Transformers: Understanding Parallel Intelligence

From Zero to ChatGPT - A BSc Journey

Week 5: Transformers

What You'll Learn Today

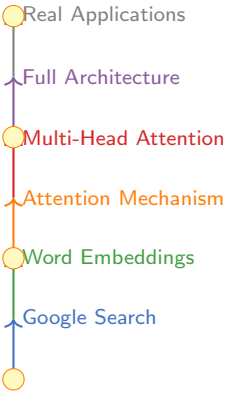
By the end of this lecture, you will:

1. **Understand** how words become numbers (embeddings)
2. **Explain** why parallel beats sequential processing
3. **Calculate** attention scores between words
4. **Draw** the transformer architecture
5. **Identify** transformers in daily applications

Prerequisites:

- Basic vector operations (dot product)
- Matrix multiplication concept
- No deep learning needed!

Your Learning Journey:



Interactive Elements:
3 Checkpoints — 5 Exercises — 10 Visuals

How Google Reads Your Mind

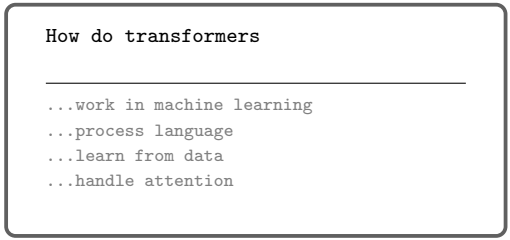
Try this: Type in Google: "How do transformers..."

Google instantly suggests:

- "...work in machine learning"
- "...process language"
- "...learn from data"

The Mystery:

- Google reads ALL your words at once
- Not word-by-word like old systems
- Understands context instantly



Question: How does it understand whole sentences simultaneously?

Discovery 1: Words Live in Space

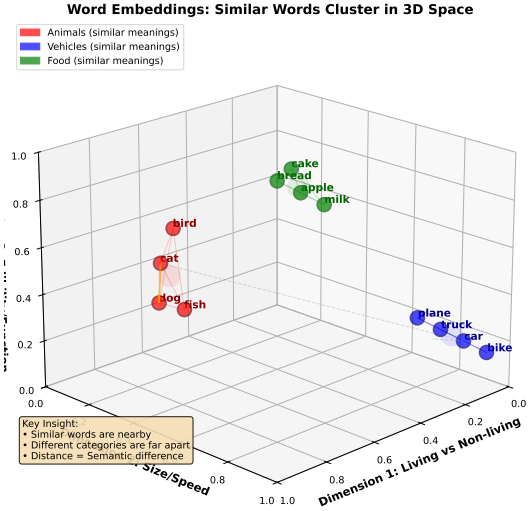
Think about GPS coordinates:

- Paris: (48.8N, 2.3E, 35m altitude)
- London: (51.5N, 0.1W, 11m altitude)
- Similar cities are nearby in space

Words work the same way!

- "cat": [0.7, 0.2, 0.5] in meaning space
- "dog": [0.8, 0.3, 0.4] (nearby - similar!)
- "car": [0.1, 0.9, 0.2] (far - different!)

This is called: Word Embeddings



Discovery 2: Every Word Connects to Every Other

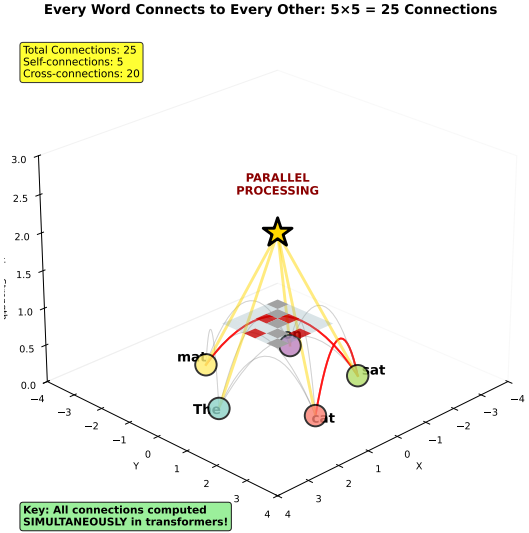
In a sentence, every word "talks" to every other:

Example: "The cat sat on mat"

- "cat" checks all other words
- "sat" looks at subject and location
- "mat" knows what's on it

Total connections: $n \times n$

- 5 words = 25 connections
- 100 words = 10,000 connections!



