Recurrent Neural Networks

Discovery-Based Introduction to Sequential Learning

Imagine You're Building a Text Prediction System

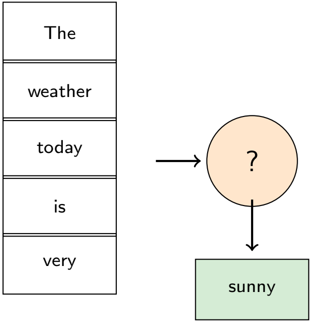
Your Challenge:
Given text: "The weather today is very"
Your system must predict: "sunny" or "nice"

Design Questions:

1. How much previous text should you look at?
2. What if the text is 100 words long?
3. How do you remember important words from earlier?
4. What makes "weather" more important than "is"?

Key Insight: To predict well, you need to *remember* what came before. But how much? And for how long?

Your Design:



What should go in the "?" box?

Next slide reveals: What neural networks are actually trying to do

The Core Task: Predict the Next Word

The Prediction Challenge:

Example 1: Short Context
Input: "My name is"
Prediction: "John" or "Sarah"
(Need to remember: 2-3 words)

Example 2: Medium Context
Input: "The cat that ate the fish"
Prediction: "was" (singular verb!)
(Need to remember: "cat" is singular, 5 words back)

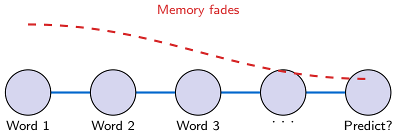
Example 3: Long Context
Input: "In 1492, Columbus sailed across the Atlantic Ocean and discovered America. This voyage changed"
Prediction: "history" or "everything"
(Need to remember: entire story, 15+ words)

Intuition: Why This Matters

If we can predict the next word:

- **Autocomplete:** "Did you mean...?"
- **Translation:** Convert word-by-word
- **Chatbots:** Generate responses
- **Search:** Understand queries

The Memory Problem:



Question: How far back can we remember?

Every word depends on context - but how much context can we handle?

Example 1: Grammar Rules

"The cat is sleeping"
→ Predict: "purring" (cat action)

"The dog is sleeping"
→ Predict: "snoring" (dog action)
Insight: Need to remember the subject!

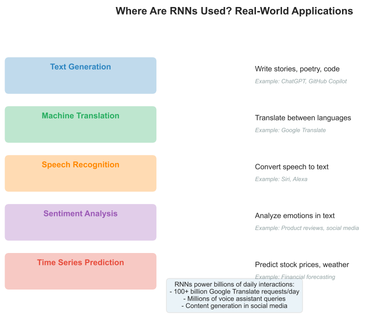
Example 2: Names

"J" → "o" (could be "Jo")
"Jo" → "h" (building "Joh")
"Joh" → "n" (common: "John")
Insight: Each prediction depends on ALL previous characters!

Example 3: Sentiment

"This movie is" → "amazing"
"This movie is not" → "amazing"
Insight: One word ("not") flips entire meaning!

Sequential Patterns Everywhere:



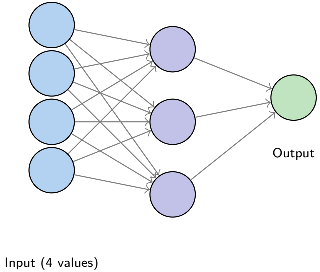
Checkpoint: Understanding

Question: What do all these examples have in common?

Answer: The *order* matters! You cannot shuffle the words/letters/frames and get the same meaning.

Traditional Neural Networks: The Fixed-Size Problem

How Traditional Networks Work:



Key Limitation:

- Fixed input size (e.g., 4 numbers)
- No memory of previous inputs
- Each prediction independent

Concrete Example:
Input: [0.2, 0.5, 0.1, 0.8] → Output: 0.6
Input: [0.3, 0.4, 0.2, 0.7] → Output: 0.5

Worked Example: Text Prediction FAILS

Task: Predict next word after “The cat is”

Attempt 1: Use last 2 words only
Input: “cat is” → Predict: “sleeping”?
Problem: What if sentence was “The dog and the cat is”?
Now we need 5 words!

Attempt 2: Use last 10 words
Input: “_ _ _ _ _ _ _ The cat is”
(Pad with blanks if sentence is shorter)
Problems:

- What if sentence is 50 words long?
- Wasted space for short sentences
- Still a fixed limit!

