Lesson 24: Factor Models

Data Science with Python – BSc Course

Data Science Program

BSc Course

45 Minutes

Previously on L21–L23...

L21 – Linear Regression:

- Fit a line through data; beta = sensitivity to one variable

L22 – Regularization:

- Ridge and Lasso prevent overfitting when predictors multiply

L23 – Regression Metrics:

- MSE, R^2, and walk-forward validation measure model quality

Now we apply ALL of this to the biggest question in finance:

"What drives stock returns?"

Module 4 arc: one variable → many variables → measuring quality → factor models

Learning Objectives

The Problem: CAPM uses only market beta, but stocks also respond to size, value, and momentum. How do we capture multiple sources of systematic risk?

After this lesson, you will be able to:

- Build multi-factor regression models
- Understand Fama-French factors (SMB, HML)
- Interpret factor loadings and alpha
- Create complete ML pipelines with sklearn

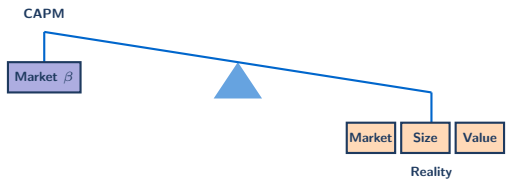
Finance application: decomposing returns into systematic factors for attribution

CAPM's Promise and Problem

CAPM says one number – beta – explains expected returns.
It won a Nobel Prize. But it's **wrong**.

The anomalies CAPM cannot explain:

- Small-cap stocks outperform large-caps (size effect)
- Cheap "value" stocks outperform expensive "growth" stocks



Nobel Prize 1990 (Sharpe). But Fama & French proved CAPM incomplete in 1993

Excess Returns: The Foundation

Risk-Free Rate (R_f):

- Return with ZERO risk (3-month U.S. Treasury bill, $\sim$4–5%)
- Excess Return = Asset Return $- R_f$
- "How much extra did you earn **for taking risk**?"

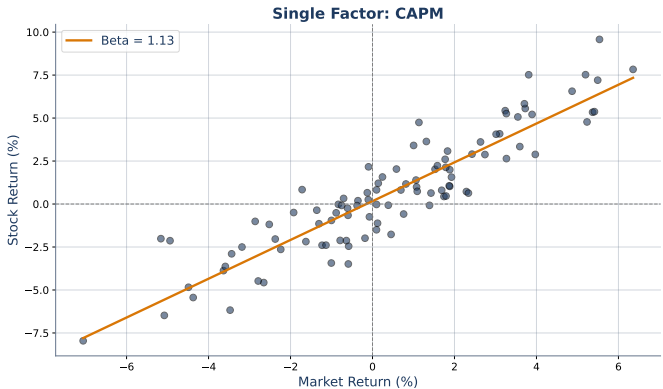
Why excess returns matter:

- Earning 5% when T-bills pay 4% is less impressive than...
- ...earning 5% when T-bills pay 1%
- Excess returns adjust for the "time value of money"

Always subtract R_f when comparing risky investments

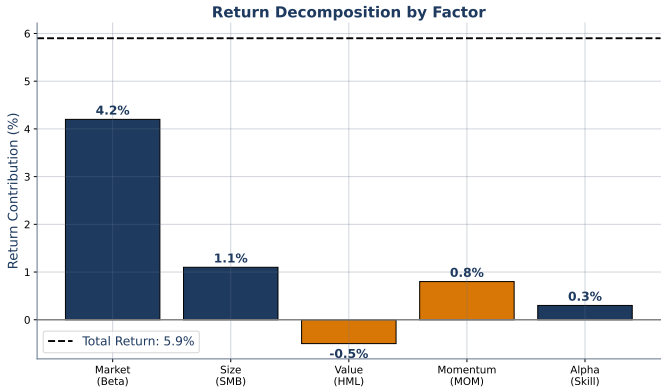
CAPM: One Factor

Market factor alone explains ∼60% of returns.
"What about the other 40%?"