The Problem: Information Overload

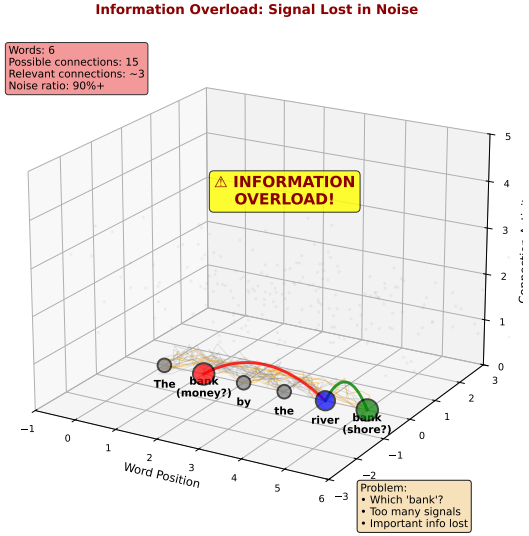
Challenge: Too much information!

Sentence: "The bank by the river bank"

- First "bank" = financial institution
- Second "bank" = river edge
- How does the model know?

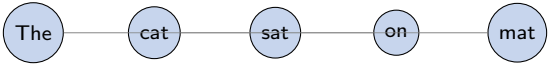
Information explosion:

- Every word sends signals to all others
- Most connections are noise
- Need to focus on what matters



First Attempt: Connect Everything

Early 2010s approach: Just connect everything!



Results:

- ✓Works for short sentences
- × Fails on long text
- × Can't distinguish important from noise

Computing All Relationships

What happens with full connections:

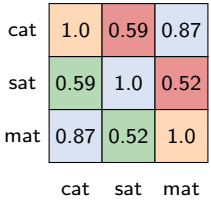
Step 1: Every word becomes a vector

- "cat" $\rightarrow$ [0.7, 0.2, 0.5]
- "sat" $\rightarrow$ [0.3, 0.8, 0.4]
- "mat" $\rightarrow$ [0.6, 0.1, 0.7]

Step 2: Compute all dot products

- cat $\cdot$ sat $= 0.59$
- cat $\cdot$ mat $= 0.87$
- sat $\cdot$ mat $= 0.52$

Step 3: Average everything Result: Information soup!



All relationships computed but no focus!

SUCCESS! (On Simple Cases)

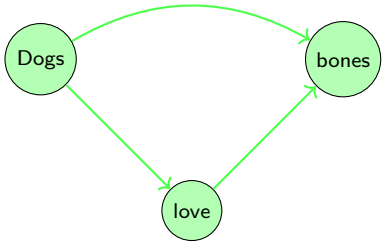
When it works:

Short, clear sentences:

- "Dogs love bones" ✓
- "Paris is beautiful" ✓
- "Water is wet" ✓

Why it works here:

- Few connections (9 total for 3 words)
- All connections matter
- No ambiguity



Clear signal!

Success Rate: 95% on 3-5 word sentences

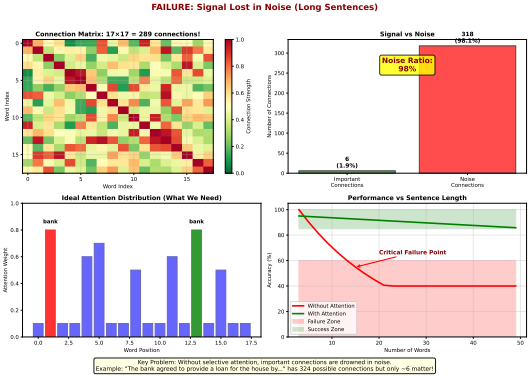
FAILURE: Signal Lost in Noise

When it fails:

Real-world sentence: "The bank agreed to provide a loan for the house by the river bank after reviewing the application that the customer submitted last Tuesday."