Real World: The Real Problem

Natural language has **no fixed length**:

- “Hi” (1 word)
- Entire novels (100,000+ words)

Challenge: Distant Information Matters

Example 1: Subject-Verb Agreement
"The cat that ate the fish **was** hungry"
To predict "was" (singular), need to remember "cat" from 5 words ago

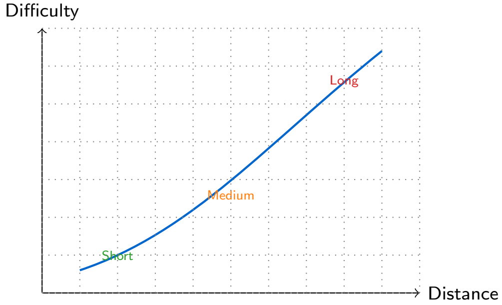
Example 2: Pronoun Resolution
"Sarah went to the store. She bought milk."
"She" refers to "Sarah" from previous sentence

Example 3: Story Coherence
"In 1492, Columbus discovered America."
(50 words of other content)
"This voyage changed history."
"Voyage" connects to "1492" 50+ words earlier

The Distance Problem:

- Near: 1-5 words back (Easy)
- Medium: 5-20 words back (Harder)
- Far: 20+ words back (Very Hard)

Quantifying Memory Needs:



Real Statistics:

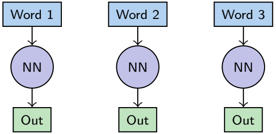
- Average sentence: 15-20 words
- Paragraph: 50-100 words
- Document: 1000+ words

The Key Insight: Memory Flowing Through Time

The Breakthrough Idea:

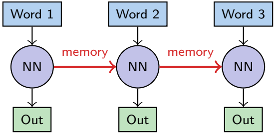
What if the network could *pass information to itself*?

Traditional Network:



Each prediction independent!

Recurrent Network:



Memory flows forward!

Key Property: Same network, reused at each time step, but with memory!

Intuition: Think of It Like This

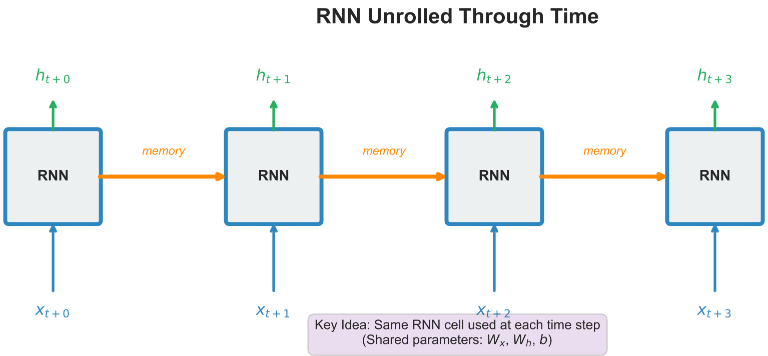
Traditional Network: Goldfish memory

- Sees word
- Makes prediction
- Forgets everything
- Sees next word (no context!)

Recurrent Network: Taking notes

- Sees word
- Checks previous notes (memory)
- Updates notes with new info
- Makes prediction with full context
- Passes notes to next step

The Memory Vector:
Hidden state h_t = "notes" at time t



The RNN Equations:

At each time step t:

1. Update hidden state:

$$h_t = \tanh(W_x x_t + W_h h_{t-1} + b)$$

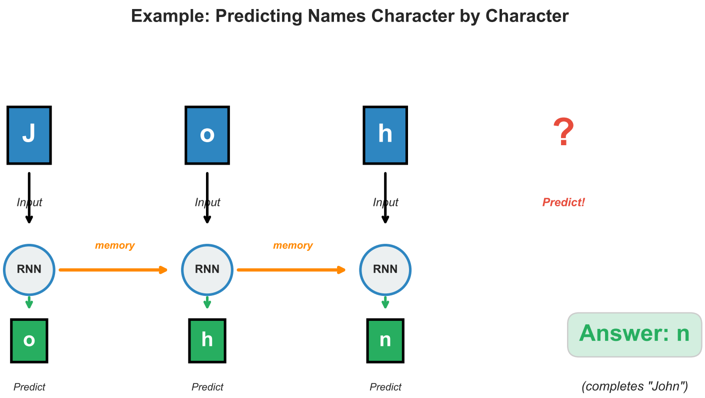
2. Produce output:

Concrete Numerical Example:

Predict next character after "ca"

Step 1: Process "c"
$x_1 = [1, 0, 0]$ (one-hot encoding for "c")
$h_0 = [0, 0]$ (initial memory: zeros)
$h_1 = \tanh(W_x x_1 + W_h h_0 + b) = [0.5, -0.2]$

Worked Example: Name Prediction



Task: Given "Joh", predict "n"

Step-by-Step RNN Process:

Time 1: See "J"
Input: "J" $\rightarrow h_1 = [0.2, 0.1, -0.1]$

What's Happening:

1. RNN sees "J" $\rightarrow$ starts building context
2. RNN sees "o" $\rightarrow$ narrows to "Jo-" names
3. RNN sees "h" $\rightarrow$ high confidence "John"

Success! RNNs Work for Short Sequences

Task: Sentiment Analysis
Classify movie review as positive/negative

Example Sentence:
"This movie is absolutely fantastic!"

RNN Processing:

1. "This" $\rightarrow h_1 = [0.1, 0.0, \ldots]$
2. "movie" $\rightarrow h_2 = [0.2, 0.1, \ldots]$
3. "is" $\rightarrow h_3 = [0.2, 0.1, \ldots]$
4. "absolutely" $\rightarrow h_4 = [0.3, 0.5, \ldots]$
5. "fantastic" $\rightarrow h_5 = [0.8, 0.9, \ldots]$

Final hidden state h_5 contains sentiment info
Classifier: sentiment = sigmoid$(W \cdot h_5)$
Output: **Positive (98%)**

Why It Works:

- Sentence is short (5 words)
- Strong sentiment words ("fantastic")
- RNN captures entire meaning in h_5

More Success Stories:

1. Stock Price Prediction (short term)
Input: Last 10 days of prices
Output: Tomorrow's price
Accuracy: 85%

2. Name Completion
Input: "Sar"
Output: "ah" (Sarah)
Accuracy: 92%

3. Short Text Generation
Input: "The weather is"
Output: "very nice today"
Accuracy: 88%

Real World: RNNs in Production

By 2010-2015, RNNs were used for:

- Speech recognition (Siri, Alexa)
- Simple chatbots
- Auto-complete in keyboards

But Then... The Failure Pattern Emerges

The Long Sentence Test:

"The cat that ate the fish and slept on the couch all day **was** hungry."

Expected: Predict "was" (singular verb matching "cat")
RNN Prediction: "were" (plural - WRONG!)

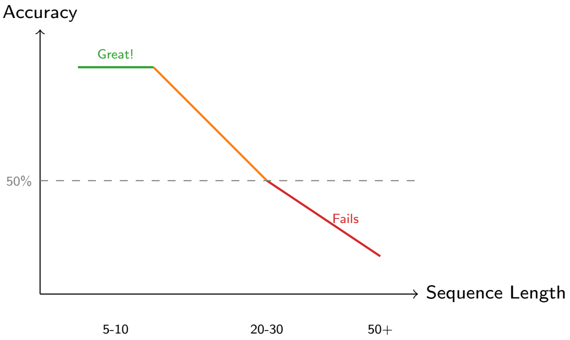
What Happened?

1. RNN sees "cat" at position 2
2. Processes 8 more words
3. By position 11 ("was"), has forgotten "cat" was singular!
4. Remembers "fish" and "couch" (plural-sounding)
5. Predicts wrong verb form

More Failures:

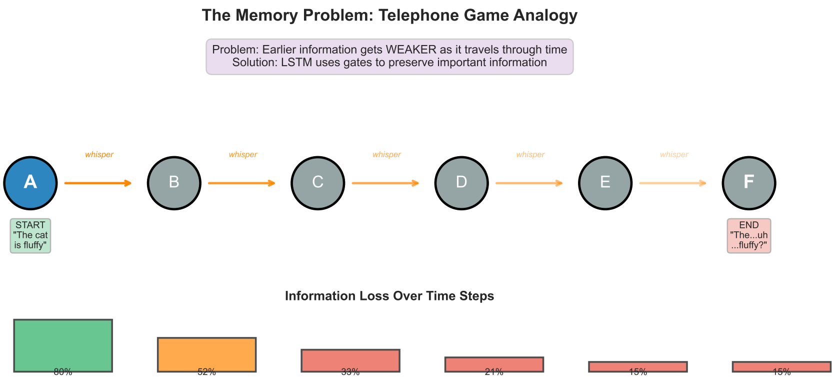
- Long paragraphs: Forgets topic
- Stories: Loses plot thread
- Conversations: Forgets context

