Day 1: What Makes Crypto Different?
Pricing and Volatility

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BSc Seminar – Digital Finance

2026

Seminar Overview: Five Days of Digital Finance

D1 **Crypto Pricing & Volatility** – what makes crypto returns wild

D2 **DeFi Explained** – Uniswap, lending, liquidity provision

D3 **Blockchain Economics** – how blockchains actually work

D4 **Stablecoins & Regulation** – pegs, crashes, and MiCA

D5 **Crypto Investing** – portfolios, risk, and the future

Format Each Day

- 45 min lecture
- 15 min worked examples
- 30 min hands-on exercise
- 15 min group discussion

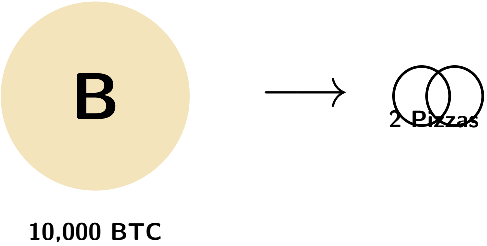
Prerequisites

Intro statistics, basic calculus, curiosity!

The Pizza That Cost $800 Million

May 22, 2010 — "Bitcoin Pizza Day"

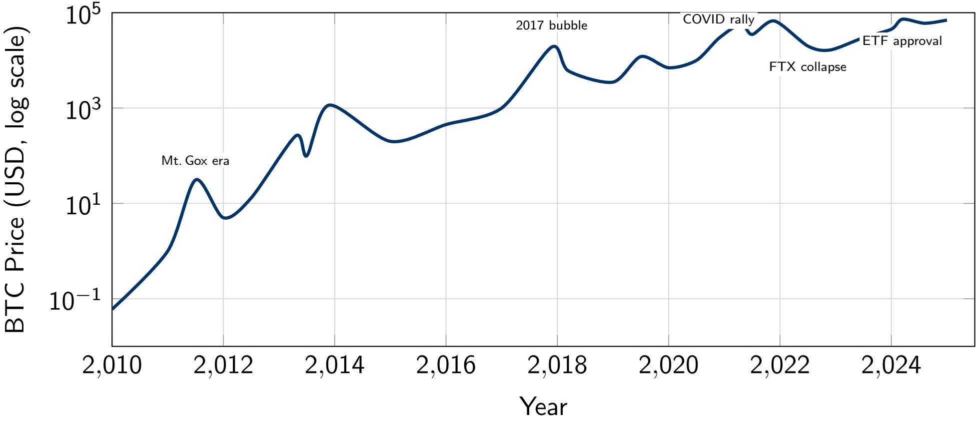
- Laszlo Hanyecz paid **10,000 BTC** for two pizzas
- At the time: worth about $41
- At BTC's all-time high ($73k, March 2024): those pizzas = $730,000,000



The Big Question

How do you value an asset that goes from $0.004 to $73,000 in 14 years?

Bitcoin Price History (2010–2025)



Is Bitcoin a Stock? Currency? Commodity? Something New?

Similarities to...

Currency	Medium of exchange, unit of account? But: extreme volatility, slow transactions
Commodity	Scarce (21M cap), mined But: no physical use, no cash flows
Stock	Tradeable, speculative But: no earnings, no company behind it

Regulators Can't Agree Either

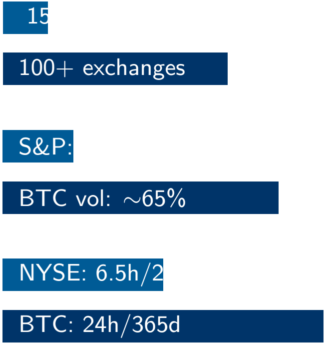
- CFTC (US): "commodity"
- SEC (US): some tokens are "securities"
- ECB: "speculative asset"
- El Salvador: "legal tender"
- EU MiCA: new category — "crypto-asset"

Key insight: We need *new tools* to analyze this new asset class.