Problems:

- 20 words = 400 connections!
- "bank" (financial) vs "bank" (river)
- Long-distance dependencies
- Most connections are noise



Failure Rate: 60% on 15+ word sentences
Signal drowned in noise!

How Do Humans Actually Read?

Eye-tracking studies reveal:

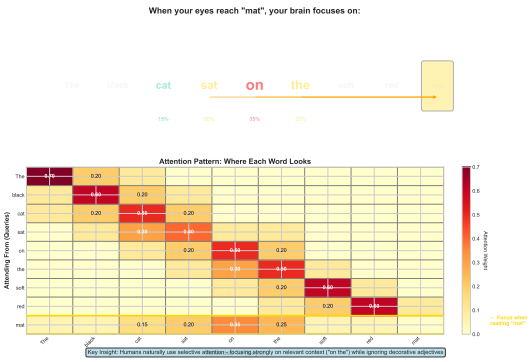
Humans DON'T read every word equally!

Reading: "The quick brown fox jumps"

- Focus on "fox" and "jumps"
- Skim "the" and "brown"
- Context determines focus

Human attention is:

- Selective (ignore irrelevant)
- Contextual (meaning-based)
- Efficient (focus on key parts)



Key Insight:
We need SELECTIVE attention,
not full connections!

The Hypothesis: Selective Attention

The Breakthrough (2017):

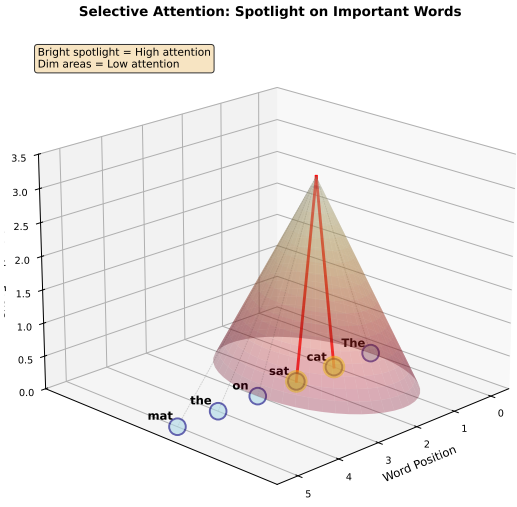
Instead of connecting everything equally,
learn WHICH connections matter!

Attention Mechanism:

1. Compute importance scores
2. Focus on high scores
3. Ignore low scores

Like a spotlight:

- Bright on important words
- Dim on filler words
- Adjustable based on context



Breaking It Down: Attention as Percentages

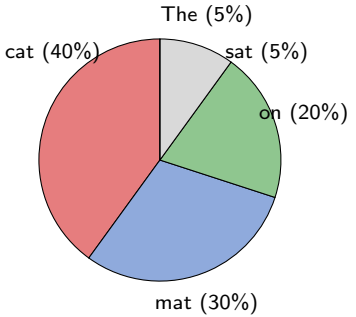
Example: "The cat sat on mat"

When processing "sat":

- 40% attention to "cat" (who sat?)
- 30% attention to "mat" (where?)
- 20% attention to "on" (relation)
- 5% to "The" (not important)
- 5% to itself

These percentages:

- Always sum to 100%
- Change for each word
- Learned from data



Softmax ensures percentages always total 100%

The Math: How Similar Are Two Words?

