Conditional Probability and Bayes – Quiz

Probability & Statistics

Question 1

What does P(A—B) represent?

A. Probability of A and B occurring together
B. Probability of A given that B has occurred
C. Probability of B given that A has occurred
D. Probability of A or B occurring

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C. Probability of B given that A has occurred
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Answer: B
P(A—B) is read as 'probability of A given B' - it represents the probability of A occurring given that we know B has occurred.

Question 2

The formula for conditional probability is:

A. P(A—B) = P(A) + P(B)
B. P(A—B) = P(A) x P(B)
C. P(A—B) = P(A and B) / P(B)
D. P(A—B) = P(A) / P(B)

Question 2

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Answer: C
P(A—B) = P(A and B) / P(B). We divide the joint probability by the probability of the conditioning event.

Question 3

Two events A and B are independent if:

A. P(A and B) = 0

B. P(A—B) = P(A)

C. P(A) + P(B) = 1

D. A and B are mutually exclusive

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D. A and B are mutually exclusive

Answer: B

Events are independent if knowing B occurred doesn't change the probability of A. Equivalently, P(A and B) = P(A) x P(B).

Question 4

If P(A) = 0.4, P(B) = 0.5, and A and B are independent, what is P(A and B)?

A. 0.9

B. 0.2

C. 0.1

D. 0.45

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B. 0.2
C. 0.1
D. 0.45

Answer: B

For independent events, P(A and B) = P(A) × P(B) = 0.4 × 0.5 = 0.2.

Question 5

In Bayes' theorem, what is the 'prior'?

A. The probability of evidence given hypothesis
B. The updated probability after seeing evidence
C. The initial probability before seeing evidence
D. The total probability of all outcomes

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Answer: C

The prior P(A) is what we believed about A before seeing any new evidence. It represents our initial belief.

Question 6

Bayes' theorem states that P(A—B) equals:

A. P(B—A) x P(A) / P(B)
B. P(A) x P(B) / P(A—B)
C. P(B) / P(A)
D. P(A and B) x P(B)

Question 6

Bayes' theorem states that P(A—B) equals:

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B. P(A) x P(B) / P(A—B)
C. P(B) / P(A)
D. P(A and B) x P(B)

Answer: A

Bayes' theorem: P(A—B) = P(B—A) x P(A) / P(B). It allows us to reverse the conditioning.

Question 7

A disease affects 1% of the population. A test has 95% sensitivity (detects disease when present) and 90% specificity (correctly identifies healthy). If you test positive, what is approximately the probability you have the disease?

A. 95%

B. 50%

C. About 9%

D. About 1%

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A. 95%

B. 50%

C. About 9%

D. About 1%

Answer: C

Using Bayes' theorem with a rare disease (1%), even a good test yields many false positives. About 9% of positive tests are true positives due to the low base rate.

Question 8

The 'base rate fallacy' refers to:

A. Ignoring conditional probabilities
B. Ignoring the prior probability when using Bayes' theorem
C. Confusing independence with dependence
D. Assuming all events are equally likely

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The 'base rate fallacy' refers to:

A. Ignoring conditional probabilities
B. Ignoring the prior probability when using Bayes' theorem
C. Confusing independence with dependence
D. Assuming all events are equally likely

Answer: B
The base rate fallacy is ignoring the prior (base rate) when evaluating probabilities. With rare events, this leads to overestimating probability after a positive test.

Question 9

The Law of Total Probability states that P(B) equals:

A. P(B—A) + P(B—A complement)

B. P(B—A) x P(A) + P(B—A complement) x P(A complement)

C. P(A and B) + P(A complement and B complement)

D. 1 - P(B complement)

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B. P(B—A) x P(A) + P(B—A complement) x P(A complement)
C. P(A and B) + P(A complement and B complement)
D. 1 - P(B complement)

Answer: B

Law of Total Probability: P(B) = P(B—A) x P(A) + P(B—A complement) x P(A complement). It breaks P(B) into weighted conditional probabilities.

Question 10

A portfolio has 60% low-risk assets (1% default) and 40% high-risk assets (5% default). What is the overall default probability?

A. 3%

B. 2.6%

C. 6%

D. 1.5%

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C. 6%

D. 1.5%

Answer: B

P(default) $= 0.01 \times 0.60 + 0.05 \times 0.40 = 0.006 + 0.020 = 0.026 = 2.6\%$. This is the weighted average.