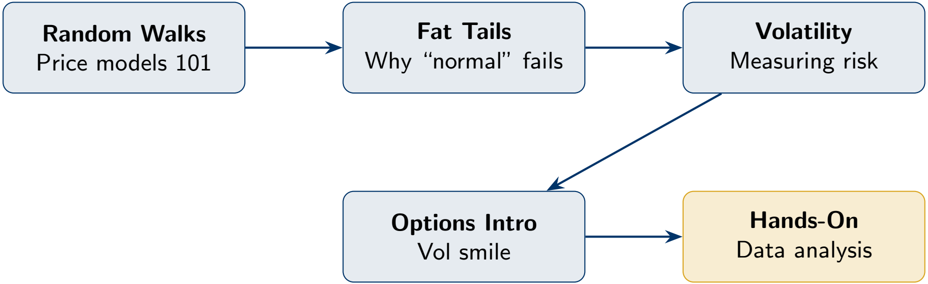
Stylized Facts: Why Crypto Markets Are Different

How crypto differs from traditional markets:

1. **24/7/365 trading** — no closing bell
2. **Extreme volatility** — 10% daily moves happen
3. **No circuit breakers** — no automatic halt
4. **Global & fragmented** — 100+ exchanges
5. **Retail-dominated** — until recently
6. **Programmable** — smart contracts



Today's Roadmap



Learning Goals

After today you will:

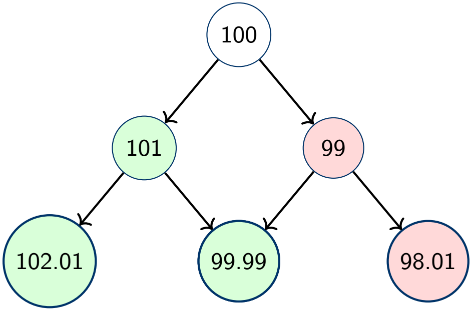
- Understand how prices are modeled as random processes
- Know why Bitcoin returns are NOT normally distributed
- Be able to measure and compare volatility across assets
- Grasp the intuition behind options pricing

Outline

The Simplest Price Model: A Coin Flip

Imagine a stock that moves by coin flip each day:

- Heads (probability $\frac{1}{2}$): price goes UP by 1%
- Tails (probability $\frac{1}{2}$): price goes DOWN by 1%



This is a **random walk** — the price has no memory of where it's been.

Random Walk: The Formula

One-Period Return

$$S_{t+1} = S_t \cdot (1 + r_t)$$

where r_t is the return at time t (a random variable).

After T periods, the price is:

$$S_T = S_0 \cdot \prod_{t=1}^{T} (1 + r_t)$$

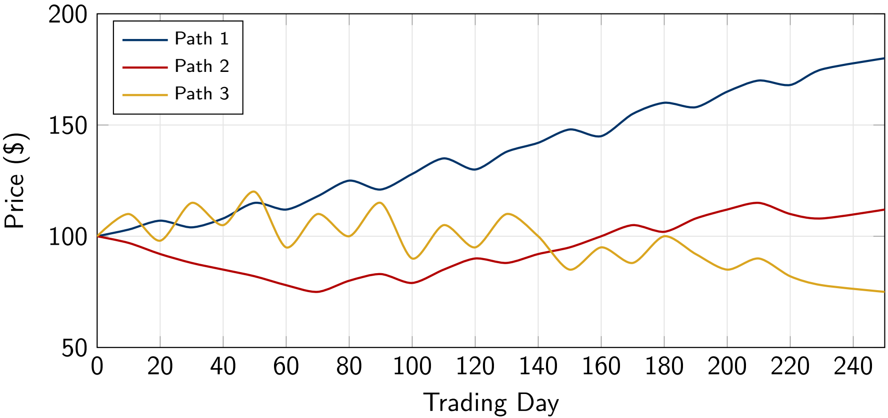
Key idea:

- Returns r_t are *random* and *independent*
- Today's return tells you nothing about tomorrow's
- This is the **Efficient Market Hypothesis** in action

Analogy

A random walk is like a drunk person stumbling on a straight road — each step is equally likely left or right. After 100 steps, where are they? *Hard to say!*

Simulated Random Walk Paths



Same model, same starting price, wildly different outcomes.
Each path is equally likely — that's randomness!