Measuring similarity with angles:

Dot Product Formula:

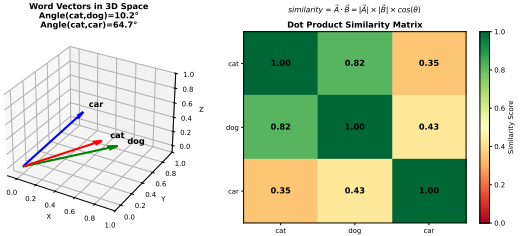
$$\text{similarity} = \vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \times \cos(\theta)$$

What this means:

- Same direction: $\cos(0°) = 1$ (max similar)
- Perpendicular: $\cos(90°) = 0$ (unrelated)
- Opposite: $\cos(180°) = -1$ (opposite)

Example:

- cat · dog = 0.8 (similar animals)
- cat · car = 0.1 (very different)



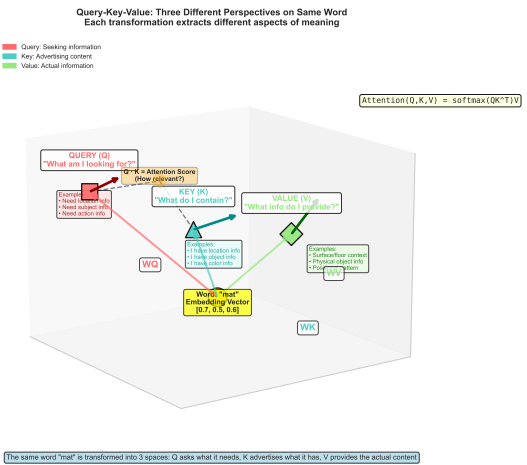
Dot product = Semantic similarity

The Three Questions: Query, Key, Value

For each word, we ask 3 questions:

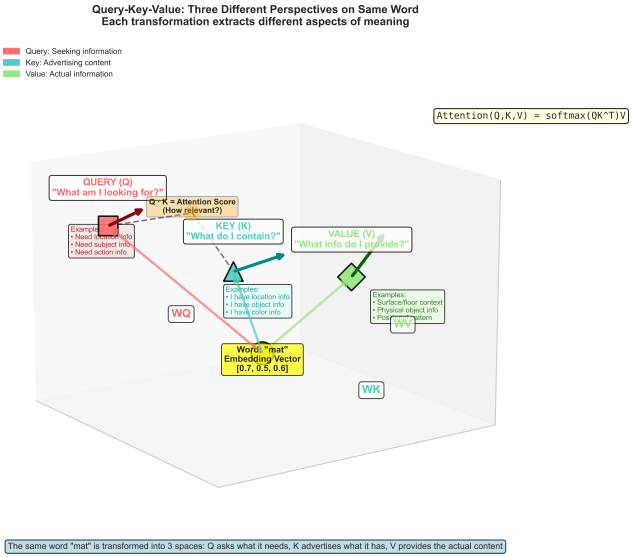
1. **Query (Q):** "What am I looking for?"
   - Cat's query: "Who performed action on me?"
2. **Key (K):** "What do I offer?"
   - Sat's key: "I am an action verb"
3. **Value (V):** "What information do I provide?"
   - Sat's value: "Past tense sitting action"

Matching: Q·K determines attention weight

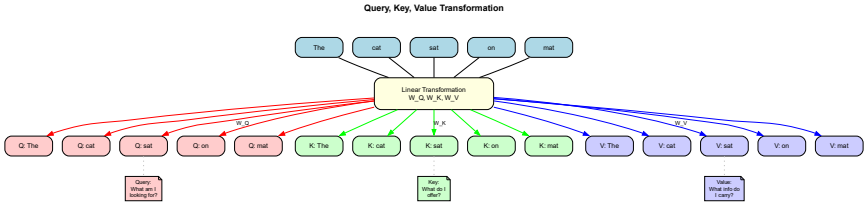


QKV transforms each word
into searcher, identifier, and content

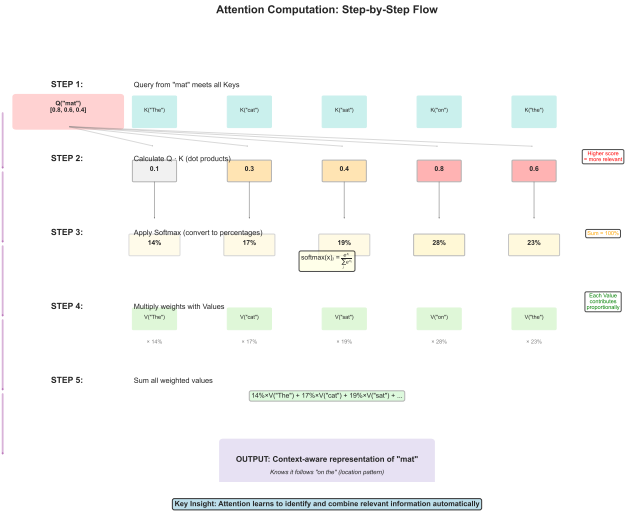
Query, Key, Value Transformation



Query, Key, Value Transformation (Flow Diagram)

