Point Estimation

Lesson 10

Digital Finance

Outline

What is Point Estimation?

Goal: Use sample data to estimate population parameters

Notation:

- θ: Unknown population parameter
- $\hat{\theta}$: Estimator (function of sample data)

Examples:

- μ estimated by $\bar{X}$
- σ^2 estimated by S^2
- p (proportion) estimated by $\hat{p}$

An estimator is a rule for computing an estimate from data.

Properties of Estimators

Think of darts:

- **Bias** = accuracy: Do you hit the bullseye on average?
- **Variance** = precision: Are your throws clustered together?

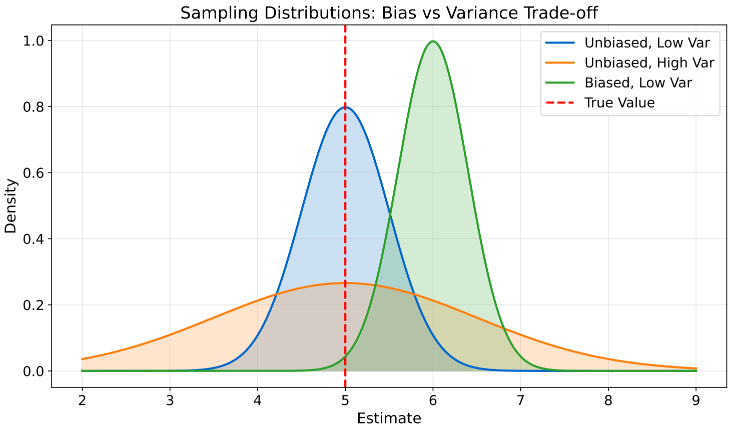
Unbiased: On average, your estimate equals the true value.
Low variance: Your estimates don't jump around much.
MSE (Mean Squared Error): Combines both

$$\text{MSE} = \text{Variance} + \text{Bias}^2$$

Trade-off: Sometimes a slightly biased estimator with much lower variance has smaller total error!

Best estimator: low bias AND low variance. MSE measures total error.

Bias-Variance Trade-off



Sometimes a biased estimator with low variance is preferable.

Method of Moments

Idea: Match sample moments to population moments

Steps:

1. Express parameters in terms of moments
2. Replace population moments with sample moments
3. Solve for parameter estimates

Example: For normal distribution

- $\hat{\mu} = \bar{X}$ (first moment)
- $\hat{\sigma}^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$ (biased; S^2 with $1/(n-1)$ is unbiased)

Simple and intuitive but not always optimal.

Maximum Likelihood Estimation

Core idea: What parameter value makes my observed data most likely?
Example: Flip a coin 10 times, get 7 heads.

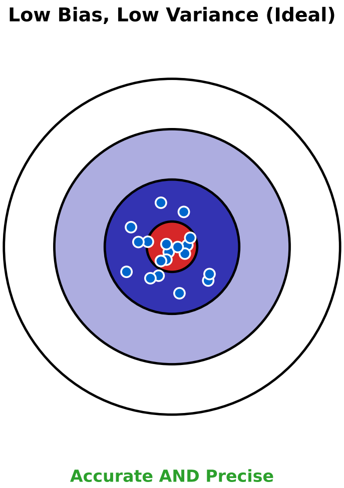
- If p = 0.5, probability of 7 heads is low
- If p = 0.7, probability of 7 heads is higher
- MLE says: estimate $\hat{p} = 0.7$ (the value that maximizes likelihood)

Why MLE is popular:

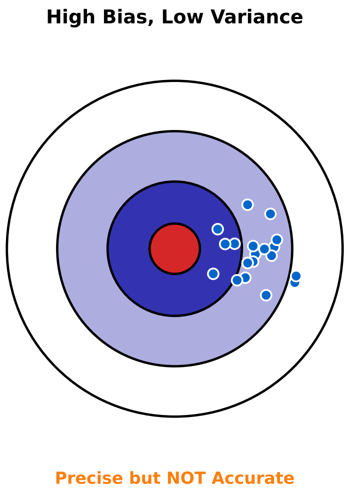
- For large samples: approximately unbiased
- For large samples: smallest possible variance among consistent estimators
- Computer software can find MLE automatically

MLE = "find the parameter that best explains the data."