The Standard Price Model: Geometric Brownian Motion

In continuous time, the standard model for asset prices is:

Log-Normal Price Formula

$$S_T = S_0 \cdot \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,Z\right], \qquad Z \sim \mathcal{N}(0,1)$$

What does this say?

- S_0 = today's price
- S_T = price at time T (in years)
- μ = expected annual return (the "drift")
- σ = volatility (how much prices fluctuate)
- Z = a random shock from the normal distribution

The exponential ensures prices stay positive (you can't have $S < 0$).

Understanding the Parameters

μ = Drift

Expected growth
$\mu = 0.10$ means
"10% per year on avg"

σ = Volatility

How wild the ride
$\sigma = 0.65$ means
"65% annual vol"

Z = Random Shock

Drawn from $\mathcal{N}(0, 1)$
$Z = +2$: great day!
$Z = -2$: terrible day

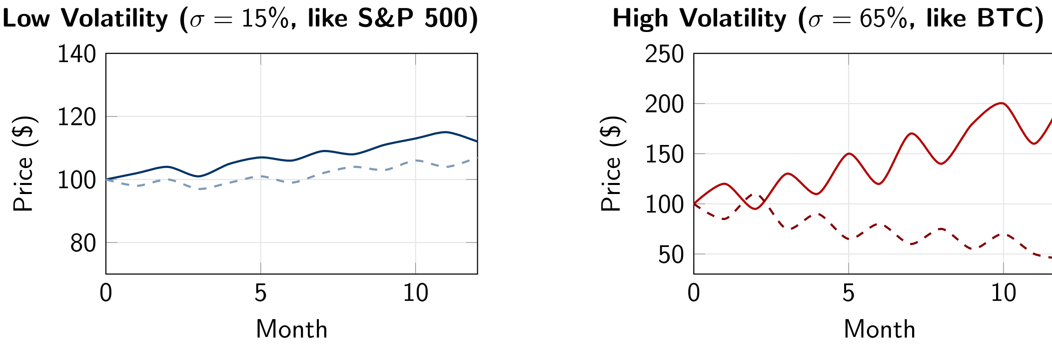
Putting It Together

With $S_0 = \$100$, $\mu = 0.10$, $\sigma = 0.65$, $T = 1$:

- If $Z = 0$ (average): $S_1 = 100 \cdot e^{0.10-0.21} \approx \$90$ (vol drag!)
- If $Z = +1$ (good luck): $S_1 = 100 \cdot e^{-0.11+0.65} \approx \$172$
- If $Z = -1$ (bad luck): $S_1 = 100 \cdot e^{-0.11-0.65} \approx \$47$

Huge range! That's what 65% volatility looks like.

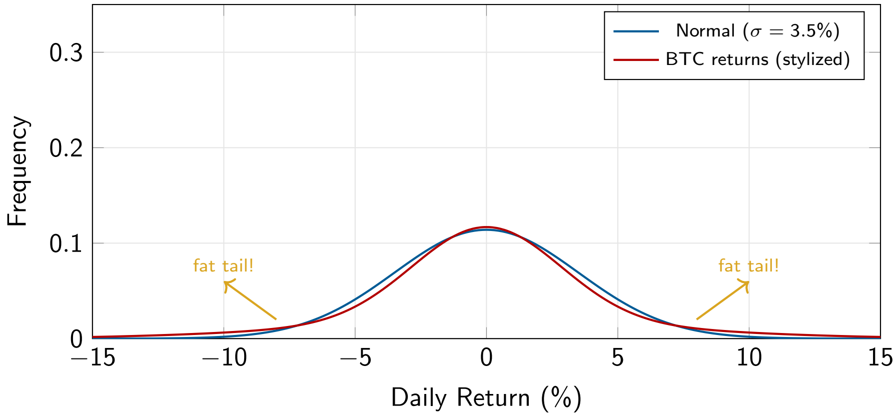
Effect of Volatility: Low σ vs. High σ



Higher $\sigma \Rightarrow$ wider range of outcomes. BTC's high vol means both huge gains AND huge losses are common.

Outline

BTC Returns vs. the Normal Distribution



Key observation: BTC has a sharper peak AND fatter tails than the normal. Extreme events happen *much more often* than a bell curve predicts.

What Are “Fat Tails”?