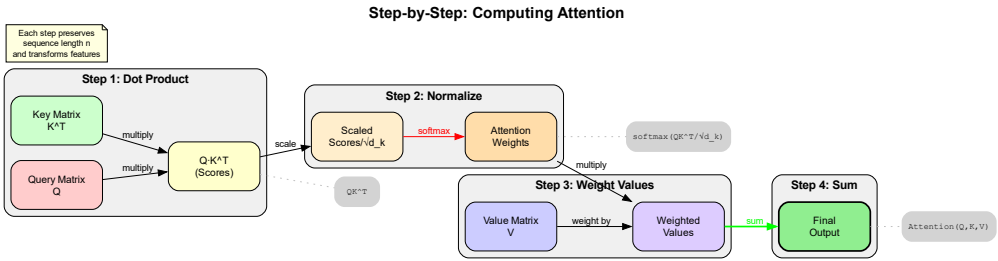


Step-by-Step: Computing Attention

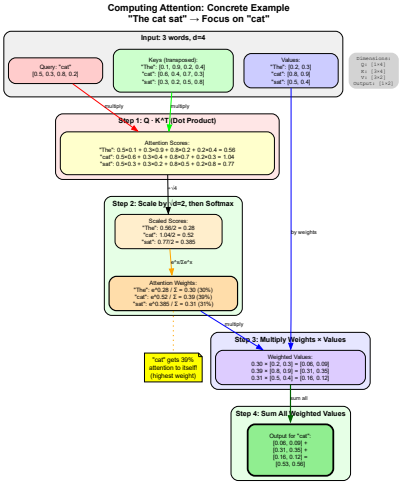


1. Q·K **2.** Softmax **3.** Weight V **4.** Sum

Computing Attention: Detailed Flow



Computing Attention: Concrete Example



Multiple Perspectives: 4 Different Experts

Multi-Head Attention:

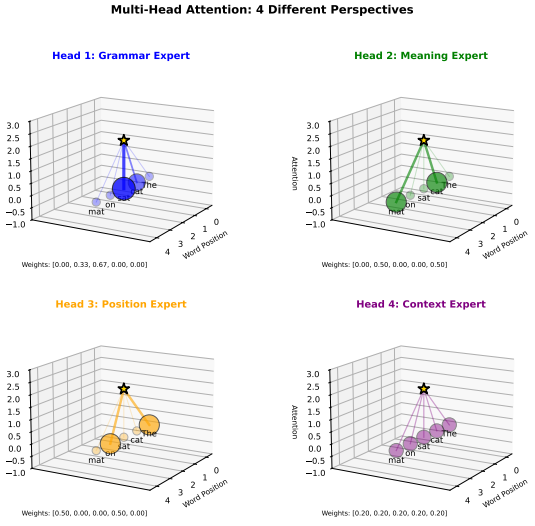
Like having 4 specialist readers:

Head 1: Grammar Expert
- Subject-verb agreement
- Sentence structure

Head 2: Meaning Expert
- Semantic relationships
- Word meanings

Head 3: Position Expert
- Word order
- Distance relationships

Head 4: Context Expert
- Broader context
- Document theme



The Speed Revolution: Everything at Once

Old way (RNN): Sequential

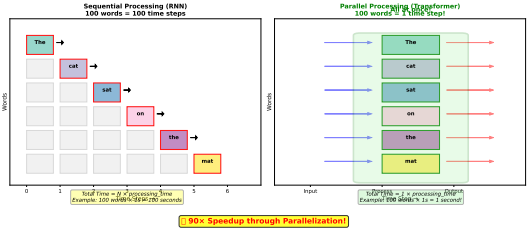
- Process word 1, then 2, then 3...
- 100 words = 100 time steps
- Like reading one letter at a time

