

Expectation and Moments – Quiz

Probability & Statistics

Question 1

The expected value $E[X]$ represents:

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- B. The probability-weighted average
- C. The maximum value
- D. The variance

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Answer: B

Expected value is the probability-weighted average of all possible values. It represents the long-run average if the experiment is repeated many times.

Question 2

If $E[X] = 5$ and $E[Y] = 3$, what is $E[X + Y]$?

- A. 15
- B. 8
- C. 2
- D. Cannot determine without knowing dependence

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Answer: B

$E[X + Y] = E[X] + E[Y] = 5 + 3 = 8$. This linearity holds even when X and Y are dependent!

Question 3

For $E[aX + b]$, the result is:

- A. $aE[X]$
- B. $E[X] + b$
- C. $aE[X] + b$
- D. $a + bE[X]$

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Answer: C

Linearity of expectation: $E[aX + b] = aE[X] + b$. Constants scale and shift the expected value.

Question 4

If X and Y are independent, then $E[XY]$ equals:

- A. $E[X] + E[Y]$
- B. $E[X] \times E[Y]$
- C. $E[X] - E[Y]$
- D. $E[X^2] \times E[Y^2]$

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Answer: B

For independent random variables, $E[XY] = E[X] \times E[Y]$. This only holds for independent variables.

Question 5

Variance measures:

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- B. The spread or dispersion around the mean
- C. The probability of success
- D. The correlation between variables

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Answer: B

Variance $\text{Var}(X) = E[(X - E[X])^2]$ measures how spread out the values are around the mean.

Question 6

The formula $\text{Var}(X) = E[X^2] - (E[X])^2$ is called:

- A. The difference formula
- B. The computational formula for variance
- C. The central limit theorem
- D. Bayes theorem

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- B. The computational formula for variance
- C. The central limit theorem
- D. Bayes theorem

Answer: B

This computational formula is equivalent to $E[(X - \mu)^2]$ but is often easier to calculate.

Question 7

If $\text{Var}(X) = 16$, what is the standard deviation?

- A. 4
- B. 256
- C. 8
- D. 2

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Answer: A

Standard deviation = $\sqrt{\text{Variance}} = \sqrt{16} = 4$.

Question 8

Var(aX + b) equals:

- A. $a \times \text{Var}(X) + b$
- B. $a^2 \times \text{Var}(X)$
- C. $(a + b)^2 \times \text{Var}(X)$
- D. $\text{Var}(X)$

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- C. $(a + b)^2 \times \text{Var}(X)$
- D. $\text{Var}(X)$

Answer: B

Adding a constant doesn't change variance. Multiplying by a scales variance by a^2 : $\text{Var}(aX + b) = a^2 \text{Var}(X)$.

Question 9

Covariance $\text{Cov}(X, Y) \neq 0$ indicates:

- A. X and Y move in opposite directions
- B. X and Y tend to move in the same direction
- C. X and Y are independent
- D. Variance of X equals variance of Y

Question 9

Covariance $\text{Cov}(X, Y) \geq 0$ indicates:

- A. X and Y move in opposite directions
- B. X and Y tend to move in the same direction
- C. X and Y are independent
- D. Variance of X equals variance of Y

Answer: B

Positive covariance means when X is above its mean, Y tends to be above its mean too - they move together.

Question 10

The correlation coefficient is bounded by:

- A. 0 and 1
- B. -1 and +1
- C. 0 and infinity
- D. -infinity and +infinity

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Answer: B

Correlation $\rho = \text{Cov}(X,Y)/(\sigma_X \times \sigma_Y)$ is always between -1 and +1.

Question 11

If $\rho = +1$, the relationship between X and Y is:

- A. No relationship
- B. Perfect positive linear relationship
- C. Perfect negative linear relationship
- D. Nonlinear relationship

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Answer: B

$\rho = +1$ means $Y = a + bX$ for some positive b . All points lie exactly on an upward-sloping line.

Question 12

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ only when:

- A. X and Y have the same mean
- B. X and Y are independent (or $\text{Cov}(X, Y) = 0$)
- C. X and Y are normally distributed
- D. Always true

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- C. X and Y are normally distributed
- D. Always true

Answer: B

In general, $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X, Y)$. The simple sum only holds when $\text{Cov}(X, Y) = 0$.

Question 13

For portfolio diversification, we want assets with:

- A. High positive correlation
- B. Low or negative correlation
- C. The same expected return
- D. Identical variances

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Answer: B

Low or negative correlation reduces portfolio variance. When assets move in opposite directions, losses in one are offset by gains in another.

Question 14

The third central moment measures:

- A. Mean
- B. Variance
- C. Skewness
- D. Kurtosis

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- B. Variance
- C. Skewness
- D. Kurtosis

Answer: C

The first moment is mean, second is variance, third is skewness (asymmetry), fourth is kurtosis (tail weight).

Positive skewness indicates:

- A. Symmetric distribution
- B. Long right tail
- C. Long left tail
- D. Fat tails

Question 15

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- A. Symmetric distribution
- B. Long right tail
- C. Long left tail
- D. Fat tails

Answer: B

Positive skewness means the right tail is longer - there are more extreme high values than extreme low values.

Question 16

High kurtosis (relative to normal) indicates:

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- B. Fat tails (more extreme values)
- C. Perfect symmetry
- D. Zero variance

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- D. Zero variance

Answer: B

High kurtosis means fat tails - extreme events (both high and low) occur more frequently than in a normal distribution.

Question 17

The covariance between X and itself, $\text{Cov}(X, X)$, equals:

- A. 0
- B. 1
- C. $\text{Var}(X)$
- D. $E[X]$

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- A. 0
- B. 1
- C. $\text{Var}(X)$
- D. $E[X]$

Answer: C

$$\text{Cov}(X, X) = E[X \times X] - E[X] \times E[X] = E[X^2] - (E[X])^2 = \text{Var}(X).$$

Question 18

If $\text{Cov}(X, Y) = 0$, then X and Y are:

- A. Always independent
- B. Uncorrelated (but not necessarily independent)
- C. Always dependent
- D. Perfectly correlated

Question 18

If $\text{Cov}(X, Y) = 0$, then X and Y are:

- A. Always independent
- B. Uncorrelated (but not necessarily independent)
- C. Always dependent
- D. Perfectly correlated

Answer: B

Zero covariance means no LINEAR relationship, but X and Y could still be related nonlinearly. Independence implies zero correlation, but not vice versa.

Question 19

For a two-asset portfolio with weights w_1 and w_2 , the portfolio variance depends on:

- A. Only the individual variances
- B. Individual variances and the covariance between assets
- C. Only the expected returns
- D. Only the covariance

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- D. Only the covariance

Answer: B

Portfolio variance = $w_1^2 \text{Var}(X_1) + w_2^2 \text{Var}(X_2) + 2 w_1 w_2 \text{Cov}(X_1, X_2)$. Both variances and covariance matter.

Question 20

If $\text{Var}(X) = 0$, then:

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- B. X is a constant (non-random)
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Answer: B

If $\text{Var}(X) = 0$, there is no spread - X takes the same value with probability 1. X is essentially a constant.