Question 11

The multiplication rule states that P(A and B) equals:

A. P(A) + P(B)
B. P(A—B) x P(B)
C. P(A) / P(B)
D. P(A) - P(B)

Question 11

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D. P(A) - P(B)

Answer: B
P(A and B) = P(A—B) x P(B) = P(B—A) x P(A). This rearranges the conditional probability formula.

Question 12

If A and B are mutually exclusive, are they independent?

A. Yes, always
B. No, they are highly dependent
C. Only if $P(A) = P(B)$
D. It depends on the sample space

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Answer: B
Mutually exclusive events are highly dependent - if one occurs, the other cannot. P(A—B) = 0 which does not equal P(A) unless P(A) = 0.

Question 13

What is P(A—A)?

A. 0
B. 0.5
C. 1
D. Cannot be determined

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What is P(A—A)?

A. 0
B. 0.5
C. 1
D. Cannot be determined

Answer: C
P(A—A) = P(A and A) / P(A) = P(A) / P(A) = 1. If A has occurred, A has certainly occurred.

Question 14

In Bayesian terminology, what is the 'likelihood'?

A. P(hypothesis)

B. P(evidence — hypothesis)

C. P(hypothesis — evidence)

D. P(evidence)

Question 14

In Bayesian terminology, what is the 'likelihood'?

A. P(hypothesis)
B. P(evidence — hypothesis)
C. P(hypothesis — evidence)
D. P(evidence)

Answer: B
The likelihood P(evidence — hypothesis) measures how probable the evidence is if the hypothesis is true.

Question 15

What is the 'posterior' in Bayes' theorem?

A. The probability before seeing evidence

B. The probability of the evidence

C. The updated probability after seeing evidence

D. The likelihood ratio

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A. The probability before seeing evidence
B. The probability of the evidence
C. The updated probability after seeing evidence
D. The likelihood ratio

Answer: C
The posterior P(A—B) is our updated belief about A after incorporating the evidence B. Prior + Evidence = Posterior.

Question 16

When drawing two cards without replacement, P(2nd ace — 1st ace) equals:

A. 4/52

B. 3/51

C. 4/51

D. 3/52

Question 16

When drawing two cards without replacement, P(2nd ace — 1st ace) equals:

A. $4/52$
B. $3/51$
C. $4/51$
D. $3/52$

Answer: B
After drawing one ace, 3 aces remain among 51 cards. P(2nd ace — 1st ace) $= 3/51$.

Question 17

A and B are conditionally independent given C means:

A. P(A and B) = P(A) x P(B)
B. P(A and B — C) = P(A—C) x P(B—C)
C. P(A—C) = P(B—C)
D. P(C—A) = P(C—B)

Question 17

A and B are conditionally independent given C means:

A. P(A and B) = P(A) x P(B)
B. P(A and B — C) = P(A—C) x P(B—C)
C. P(A—C) = P(B—C)
D. P(C—A) = P(C—B)

Answer: B
Conditional independence given C means that within the context of C, A and B are independent: P(A and B — C) = P(A—C) x P(B—C).

Question 18

If P(A) = 0.3 and P(B—A) = 0.5, what is P(A and B)?

A. 0.8

B. 0.15

C. 0.2

D. 0.6

Question 18

If P(A) = 0.3 and P(B—A) = 0.5, what is P(A and B)?

A. 0.8
B. 0.15
C. 0.2
D. 0.6

Answer: B
Using the multiplication rule: P(A and B) = P(B—A) × P(A) = 0.5 × 0.3 = 0.15.

Question 19

The likelihood ratio is defined as:

A. P(evidence — hypothesis) / P(evidence — not hypothesis)
B. P(hypothesis) / P(not hypothesis)
C. P(hypothesis — evidence) / P(hypothesis)
D. P(evidence) / P(hypothesis)

Question 19

The likelihood ratio is defined as:

A. P(evidence — hypothesis) / P(evidence — not hypothesis)
B. P(hypothesis) / P(not hypothesis)
C. P(hypothesis — evidence) / P(hypothesis)
D. P(evidence) / P(hypothesis)

Answer: A
The likelihood ratio = P(evidence — H) / P(evidence — not H). A ratio ¿ 1 means the evidence supports the hypothesis.

Question 20

Why does the 'false positive paradox' occur with rare diseases?

A. Tests are always inaccurate
B. Healthy people vastly outnumber sick people, so false positives exceed true positives
C. Doctors make mistakes
D. The disease is contagious

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Answer: B
With rare diseases, the large number of healthy people (even with low false positive rate) produces more false positives than the small number of sick people produces true positives.