Plain English

“Fat tails” means **extreme events happen far more often** than a normal distribution predicts.

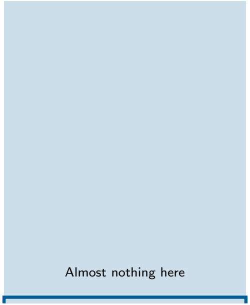
Under the normal distribution:

- A 3-sigma event happens once every 1.5 years
- A 5-sigma event: once every 14,000 years
- A 10-sigma event: *never* (once per 10^{23} years)

Bitcoin reality:

- 5-sigma daily moves: happens multiple times per year!
- March 12, 2020: BTC fell **37%** in one day
- That is a >10-sigma event under the normal

What normal predicts



What actually happens

Measuring Fat Tails: Kurtosis

Kurtosis = "Tailedness" of a Distribution

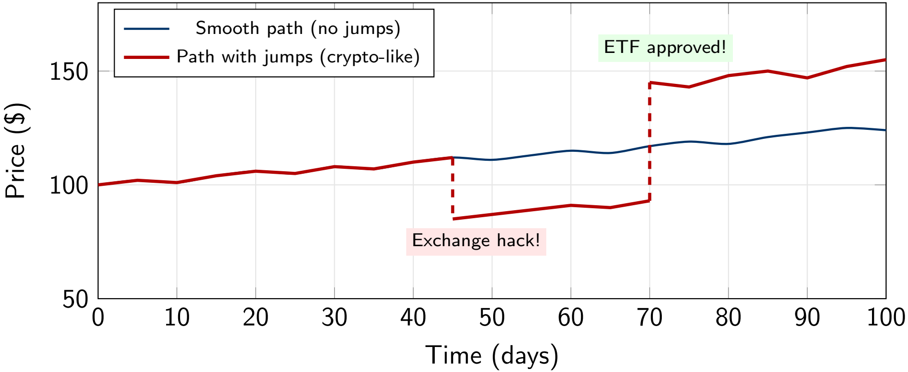
$$\text{Kurtosis} = \frac{\mathbb{E}\left[(r-\mu)^4\right]}{\left(\mathbb{E}\left[(r-\mu)^2\right]\right)^2}$$

Higher kurtosis $\Rightarrow$ more extreme observations.

Asset	**Kurtosis**	**Interpretation**
Normal distribution	3	Baseline (no excess tails)
S&P 500	∼4–5	Mild fat tails
Gold	∼5–7	Moderate fat tails
Bitcoin	**10–20**	**Extreme fat tails**
Ethereum	**12–25**	**Even more extreme**

Takeaway: If you use normal-distribution models for crypto, you will *massively* underestimate risk.

Jumps: When Prices Don't Move Smoothly



Real crypto prices don't move smoothly — they have sudden jumps caused by news events.

Why Do Crypto Prices Jump?

Negative Jumps (Crashes)

- Exchange hacks (Mt. Gox 2014)
- Regulatory crackdowns (China ban)
- Stablecoin de-pegs (Terra/Luna 2022)
- Fraud revealed (FTX collapse 2022)
- Whale liquidations

Positive Jumps (Rallies)

- ETF approvals (Jan 2024)
- Institutional adoption announcements
- Halving events (supply shock)
- Favorable regulation news
- Major partnerships

Key Point

The basic random walk model assumes prices move *smoothly*. Crypto needs models that allow for **sudden jumps**. At the PhD level, this leads to *jump-diffusion models* (Merton 1976).

Outline

What Is Volatility?

Definition

Volatility $= \sigma =$ the **standard deviation** of returns.
It measures how much prices fluctuate around their average.