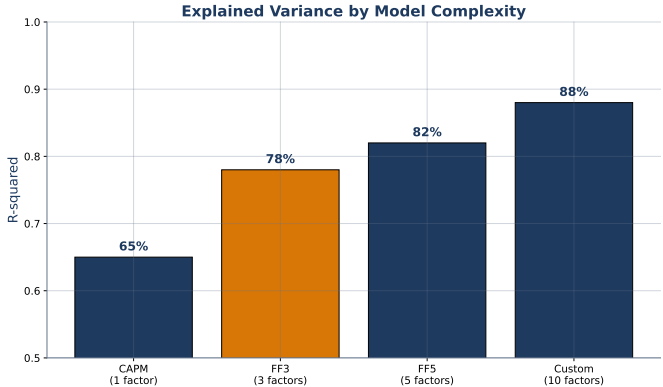
CAPM: $R_i - R_f = \alpha + \beta(R_m - R_f) + \varepsilon$

Return Decomposition



Multi-factor models split returns into factor contributions + alpha

Factor Concept: Explained Variance



More factors explain more variance – but beware overfitting

Enter Fama and French (1993)

Eugene Fama & Kenneth French – Nobel Prize 2013

Their insight: Two additional factors explain the anomalies.

What they did:

- Collected decades of US stock return data
- Sorted stocks into portfolios by **size** and **book-to-market**
- Showed CAPM systematically misprices small and value stocks
- Proposed the **3-Factor Model**: Market + SMB + HML

$$R_i - R_f = \alpha + \beta_1(R_m - R_f) + \beta_2 \cdot \text{SMB} + \beta_3 \cdot \text{HML} + \varepsilon$$

Fama & French (1993): "Common Risk Factors in the Returns on Stocks and Bonds"

SMB: Small Minus Big

SMB = Small Minus Big (Size Factor)

- Construction: return of small-cap − return of large-cap stocks
- Size premium: small stocks outperform (riskier, less diversified)
- Positive SMB loading = stock behaves like small caps

Why does the size premium exist?

- **Risk:** Small firms are more volatile, less liquid
- **Behavioral:** Investors overlook small stocks
- Premium has shrunk since discovery (1981) – markets adapted

---

SMB = Small Minus Big. Positive loading = small-cap-like behavior

HML: High Minus Low

HML = High Minus Low (Value Factor)

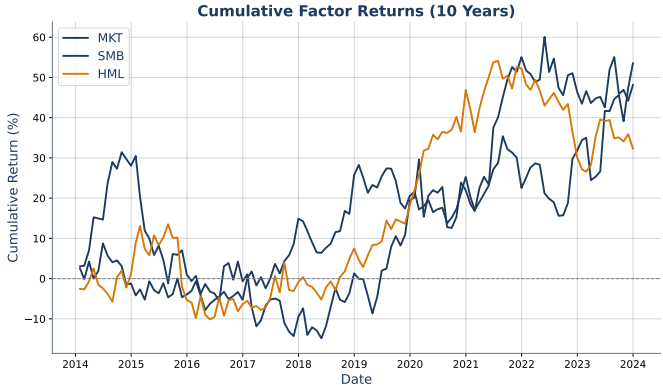
- Construction: return of high book-to-market − low book-to-market
- Positive HML = value-like behavior (cheap stocks)
- Negative HML = growth-like behavior (expensive stocks)

Why does the value premium exist?

- **Risk:** Value stocks are distressed, may face bankruptcy
- **Behavioral:** Investors overpay for "glamour" growth stocks
- Value struggled 2010–2020 but historically strong