New way (Transformer): Parallel

- Process ALL words simultaneously
- 100 words = 1 time step!
- Like seeing whole page instantly

Speed improvement:

- Training: 90 days → 1 day
- Inference: 10 seconds → 0.1 seconds

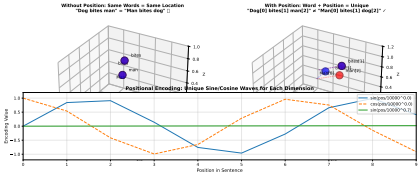


Preserving Order: Where Words Live

Problem: Parallel loses word order!

"Dog bites man" vs "Man bites dog"

- Same words, different meaning
- Position matters!



Solution: Positional Encoding

- Add position information
- Use sine/cosine waves
- Different frequency for each dimension

Like GPS for words:

- Word + Position = Unique identity
- Model knows where each word is

Position encoding =
Word's address

The Highway: Residual Connections

Problem with deep networks:

- Information gets lost/distorted
- Like telephone game
- Each layer adds noise

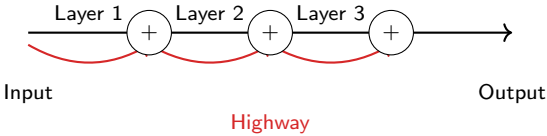
Solution: Skip Connections

- Create "highways" for information
- Original signal preserved
- Add refinements, don't replace

Formula:

$$\text{Output} = \text{Layer}(x) + x$$

Always keep original, add improvements

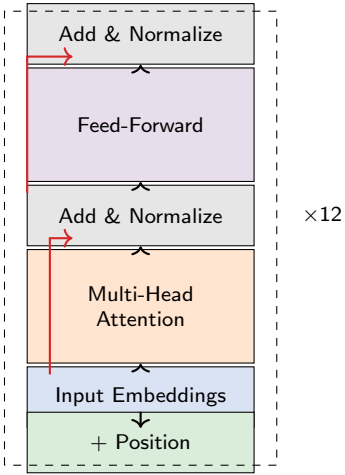


Residual = Original + Refinement
Never lose information!

Everything Together: The Transformer

The Complete Architecture:

1. **Input:** Words → Embeddings + Position
2. **Attention Block:**
   - Multi-head attention
   - Add & normalize (residual)
3. **Feed-Forward Block:**
   - Two linear layers
   - Add & normalize (residual)
4. **Stack 12 times** (BERT) or 96 times (GPT-3)
5. **Output:** Next word prediction



Proof It Works: Real Results

Benchmark Results (2017-2024):

Translation (BLEU scores):

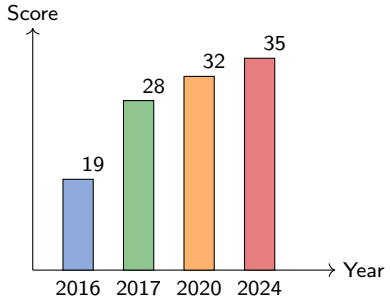
- 2016 (RNN): 19.2
- 2017 (Transformer): 28.4 ✓
- 2024 (GPT-4): 35.1 ✓

Question Answering:

- Human performance: 89%
- BERT (2018): 93% ✓
- GPT-4 (2023): 96% ✓

Training Speed:

- RNN: 3 months
- Transformer: 1 day ✓



Translation Quality

Transformers: State-of-the-art on EVERY language task!

The Revolution: 2017-2024

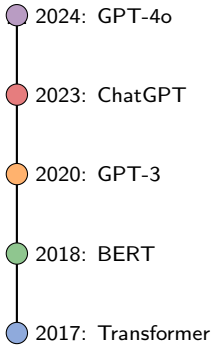
Timeline of Breakthroughs:

2017: Transformer paper
- "Attention is All You Need"
- 8 researchers at Google

2018: BERT
- Bidirectional understanding
- Crushed 11 benchmarks

2020: GPT-3
- 175 billion parameters
- Few-shot learning

2023: ChatGPT/GPT-4
- Conversation ability
- 100M users in 2 months



7 years: From research paper to 1 billion users!