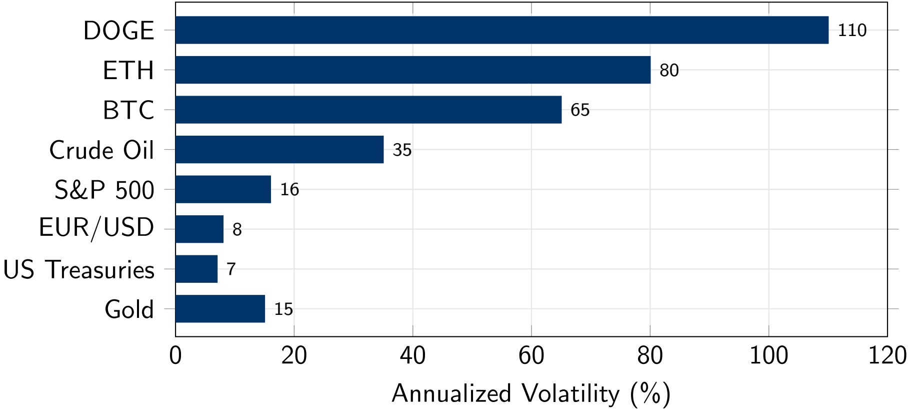
Calculating daily volatility:

1. Compute daily log-returns: $r_t = \ln(S_t/S_{t-1})$
2. Calculate the standard deviation: $\hat{\sigma}_{\text{daily}} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t - \bar{r})^2}$
3. Annualize: $\hat{\sigma}_{\text{annual}} = \hat{\sigma}_{\text{daily}} \times \sqrt{365}$ (crypto trades every day!)

Why $\sqrt{365}$?

Volatility scales with the *square root* of time. If daily vol is 3.4%, annual vol $\approx 3.4\% \times \sqrt{365} \approx 65\%$.
For stocks (252 trading days): $\hat{\sigma}_{\text{annual}} = \hat{\sigma}_{\text{daily}} \times \sqrt{252}$

Volatility Comparison: Crypto vs. Traditional Assets



Crypto is 4–7× more volatile than traditional assets. DOGE is in a league of its own!

Two Types of Volatility

Historical Volatility

- Computed from **past** returns
- "What happened"
- Formula: standard deviation of recent returns
- Example: 30-day rolling vol
- Backward-looking

Implied Volatility (IV)

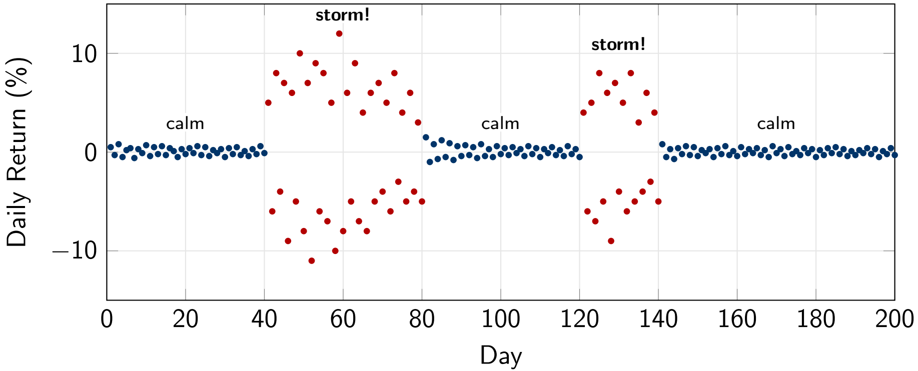
- Extracted from **option prices**
- "What the market expects"
- Backed out from Black-Scholes formula
- Example: Deribit BTC options IV
- Forward-looking

Analogy

- Historical vol = looking in the **rearview mirror**
- Implied vol = looking through the **windshield**

Both are useful, but traders care more about where they're going!

Volatility Clustering: Calm Periods and Storms



“Big moves follow big moves, small moves follow small moves.”

Modeling Volatility Clustering: GARCH(1,1)

GARCH(1,1) — "Tomorrow's Volatility" Formula

$$\sigma_t^2 = \underbrace{\omega}_{\text{base level}} + \underbrace{\alpha\, r_{t-1}^2}_{\text{yesterday's surprise}} + \underbrace{\beta\, \sigma_{t-1}^2}_{\text{yesterday's vol}}$$

In plain English:

- ω = a baseline level of volatility (always present)
- $\alpha\, r_{t-1}^2$ = if yesterday had a BIG return (positive or negative), volatility goes up $\Rightarrow$ "shock effect"
- $\beta\, \sigma_{t-1}^2$ = if yesterday was already volatile, today probably is too $\Rightarrow$ "persistence effect"

Typical BTC Estimates

$\alpha \approx 0.10$, $\beta \approx 0.85$, so $\alpha + \beta = 0.95$

$\Rightarrow$ Volatility shocks decay slowly. A spike in vol takes weeks to calm down.