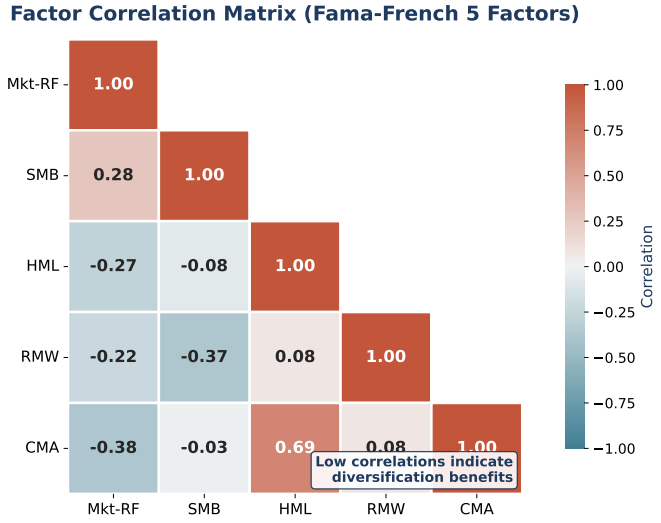
HML = High Minus Low (book-to-market). Value stocks have positive HML loading

Factor Returns Over Time



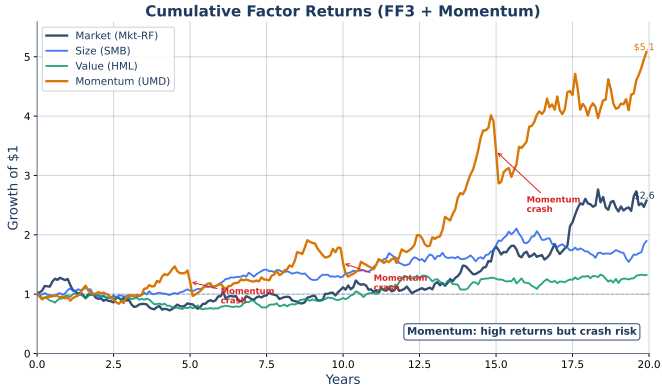
All three premia positive long-term, but with periods of underperformance

Factor Correlation Matrix



Low factor correlations mean each factor captures distinct risk

Beyond FF3: Five Factors and Momentum

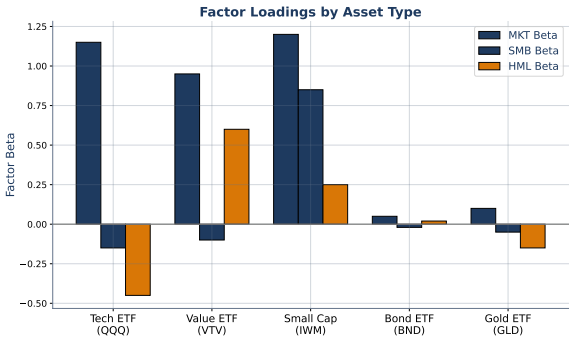


Fama-French 5-Factor model adds profitability (RMW) and investment (CMA)

Factor Loadings: Every Stock Has a Fingerprint

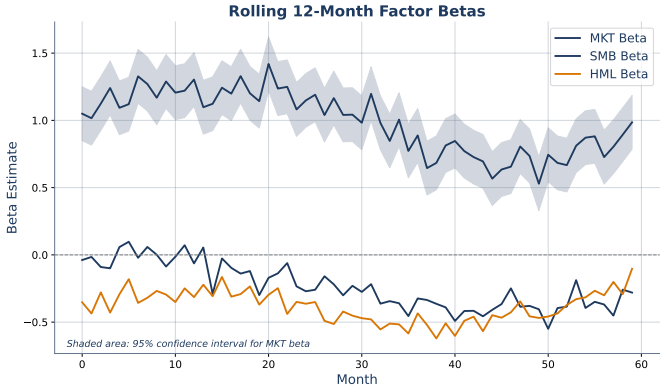
Different stocks have different factor exposures:

- Tech: high β, negative SMB (large), negative HML (growth)
- Small bank: moderate β, positive SMB, positive HML (value)



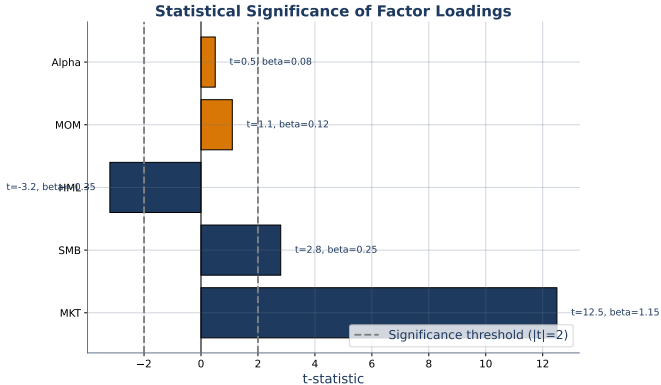
Factor loadings = the stock's "fingerprint" of systematic risk exposures