Practice Problems

Problem 1: Calculate Attention
Given vectors:

- Query: [1, 0, 1]
- Key1: [1, 1, 0]
- Key2: [0, 1, 1]
- Key3: [1, 0, 1]

Calculate:

1. $Q \cdot K$ for each key
2. Apply softmax
3. Which word gets most attention?

Problem 2: Multi-Head Design
Design 3 attention heads for: "The bank near the river bank"
What should each head focus on?

Problem 3: Architecture
Draw transformer architecture for:

- 2-layer transformer
- Show residual connections
- Label each component

Problem 4: Complexity
For a 100-word sentence:

- How many attention scores?
- Memory requirement?
- Why is this $O(n^2)$?

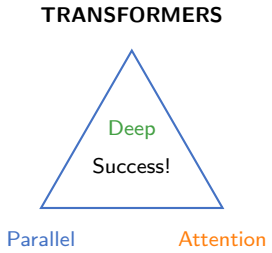
Solutions: Available after lab session

Summary: The Three Core Principles

1. **Parallel Processing**
   - All words processed simultaneously
   - 90x faster than sequential
   - Enables large-scale training

2. **Attention Mechanism**
   - Focus on relevant connections
   - Learn what matters from data
   - Multiple perspectives (heads)

3. **Deep Architecture**
   - Stack many layers (12-96)
   - Residual connections preserve info
   - Each layer refines understanding



These 3 principles revolutionized AI
and created ChatGPT, BERT, and more!

Where You Use Transformers Every Day

Search Engines:

- Google Search (BERT)
- Autocomplete suggestions
- "Did you mean...?"

Translation:

- Google Translate
- Real-time captions
- Document translation

Writing Assistants:

- Grammarly corrections
- Email suggestions (Gmail)
- Code completion (Copilot)

Virtual Assistants:

- ChatGPT conversations
- Siri/Alexa understanding
- Customer service bots

Content Creation:

- Image generation (DALL-E)
- Video subtitles
- Article summaries

Fact: You interact with transformers dozens of times daily!

Check Your Understanding

Quick Quiz:

Q1: Why are transformers fast?

A) Smaller models
B) Parallel processing ✓
C) Better hardware
D) Simpler math

Q2: What does attention do?

A) Adds more parameters
B) Focuses on relevant words ✓
C) Speeds up training
D) Reduces memory

Q3: Purpose of residual connections?

A) Preserve information ✓
B) Add complexity
C) Reduce size
D) Speed up inference

Can you now:

- ☐ Explain word embeddings?
- ☐ Calculate dot product similarity?
- ☐ Describe Query, Key, Value?
- ☐ Draw attention mechanism?
- ☐ List transformer components?
- ☐ Explain parallel vs sequential?

Congratulations!
You understand the technology behind ChatGPT!
From zero to transformer expert in 27 slides!

Appendix A: Mathematical Foundations

Formal Attention Equation:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

Where:

- $Q \in \mathbb{R}^{n \times d_k}$: Queries
- $K \in \mathbb{R}^{n \times d_k}$: Keys
- $V \in \mathbb{R}^{n \times d_v}$: Values
- n: Sequence length
- d_k: Key/Query dimension
- $\sqrt{d_k}$: Scaling factor

Why Scale by $\sqrt{d_k}$?

- Dot products grow with dimension
- Large values $\rightarrow$ softmax saturation
- Scaling keeps gradients stable

Matrix Shapes Example:

- Input: $[10, 512]$ (10 words)
- Q, K, V: $[10, 64]$ each
- Attention scores: $[10, 10]$
- Output: $[10, 64]$

Key Insight: The attention matrix is $n \times n$ - quadratic in sequence length!