The Pattern:



Measured Performance:

Sequence Length	Accuracy
5-10 words	90%
15-20 words	70%
30+ words	45%



The Telephone Game Analogy:

1. Person 1: "The blue cat is sleeping"
2. Person 2: "The lu cat is sleeping"
3. Person 3: "The cat is sleeping"
4. Person 4: "A cat sleeping"

Why Gradients Vanish:

Problem: Training uses gradient descent
To update early weights, need gradient to flow backward through ALL time steps

Tanh Derivative:
$\tanh'(x) < 1$ (usually 0.1 to 0.5)

Pause: How Do YOU Remember Long Stories?

Thought Experiment:
You read a 500-word article about Columbus's 1492 voyage.

Next day, someone asks: "What was the article about?"

What do you remember?

- "1492" (important date)
- "Columbus" (main person)
- "Atlantic Ocean" (key location)
- "discovered America" (main event)
- NOT every single word!

Human Memory Strategy:

1. **Selective:** Keep important info, forget details
2. **Protected:** Important facts stay in memory
3. **Updated:** Add new important info, remove old

You don't remember every word - you remember *what matters*!

RNN vs Human Memory:

	RNN	Human
Selective?	No	Yes
Protected?	No	Yes
Updated?	Sort of	Yes

The Key Insight:

RNN problem: $h_t = \tanh(W_x x_t + W_h h_{t-1} + b)$
Everything gets mixed together and fades!

What we need:

1. **Selective Gate:** Decide what to remember
2. **Protected Memory:** Keep important info safe
3. **Update Control:** Add new info carefully

Intuition: The Hypothesis

What if we add **explicit control** over memory?

- Gate 1: "Should I forget this?"
- Gate 2: "Should I add this new info?"

The Hypothesis: Protect Important Memories

The Problem with RNN:

$h_t = \tanh(W_x x_t + W_h h_{t-1} + b)$

Multiplication at each step → gradient vanishes

The Breakthrough Idea:

What if we use ADDITION instead?

$C_t = C_{t-1} + \text{new info}$

Addition preserves gradients → memory lasts!

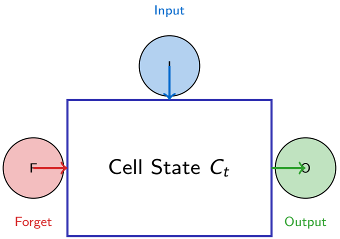
But we need control:

- **What to forget:** Remove outdated info
- **What to add:** Add new important info
- **What to output:** Show relevant info

Solution: Three "gates" (valves) controlling memory flow!

Conceptual Design:

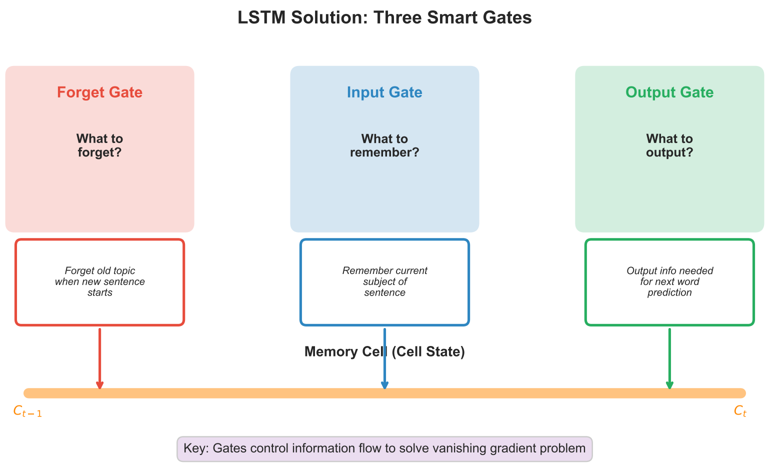
Imagine a protected memory cell C_t:



How Gates Work:

1. **Forget Gate:** "Remove outdated info"
   $f_t = \sigma(\ldots)$ ($0 =$ forget all, $1 =$ keep all)
2. **Input Gate:** "Add new important info"
   $i_t = \sigma(\ldots)$ ($0 =$ ignore, $1 =$ add fully)
3. **Output Gate:** "Show relevant info"
   $o_t = \sigma(\ldots)$ ($0 =$ hide, $1 =$ show)