Factor Loadings Change Over Time



Rolling windows reveal that factor exposures shift as companies evolve

Factor Loadings: Statistical Significance

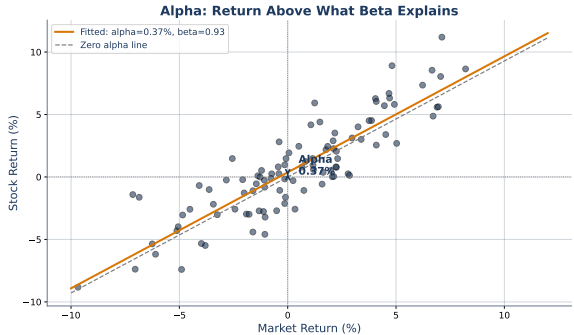


Check t-stats: is the factor loading significantly different from zero?

Alpha: The Holy Grail

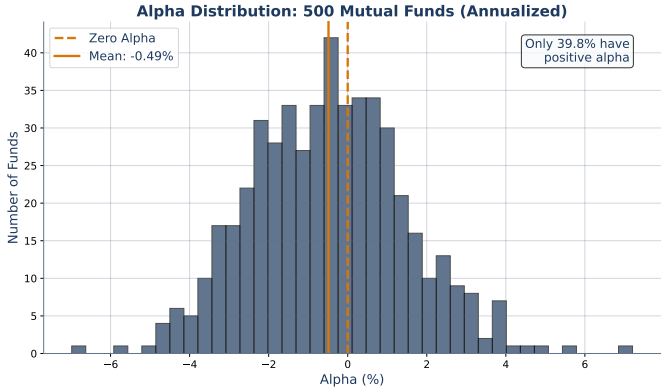
Alpha = return unexplained by factors = "skill" (or luck)

- Most alphas cluster near zero after controlling for factors
- True persistent alpha is extremely rare



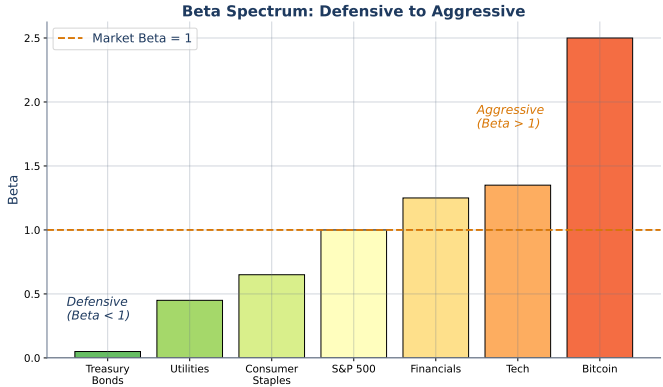
"In search of alpha" – the central quest of active management

Alpha: Distribution Across Assets



Most alphas cluster around zero; few stocks consistently outperform

Beta: Spectrum Across Assets



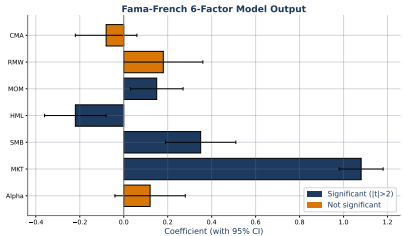
Market beta varies: defensive stocks < 1, aggressive stocks > 1

Multi-Factor Regression in Python

Code:

- `X = factors[['Mkt-RF','SMB','HML']]`
- `y = stock_returns - rf` # excess returns!
- `model = LinearRegression().fit(X, y)`