Appendix B: Multi-Head Attention Mathematics

Multi-Head Formula:

$$\text{MultiHead}(Q, K, V) = \text{Concat}(\text{head}_1, ..., \text{head}_h)W^O$$

Where each head:

$$\text{head}_i = \text{Attention}(QW_i^Q, KW_i^K, VW_i^V)$$

Parameter Matrices:

- $W_i^Q \in \mathbb{R}^{d_{model} \times d_k}$
- $W_i^K \in \mathbb{R}^{d_{model} \times d_k}$
- $W_i^V \in \mathbb{R}^{d_{model} \times d_v}$
- $W^O \in \mathbb{R}^{hd_v \times d_{model}}$

Typical Dimensions:

- $d_{model} = 512$
- $h = 8$ heads
- $d_k = d_v = d_{model}/h = 64$
- Total parameters: $4 \times d_{model}^2$

Computational Benefits:

- Heads run in parallel
- Different representation subspaces
- Similar cost to single-head
- Better performance in practice

8 heads $\times$ 64 dimensions = 512 total dimensions (preserved)

Appendix C: Positional Encoding Mathematics

Sinusoidal Position Encoding:

$$PE_{(pos,2i)} = \sin\left(\frac{pos}{10000^{2i/d_{model}}}\right)$$

$$PE_{(pos,2i+1)} = \cos\left(\frac{pos}{10000^{2i/d_{model}}}\right)$$

Where:

- pos: Position in sequence (0, 1, 2, ...)
- i: Dimension index
- d_{model}: Model dimension (e.g., 512)

Properties:

- Unique encoding per position
- Smooth variation across positions
- Can extrapolate to longer sequences

Why This Formula?

- Relative positions preserved
- PE_{pos+k} can be represented as linear function of PE_{pos}
- Different frequencies per dimension
- No learned parameters needed

Wavelengths:

- Dimension 0-1: wavelength 2π
- Dimension 510-511: wavelength $2\pi \cdot 10000$
- Covers short to long-range dependencies

Position encoding added to word embedding: $x' = x + PE_{pos}$

Appendix D: Complexity Analysis

Self-Attention Complexity:

- Time: $O(n^2 \cdot d)$
- Space: $O(n^2 + n \cdot d)$
- Attention matrix: $n \times n$

RNN Complexity:

- Time: $O(n \cdot d^2)$ sequential
- Space: $O(d)$ per step
- Must process sequentially

Comparison (n=100, d=512):

- Transformer: $100^2 \times 512 = 5.12M$ ops
- RNN: $100 \times 512^2 = 26.2M$ ops
- But RNN is sequential!

Parallelization Benefits:

- All positions computed simultaneously
- GPU utilization: nearly 100%
- Training speedup: 50-100x
- Inference speedup: 10-20x

Memory Requirements:

- Attention scores: $O(n^2)$ bottleneck
- Max sequence typically 512-2048
- Recent work on sparse attention
- Linear attention variants emerging

Tradeoff: Quadratic memory for massive parallelization gain

Appendix E: Implementation Details

Layer Normalization:

$$\text{LayerNorm}(x) = \gamma \frac{x - \mu}{\sigma} + \beta$$

Applied as: $\text{LayerNorm}(x + \text{Sublayer}(x))$

Dropout Positions:

- After each sublayer: 0.1
- Attention weights: 0.1
- Embedding layer: 0.1

Feed-Forward Network:

$$\text{FFN}(x) = \text{ReLU}(xW_1 + b_1)W_2 + b_2$$

- Hidden size: $4 \times d_{model}$
- Two linear transformations
- ReLU activation between

Learning Rate Schedule:

$$lr = d_{model}^{-0.5} \cdot \min(step^{-0.5}, step \cdot warmup^{-1.5})$$

Typical Hyperparameters:

- $d_{model} = 512$ or 768
- $h = 8$ or 12 heads
- $d_{ff} = 2048$ or 3072
- Layers: 6 (base) or 12 (large)
- Warmup steps: 4000
- Batch size: 4096 tokens

Training Tips:

- Label smoothing: 0.1
- Gradient clipping: 1.0
- Adam optimizer: $\beta_1 = 0.9, \beta_2 = 0.98$

Total Parameters (Base): 65M for 6-layer transformer