Low Bias, Low Variance (Ideal)

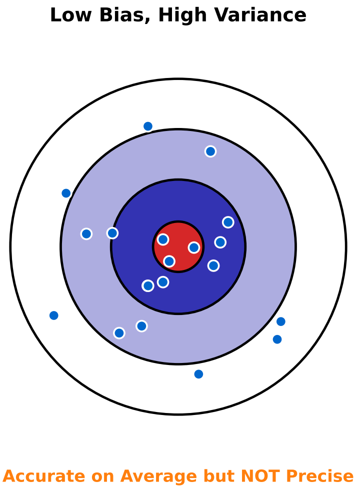


The ideal estimator: accurate AND precise.



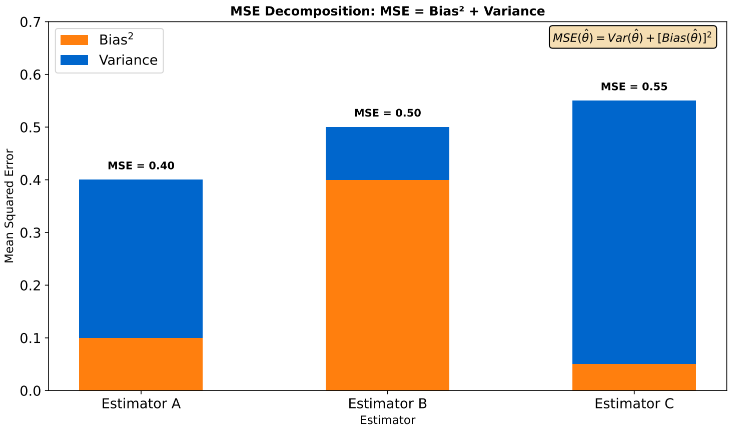
Consistently wrong: precise but systematically off-target.

Low Bias, High Variance



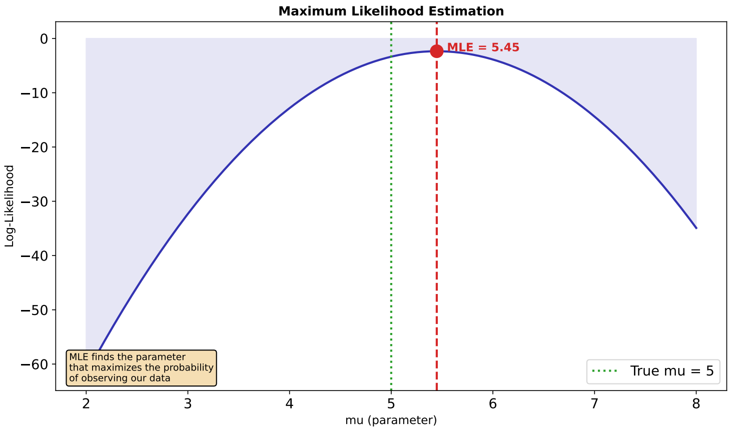
Right on average but imprecise: individual estimates vary widely.

MSE = Variance + Bias Squared



Total error decomposes into two components.

Maximum Likelihood Estimation



Find the parameter that maximizes data probability.

Summary

Estimators:

- Rules for computing estimates from data
- Evaluated by bias, variance, MSE

Method of Moments:

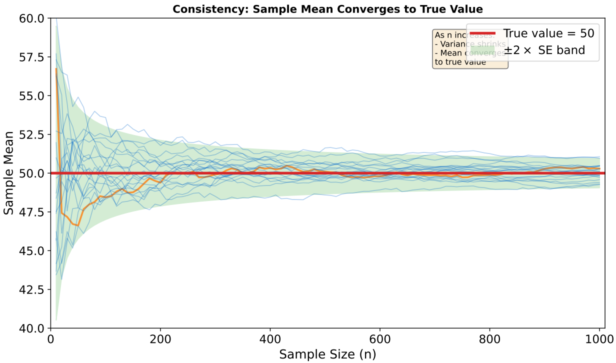
- Match sample and population moments
- Simple but not always efficient

Maximum Likelihood:

- Find parameter that maximizes probability of data
- Optimal large-sample properties

