CFMM geometry, IL	Optimal LP strategies
3	Blockchain Economics	Mechanism design, MEV	MEV redistribution
4	ML & Microstructure	DeepLOB, RL	On-chain vs. off-chain signals
5	Risk & Regulation	Network theory, EVT	DeFi-SIFI identification
6	DeFi Derivatives	Perp funding, cash-carry	Funding rate dynamics
7	AI Agents	Autonomous finance	Agent alignment in DeFi
8	ZK Proofs	KZG, FRI, simulation	Post-quantum ZK for finance
9	Prediction Markets	LMSR, Milgrom–Stokey	Manipulation-proof design
10	RWA Tokenization	Amihud, Vickrey, DCF	\$600T market design

# Unifying Themes Across the Seminar

## Four Pillars of Digital Finance Research

- 1 **Mechanism Design:** From MEV auctions (Day 3) to prediction market scoring rules (Day 9) to token issuance auctions (Day 10) — *how do we design markets that align incentives?*
- 2 **Information Economics:** From Kyle's lambda (Day 4) to Grossman–Stiglitz (Day 9) to price discovery in thin markets (Day 10) — *how is information reflected in prices?*
- 3 **Risk Measurement:** From jump-diffusion models (Day 1) to network contagion (Day 5) to tokenization risk premia (Day 10) — *how do we quantify novel risks?*
- 4 **Privacy & Trust:** From pseudonymous markets (Day 3) to ZK proofs (Day 8) to legal wrappers (Day 10) — *how do we balance transparency and privacy?*

# Research Opportunities for PhD Students

## High-Impact Open Questions

- 1 **Empirical:** Estimate the tokenization risk premium term structure using matched-pair data (tokenized vs. traditional bonds)
- 2 **Theoretical:** Optimal market design for assets with  $ILLIQ > 10^{-5}$  — when does AMM dominate CLOB?
- 3 **Applied:** Build a factor model for tokenized assets with DeFi-specific risk factors ( $\lambda_{DeFi}$ )
- 4 **Legal–Economic:** Cross-jurisdictional analysis of legal wrapper costs and their impact on tokenization adoption
- 5 **Computational:** Efficient ZK circuits for proving RWA cash flow validity (privacy-preserving bond pricing)

The field is young. First-mover advantage in research is substantial. Most questions above have <10 published papers.

# Day 10: Key Takeaways

## Core Results

- ① **Tokenization discount** decomposes into smart contract, liquidity, and regulatory risk premia — each independently calibratable
- ② **Amihud illiquidity:** Tokenized assets span 10 orders of magnitude in liquidity — from Treasuries ( $10^{-8}$ ) to real estate ( $10^{-2}$ )
- ③ **Vickrey auctions** with on-chain commit-reveal achieve incentive compatibility for token issuance; participation costs are 100× lower
- ④ **CLOB vs. AMM:** CLOB dominates for liquid tokenized assets; AMM (or hybrid) dominates for illiquid ones
- ⑤ **DeFi composability** directly compresses the liquidity premium by enabling borrowing-as-liquidity

# Final Reflection

*“Tokenization is where crypto meets the real economy.  
DeFi started with native crypto assets. Now it incorporates  
real-world assets: bonds, real estate, commodities.  
If the legal, technical, and liquidity challenges are solved,  
it is not just digital finance.  
It is the future of all finance.  
The \$600 trillion global asset market,  
programmable, composable, and accessible 24/7.  
That is the vision. Whether it becomes reality  
depends on the problems you will solve.”*

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- [3] Albert S. Kyle. “Continuous Auctions and Insider Trading”. In: *Econometrica* 53.6 (1985), pp. 1315–1335.