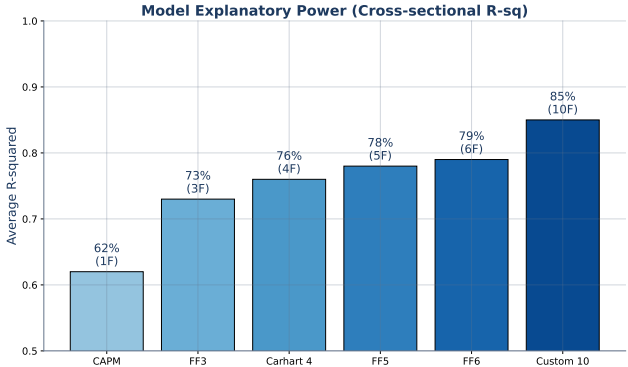
Interpreting: each coefficient = effect **holding others constant**.
β_{Mkt}=1.2, SMB=0.3, HML=−0.1: aggressive, small-cap tilt, growth tilt.



Alpha = intercept = return unexplained by factors ("skill")

Model Comparison: More Factors = Better?

R^2: CAPM ∼60% vs FF3 ∼75%. But check **adjusted** R^2 – diminishing returns from adding factors.



Adjusted R^2 penalizes model complexity – more factors is not always better

Statistical Significance Primer

What is a t-statistic? (Review L15)

- t-stat = coefficient / standard error
- Measures: "How many standard errors is the coefficient from zero?"
- Large $|t|$ = strong evidence the coefficient is nonzero

Decision Rule:

- $|t| > 1.96 \rightarrow$ significant at 5% ($p < 0.05$)
- $|t| > 2.58 \rightarrow$ significant at 1% ($p < 0.01$)
- Rule of thumb: $|t| > 2$ means "probably significant"

For Factor Models:

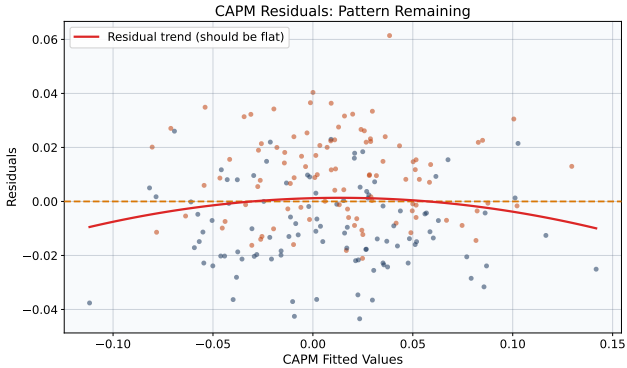
- Non-significant loading? Stock doesn't respond to that factor
- Example: HML t-stat = 0.5 means value factor irrelevant for this stock

---

t-stat > 2 (or < -2) indicates statistically significant factor exposure

CAPM Residuals: The Evidence

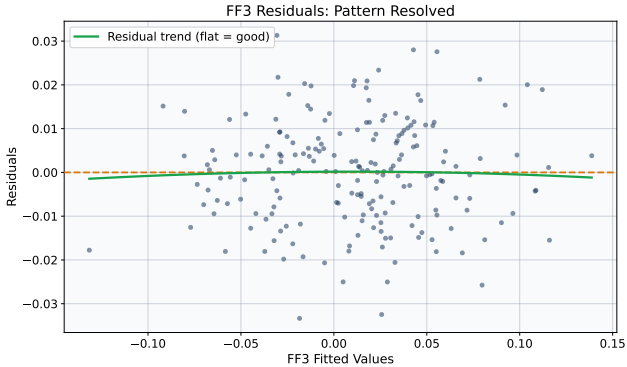
CAPM residuals show a systematic pattern:
Small/value stocks consistently under- or over-predicted.



Pattern in residuals = model misspecification. CAPM misses size and value effects

FF3 Residuals: Pattern Resolved

Adding SMB and HML captures what CAPM missed.
Residuals now look random – this is a well-specified model.