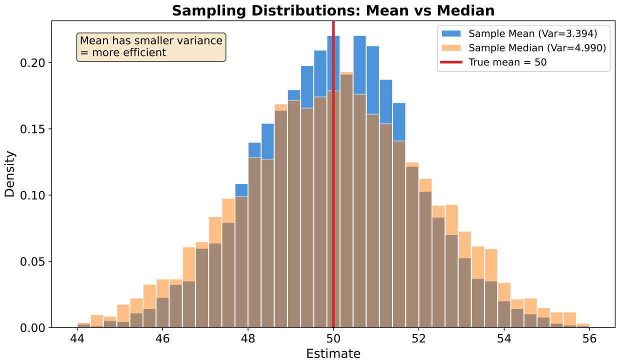
Next lesson: Interval Estimation

Consistency: Convergence to True Value



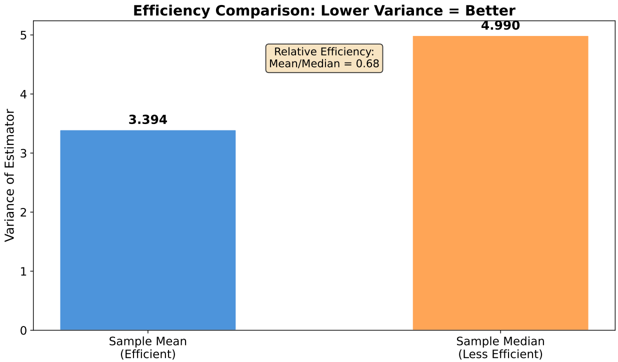
Consistent estimators converge to the true parameter as n grows.

Efficiency: Sampling Distributions



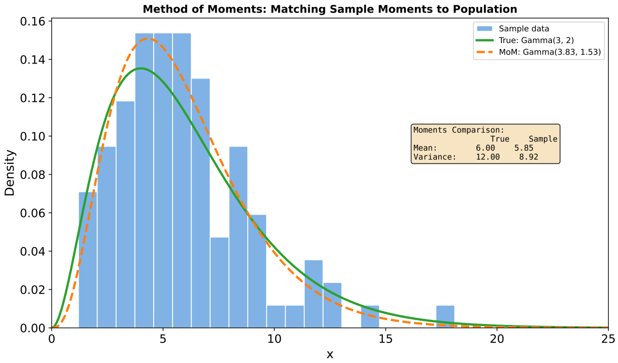
Mean has smaller variance than median for normal data.

Efficiency: Variance Comparison



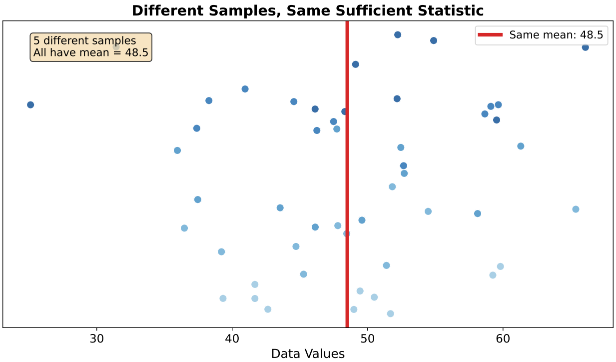
More efficient = smaller variance = more precise estimates.

Method of Moments Estimation



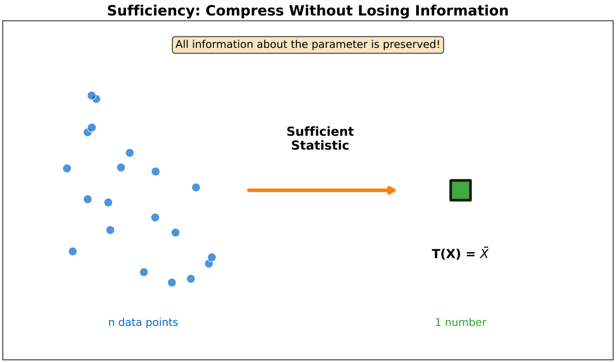
Match sample moments to population moments to estimate parameters.

Sufficiency: Same Statistic, Different Data



Different raw data can produce the same sufficient statistic.

Sufficiency: Data Reduction



Sufficient statistic compresses data without losing information about parameter.