Outline

What Is an Option?

Call Option

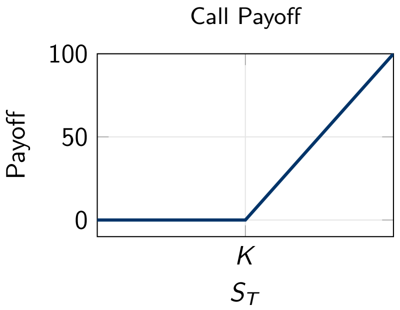
The **right** (not obligation) to **buy** an asset at a fixed price K ("strike") on date T.

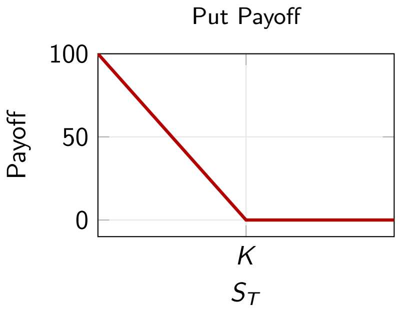
$\text{Payoff} = \max(S_T - K, 0)$

Put Option

The **right** (not obligation) to **sell** an asset at a fixed price K on date T.

$\text{Payoff} = \max(K - S_T, 0)$





Options are like insurance — you pay a premium for protection.

The Black-Scholes Formula (1973)

Call Price under Black-Scholes [ ]

$$C = S\,\Phi(d_1) - K\,e^{-rT}\,\Phi(d_2)$$

where $d_1 = \frac{\ln(S/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$, $\quad d_2 = d_1 - \sigma\sqrt{T}$

What each piece means:

- S = current price, $\quad K$ = strike price, $\quad T$ = time to expiry
- r = risk-free interest rate, $\quad \sigma$ = volatility
- $\Phi(\cdot)$ = cumulative normal distribution ("probability of being below")
- $S\,\Phi(d_1)$ = expected value of the asset if the option pays off
- $K\,e^{-rT}\Phi(d_2)$ = discounted cost of exercising

Nobel Prize!

Scholes and Merton won the 1997 Nobel Prize for this. It revolutionized finance — but it has a critical flaw for crypto...

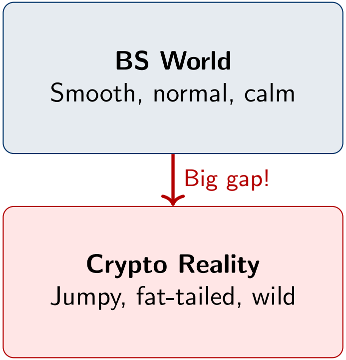
The Problem: Black-Scholes Assumes Normality

Black-Scholes assumes:

1. Returns are **normally distributed** ✗
2. Volatility is **constant** over time ✗
3. Prices move **smoothly** (no jumps) ✗
4. Trading is **continuous** ✗ for crypto

We already know from today:

- BTC returns have fat tails (kurtosis $\gg 3$)
- Volatility clusters (GARCH effects)
- Prices jump (hacks, news, liquidations)



The Volatility Smile

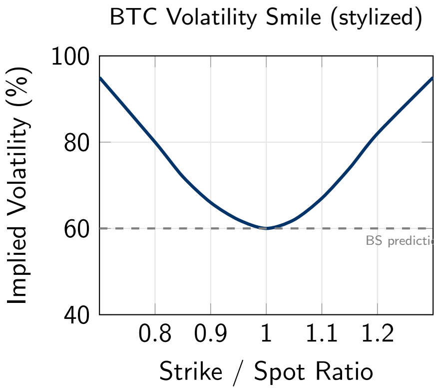
If Black-Scholes were correct:

- Implied volatility should be the *same* for all strikes
- The IV plot would be a flat horizontal line