Flat residual trend = model captures the relevant risk factors

Checkpoint: Reading Factor Loadings

AAPL has β_{Mkt}=1.2, SMB=$-$0.3, HML=$-$0.4

Quiz yourself:

1. Is Apple value or growth?
   $\rightarrow$ Growth (negative HML = growth-like behavior)
2. Large-cap or small-cap behavior?
   $\rightarrow$ Large-cap (negative SMB = large-cap-like)
3. Aggressive or defensive?
   $\rightarrow$ Aggressive ($\beta_{Mkt} > 1$, amplifies market moves)

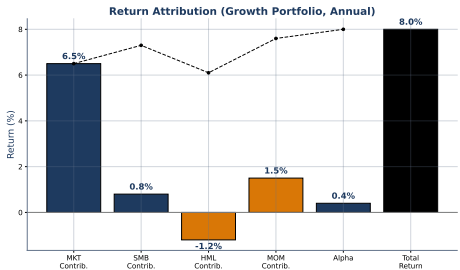
Summary: Apple is a large-cap growth stock with above-average market sensitivity.

Factor loadings tell you *what kind* of risk a stock carries

Portfolio Attribution: Worked Example

Given: α=0.2%, β_{Mkt}=1.1, β_{SMB}=0.3, β_{HML}=−0.2. **Month:** Mkt-RF=2%, SMB=1%, HML=−0.5%.

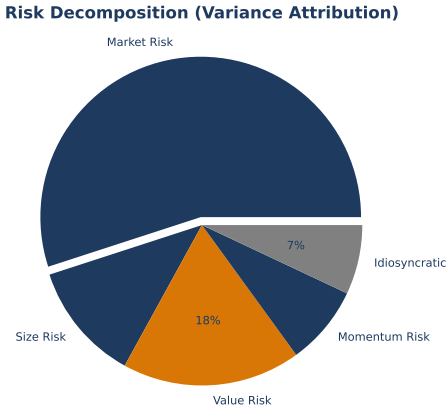
Attribution: Market $1.1 \times 2\% = 2.2\%$ + Size 0.3% + Value 0.1% + Alpha 0.2% = **2.8%**.
Most return (2.2%) came from market exposure, not skill.



Attribution: Return $= \alpha + \sum \beta_j \times F_j$

Risk Decomposition

Factor risk vs idiosyncratic risk: where does volatility come from?



Factor risk contribution shows where volatility comes from

Pipelines: Preventing Data Leakage

The problem:

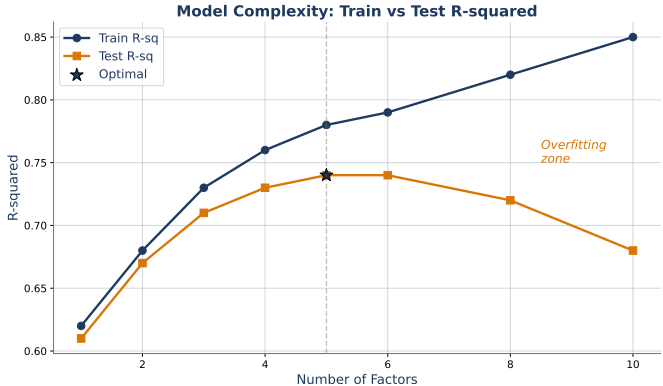
- Preprocessing (scaling, imputation) must fit ONLY on training data
- If you scale on ALL data, then split → test info leaks into training

Pipeline bundles preprocessing + model:

- `Pipeline([('scaler', StandardScaler()),`
- `          ('model', LinearRegression())])`
- Cross-validation respects this automatically

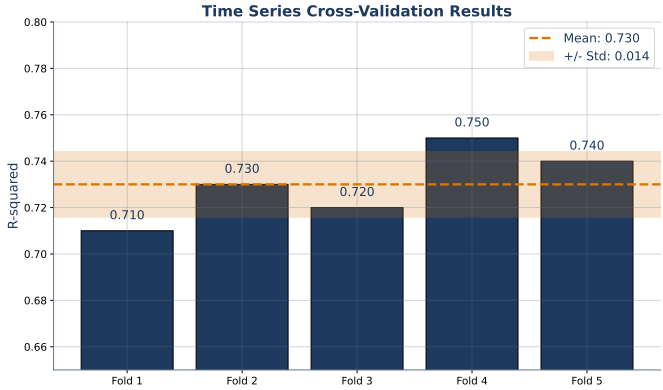
Pipeline = preprocessing + model in one object. Prevents data leakage