Preview: LSTM (Long Short-Term Memory)



The LSTM Solution:

Two memory streams:

Checkpoint: Breakthrough Comparison

Next: Deep Dive into LSTM

What You Learned Today (RNN):

1. **The Challenge:** Variable-length sequences
2. **RNN Insight:** Memory through recurrence
3. **RNN Success:** Works for short sequences (5-10)
4. **RNN Failure:** Vanishing gradient kills long memory
5. **Root Cause:** Repeated multiplication
6. **Human Insight:** Selective, protected memory
7. **LSTM Idea:** Gates + Addition = Long memory

Real World: Where RNNs Still Used

Despite limitations, RNNs work well for:

- **Short sequences:** 5-15 steps
- **Real-time audio:** Low latency needed
- **Simple patterns:** Limited context required
- **Embedded systems:** Memory constraints

What's Next (LSTM Slides):

1. **Water Tank Analogy:** Concrete intuition for gates
2. **Gate Mechanics:** How forget/input/output gates work
3. **Worked Examples:** Step-by-step LSTM processing
4. **Vector Mathematics:** Full equations explained
5. **Training Details:** BPTT through LSTM
6. **Applications:** Where LSTMs excel
7. **Implementation:** PyTorch code

Intuition: The Learning Journey

Today: Discovered WHY we need better memory (Vanishing gradient problem)

LSTM Slides: Learn HOW gates solve it (Discovery-based approach)

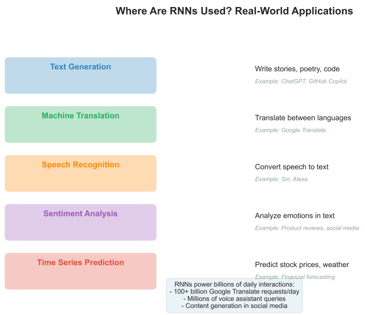
Lab: Build your own LSTM (Hands-on implementation)

Where RNNs Work Well Today

Successful Applications:

1. **Short-Sequence Tasks**
   - **Name prediction:** "Joh" → "n"
   - **Next character:** Autocomplete (5-10 chars)
   - **Sentiment:** Short reviews (1-2 sentences)
   - **Simple classification:** Fixed-length input
2. **Real-Time Systems**
   - **Voice recognition:** Process audio frame-by-frame
   - **Gesture detection:** Short movement sequences
   - **Sensor data:** Last 10 readings for prediction
3. **When Speed Matters**
   - RNN: Simpler, faster than LSTM
   - Fewer parameters to train
   - Good for resource-constrained devices

Application Comparison:



Limitations Summary:

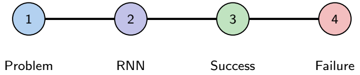
Task	Max Length
Character prediction	10-15
Sentiment (sentence)	15-20
Short translation	10-15
Paragraph understanding	FAILS

Summary: What We Learned About RNNs

Key Concepts:

1. **Sequential Data:** Order matters (text, speech, time series)
2. **Traditional Networks Fail:** Fixed-size input, no memory
3. **RNN Innovation:** Recurrent connections create memory
   $h_t = \tanh(W_x x_t + W_h h_{t-1} + b)$
4. **Hidden State:** Carries context forward through time
5. **Success:** Works well for short sequences (5-20 steps)
6. **Failure:** Vanishing gradient for long sequences
7. **Root Cause:** Repeated multiplication fades gradients
   gradient $\propto 0.5^{20} \approx 0$
8. **Solution Preview:** LSTM uses addition + gates

The Discovery Journey:



Worked Examples Covered:

- **Name prediction:** "Joh" $\to$ "n"
- **Sentiment analysis:** Movie reviews
- **Long sentences:** Subject-verb agreement fails

Mathematical Insights:

- Recurrence creates memory
- Tanh activation bounds values
- Backpropagation through time (BPTT)
- Gradient vanishing from repeated multiplication

Checkpoint: Final Check

Next Steps: From RNN to LSTM

Learning Path:

1. **Today (RNN):** [Completed]
   - Understand sequential data challenge
   - Learn RNN architecture
   - Discover vanishing gradient problem
2. **Next (LSTM):** Continue reading
   - Discovery-based water tank analogy
   - Learn three gates (forget, input, output)
   - See how addition solves vanishing gradient
   - Worked examples with actual numbers
3. **Lab Session:** Hands-on implementation
   - Build RNN from scratch
   - Implement LSTM
   - Train on real text data
   - Compare performance
4. **Week 4 (Seq2seq):** Advanced architectures
   - Encoder-decoder models
   - Attention mechanism
   - Machine translation

Resources:

Slides:

- LSTM slides (next in sequence)
- Week 4: Seq2seq models
- Week 5: Transformers

Labs:

- Week 3 Lab: RNN/LSTM implementation
- Shakespeare generation notebook
- Sentiment classification exercise

Code Examples:

- PyTorch RNN tutorial
- Character-level generation
- Name prediction model

Key Questions for Next Session:

1. How do gates actually work? (Sigmoid functions)
2. Why does addition preserve gradients? (Math proof)

Forward Pass (Inference):

At each time step $t = 1, 2, \ldots, T$:

1. Hidden state update:

$$h_t = \tanh(W_x x_t + W_h h_{t-1} + b_h)$$

where:

- $x_t \in \mathbb{R}^{d_x}$: input at time t
- $h_t \in \mathbb{R}^{d_h}$: hidden state
- $W_x \in \mathbb{R}^{d_h \times d_x}$: input weight matrix
- $W_h \in \mathbb{R}^{d_h \times d_h}$: recurrent weight matrix
- $b_h \in \mathbb{R}^{d_h}$: hidden bias

2. Output computation:

$$y_t = W_y h_t + b_y$$

where:

- $W_y \in \mathbb{R}^{d_y \times d_h}$: output weight matrix
- $b_y \in \mathbb{R}^{d_y}$: output bias

Backward Pass (Training - BPTT):

Loss function:

$$L = -\sum_{t=1}^{T} \log p(w_t^* \mid w_{<t})$$

where w_t^* is the true next word.

Backpropagation through time (BPTT):

Output gradient:

$$\frac{\partial L}{\partial y_t} = \hat{y}_t - y_t^*$$

Hidden state gradient at time t:

$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial y_t} \frac{\partial y_t}{\partial h_t} + \frac{\partial L}{\partial h_{t+1}} \frac{\partial h_{t+1}}{\partial h_t}$$

Expanded:

$$\frac{\partial L}{\partial h_t} = W_y^T \frac{\partial L}{\partial y_t} + W_h^T \frac{\partial L}{\partial h_{t+1}} \odot \tanh'(z_{t+1})$$

Gradient Flow Analysis:

To update weights based on early time steps, gradients must flow backward through all intermediate steps.

Chain rule for gradient at time t:

$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial h_T} \prod_{k=t+1}^{T} \frac{\partial h_k}{\partial h_{k-1}}$$

Jacobian of hidden state:
Recall: $h_t = \tanh(W_x x_t + W_h h_{t-1} + b_h)$

$$\frac{\partial h_t}{\partial h_{t-1}} = \mathrm{diag}(\tanh'(z_t)) \cdot W_h$$

where $\tanh'(z) = 1 - \tanh^2(z)$

Product expansion:

$$\frac{\partial L}{\partial h_t} = \frac{\partial L}{\partial h_T} \prod_{k=t+1}^{T} \mathrm{diag}(\tanh'(z_k)) \cdot W_h$$

Why Gradients Vanish:

1. Tanh derivative bounds:

$$0 < \tanh'(z) = 1 - \tanh^2(z) \leq 1$$

Typically: $\tanh'(z) \approx 0.2$ to 0.5 for most z

2. Spectral radius of W_h:
Let $\lambda_{\max}$ be the largest eigenvalue of W_h.
For gradient flow to be stable:

$$\left\| \frac{\partial h_t}{\partial h_{t-1}} \right\| = \|\mathrm{diag}(\tanh'(z_t)) W_h\| \lesssim 1$$

3. Long-term gradient magnitude:

$$\left\| \frac{\partial L}{\partial h_t} \right\| \approx \left\| \frac{\partial L}{\partial h_T} \right\| \cdot \gamma^{T-t}$$

where $\gamma = \|\mathrm{diag}(\tanh'(z)) W_h\|$

Typical case: $\gamma \approx 0.5$

For $T - t = 20$ steps back:

$$\text{gradient magnitude} \approx 0.5^{20} \approx 9.5 \times 10^{-7}$$