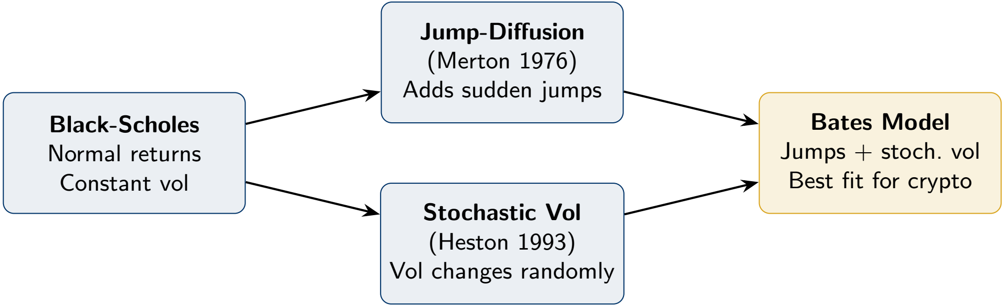
What we actually see:

- Deep out-of-the-money puts have *higher* IV (crash protection is expensive!)
- Deep out-of-the-money calls also have higher IV (people pay for moonshot bets)
- Result: a **smile** or **smirk** shape

This tells us the market "knows" that extreme moves are more likely than the normal distribution predicts.



Beyond BSc: How the PhD Version Fixes This



For This Course

We focus on **understanding the phenomena**: fat tails, volatility clustering, jumps, and the smile. The PhD seminar builds the stochastic calculus machinery to model them.

Outline

Hands-On Exercise

Analyzing BTC vs. S&P 500 Returns

Exercise: Comparing Crypto and Stock Returns

Goal

Use real data to verify the stylized facts we discussed today. Compare BTC and S&P 500 returns side by side.

Tools: Python with `pandas`, `numpy`, `matplotlib` (or Excel / Google Sheets if you prefer)

Data:

- BTC daily prices: Yahoo Finance (`BTC-USD`)
- S&P 500 daily prices: Yahoo Finance (`^GSPC`)
- Period: Jan 2020 – Dec 2025

Time: 30 minutes

Steps

1. **Download data:**
   - Use `yfinance` in Python or download CSV from Yahoo Finance
2. **Calculate daily log-returns:**

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

3. **Plot histograms** of BTC and S&P 500 returns (overlay them!)
4. **Compute summary statistics:** mean, std, skewness, kurtosis
5. **Rolling volatility:** 30-day rolling standard deviation
6. **Extreme events:** count days with $|r_t| > 5\%$

Python Hint

```
import yfinance as yf
btc = yf.download("BTC-USD", start="2020-01-01")
```

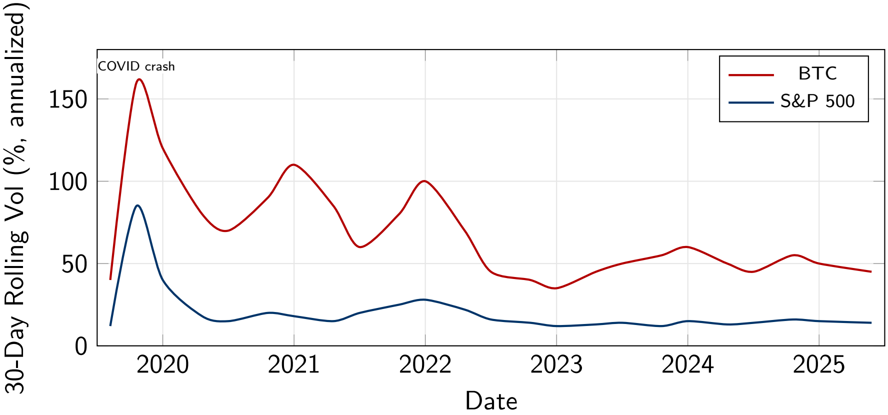
What You Should Find: Summary Statistics

Statistic	BTC	S&P 500
Mean daily return	$\sim 0.10\%$	$\sim 0.05\%$
Std (daily)	$\sim 3.5\%$	$\sim 1.1\%$
Annualized vol	$\sim 65\%$	$\sim 17\%$
Skewness	~ -0.3	~ -0.5
Kurtosis	$\sim$ **12**	$\sim$ **4.5**
Days with $\lvert r \rvert > 5\%$	~ 40	~ 5

Key observations:

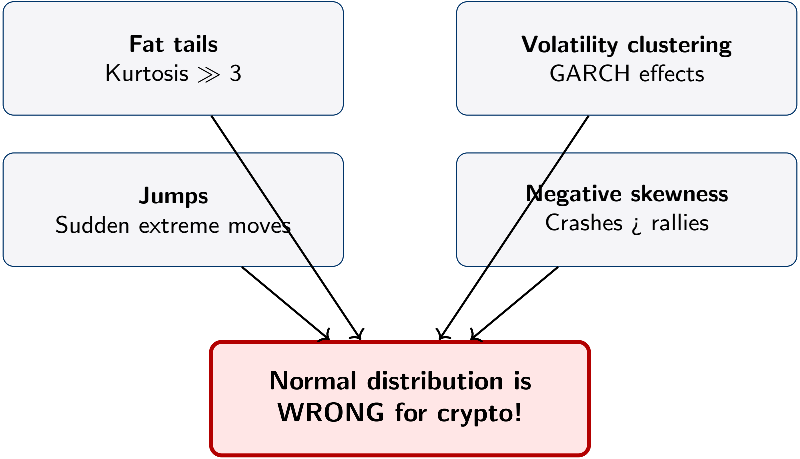
- BTC is $\sim 4\times$ more volatile
- Kurtosis is $\sim 3\times$ higher (extreme moves are NOT rare)
- BTC has $\sim 8\times$ more extreme days
- Both are negatively skewed (crashes ¿ rallies)

Rolling 30-Day Volatility



Notice the clustering: BTC vol spikes and stays high for weeks.
March 2020 (COVID): both assets spiked, but BTC reached ∼160%.

Key Finding: Crypto Is NOT Normally Distributed



Practical implication: Any risk model, portfolio tool, or pricing formula that assumes normality will *underestimate* the true risk of holding crypto.

Discussion: Is Bitcoin a Good Investment?

Arguments FOR

- Highest-returning asset class (2010–2025)
- Scarce: only 21 million will ever exist
- Uncorrelated with stocks (sometimes)
- Inflation hedge narrative
- Growing institutional adoption (ETFs)

Arguments AGAINST

- Extreme volatility = extreme risk
- No fundamental cash flows
- Environmental concerns (PoW energy)
- Regulatory uncertainty
- History of fraud and hacks

Think About

Would you put 1% of your portfolio in BTC? 5%? 50%? What information would change your answer?

Discussion: Should Crypto Be Regulated Like Stocks?

Arguments for regulation:

- Investor protection (FTX, Terra/Luna)
- Market integrity (manipulation, wash trading)
- Level playing field with TradFi
- Anti-money laundering (AML)
- EU's MiCA shows it's possible [3]

Arguments against (heavy) regulation:

- Innovation may flee to crypto-friendly jurisdictions
- DeFi is borderless — how do you regulate code?
- Pseudonymity is a feature, not a bug
- Self-custody = financial sovereignty
- Overregulation killed prior innovations

Key question: Can regulation keep pace with 24/7 global markets running on smart contracts? [1]

Day 1 Summary and Key Takeaways

What We Learned Today

1. **Random walks** model prices as unpredictable sequences of returns
2. **Crypto returns are NOT normal:** fat tails, kurtosis $\gg 3$, jumps
3. **Volatility** is 4–7$\times$ higher than stocks, and it clusters
4. **GARCH** captures volatility persistence: $\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$
5. **Options** exist on BTC — the **volatility smile** reveals the market knows normality fails

Day 2 Preview: DeFi Explained

How do decentralized exchanges like Uniswap work?
What is the math behind $x \times y = k$?
What is "impermanent loss" and why should you care?

References I

[1] Bank for International Settlements. *Cryptocurrencies and Decentralised Finance*. BIS Annual Economic Report, Chapter III. Bank for International Settlements, 2024.

[2] Fischer Black and Myron Scholes. "The Pricing of Options and Corporate Liabilities". In: *Journal of Political Economy* 81.3 (1973), pp. 637–654.

[3] European Commission. *Regulation (EU) 2023/1114 on Markets in Crypto-Assets (MiCA)*. Official Journal of the European Union, L 150/52. 2023.