Pipeline: Train-Test Complexity



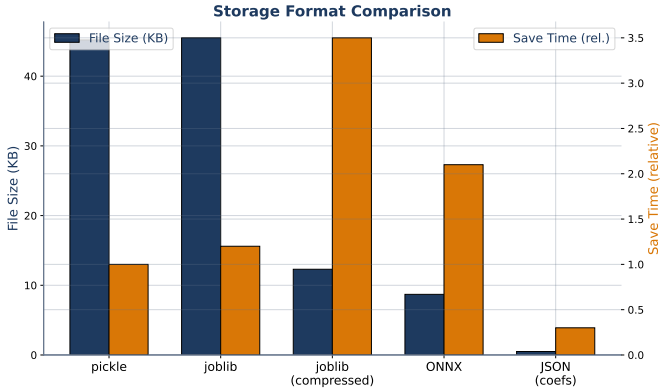
Pipeline ensures preprocessing fits only on training data

Pipeline: Cross-Validation Results



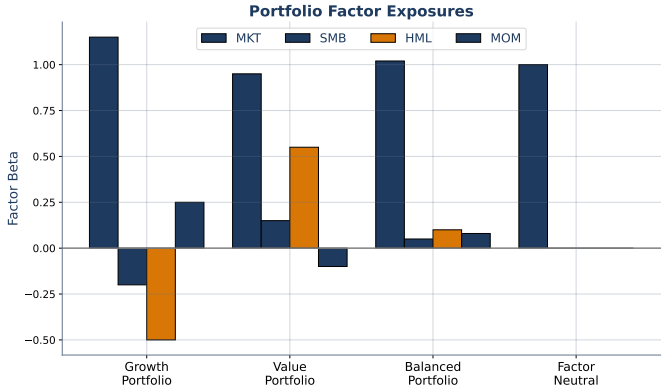
CV with pipeline prevents data leakage in scaling/encoding

Model Persistence: Storage Formats



joblib recommended for sklearn; pickle for general Python objects

Portfolio: Factor Exposures



Portfolio-level factor betas = weighted average of asset betas

Common Mistakes in Factor Models

- ✗ Using **raw returns** instead of excess returns ($R_i - R_f$)
- ✗ Not checking **statistical significance** of factor loadings
- ✗ Assuming factor loadings are **constant over time**
- ✗ Confusing alpha with **luck** – need a statistical test!

Avoid these pitfalls when building and interpreting factor models

Hands-On Exercise (25 min)

Task: Build a Fama-French 3-Factor Model

1. Download FF3 data from Kenneth French's website
2. Merge with your stock returns (AAPL or similar)
3. Fit multi-factor regression: stock excess returns vs [Mkt-RF, SMB, HML]
4. Interpret: What are the factor loadings? Is there significant alpha?
5. Compare R^2 to single-factor CAPM model

Deliverable: Table of factor loadings + R^2 comparison.

Extension: Add momentum (UMD) as a 4th factor – does R^2 improve?

Reference: `inclass/L24_inclass_factor_attribution.ipynb`

Use `pandas_datareader` or download CSV from Kenneth French Data Library

Module Wrap-Up: The Regression Journey

Module 4 arc (L21–L24):

- **L21:** One variable → best-fit line, slope = beta
- **L22:** Many variables → regularization prevents overfitting
- **L23:** Measuring quality → MSE, R^2, walk-forward
- **L24:** Multi-factor finance models → decompose risk and return

"From Galton's sweet peas to Fama-French factor models –
150 years of the best-fit line."

Next module preview:
What if Y is not a number but a **category**?
Welcome to **Classification** (L25–L28).

Module 4 complete. Next: L25 Logistic Regression – predicting categories