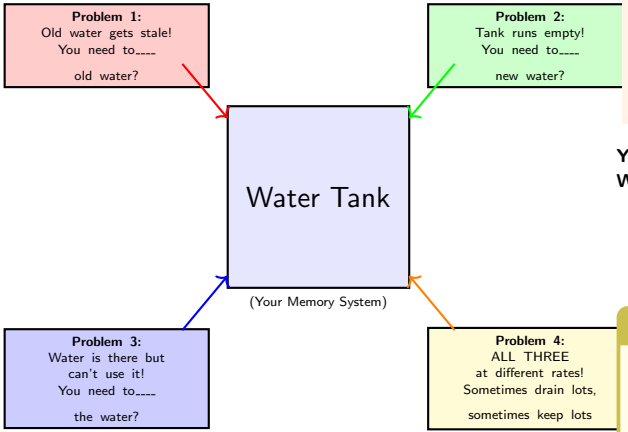
LSTM - Long Short-Term Memory

Understanding Through a Complete Example

Imagine You're Designing a Memory System

The Challenge:



Why Do We Care About This Tank?

Imagine: It rained yesterday
Question: Should we buy water tomorrow?
To predict, you need to know:

- How much was **DRAINED**? (old water out)
- How much rain was **ADDED**? (new water in)
- How much is **AVAILABLE** now? (can use it?)

→ **Memory state helps make predictions!**

Your Design Task:
What THREE controls would YOU design?

1. **Control #1:** To handle stale water (Hint: Think about draining...)
2. **Control #2:** To handle empty tank (Hint: Think about adding...)
3. **Control #3:** To handle using water (Hint: Think about tapping...)

Checkpoint: Think First!

Key Question: Do these three controls need to be INDEPENDENT?
Can you drain a lot while adding a little? Can you add a lot while using only some?
Your answer: _____

The Core Task: Predict the Next Word

Simple Example:

Input: "The cat was"

↓

LSTM

↓

Predictions:

hungry	35%
sleeping	28%
running	15%
small	8%

How does it work?
Must remember "cat" to predict appropriate adjective/verb!

Why This Matters:

- **Autocomplete**
  Your phone keyboard
- **Translation**
  Google Translate (2016)
- **Text Generation**
  Write stories, code
- **Voice Assistants**
  Siri, Alexa
- **Chatbots**
  Customer service

Checkpoint: The Challenge

To predict well, the model must REMEMBER earlier words in the sentence.
That's what LSTM does brilliantly!

The Challenge and The Solution

THE PROBLEM

RNNs Cannot Remember

- Vanishing gradients
- $0.5^{50} \approx 0$ (information dies)
- Forgets early context
- Cannot handle long sentences

Example:
"I grew up in Paris. I speak fluent ___"

RNN forgets "Paris" after 20 words
Cannot predict "French"

THE SOLUTION

LSTM Controls Memory

- Three gates (0-1 values)
- Addition path (not multiplication)
- Preserves gradients
- Remembers 100+ steps

The Method:

Three Independent Gates:

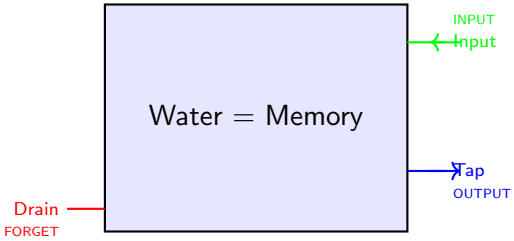
- FORGET: What to erase
- INPUT: What to add
- OUTPUT: What to use

Let's see how this works with a simple analogy...

Understanding The Idea: Water Tank Analogy

Think of Memory as a Water Tank with Three Valves

The Tank System:



How Each Valve Works:

FORGET = Drain Valve
Controls how much water flows OUT
0.1 = Open 10% → 90% drains away
Removes old water (old memory)

INPUT = Input Valve
Controls how much new water flows IN
0.9 = Open 90% → lots added
Adds fresh water (new memory)

OUTPUT = Output Tap

Real Examples:

At period "." in sentence:

- Drain: 90% (0.1 forget)
- Input: 40% (0.4 input)
- Tap: 30% (0.3 output)

→ Tank mostly empties!

At noun "dog":

- Drain: 30% (0.7 forget)
- Input: 90% (0.9 input)
- Tap: 90% (0.9 output)

→ Tank fills up!

Intuition: The Key Insight

Three INDEPENDENT valves on ONE tank!

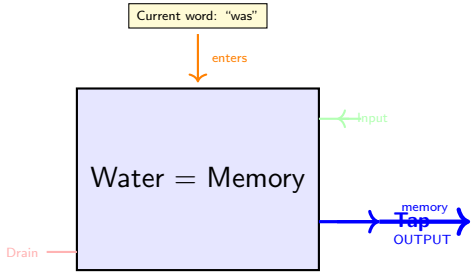
Each valve controls a different aspect:

- Drain: How much OLD to remove
- Input: How much NEW to add
- Tap: How much to USE now

This is EXACTLY what LSTM does!

Continuing The Analogy: From Tank to Predictions

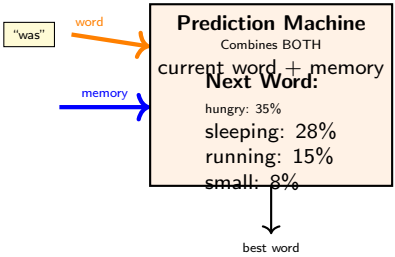
The Complete System:



Two Things Happening:

- Current word enters system
- Memory (tap water) flows out
- BOTH go to prediction!

Where They Go:



The Complete Flow:

Current word: "was"
+
Memory (tap water)
↓
Prediction Machine
↓
Next word: "hungry"!

Checkpoint: Key Insight!

What Are "Gates"?

Three Gates Control Memory Like Volume Knobs (0 to 1)

FORGET

$\times$

REMOVES
old information

Value 0-1:

- 0.0 = erase all
- 0.5 = keep half
- 1.0 = keep all

Example:
0.1 at period
$\rightarrow$ Erase 90%!

INPUT

$+$

ADDS
new information

Value 0-1:

- 0.0 = add nothing
- 0.5 = add half
- 1.0 = add all

Example:
0.9 on "cat"
$\rightarrow$ Store lots!

OUTPUT

$\rightarrow$

REVEALS
stored information

Value 0-1:

- 0.0 = hide all
- 0.5 = show half
- 1.0 = show all

Example:
0.9 at "was"
$\rightarrow$ Use memory!

How They Work Together:

New Memory = (Forget $\times$ Old Memory) + (Input $\times$ New Info)
Output = Output Gate $\times$ Memory

Now let's see these gates in action with concrete numbers...

The 4-Step Process (Part 1): Erase Old, Add New

Updating Memory From "cat" to "dog"

Starting Point:

Old Memory: [0.8, 0.6, 0.4]

Contains "cat" information

Step 1: FORGET Gate = 0.1

What it does: Multiply old memory by 0.1

[0.8, 0.6, 0.4] $\times$ 0.1 = [0.08, 0.06, 0.04]

Result: 90% erased! "cat" mostly removed.

Why? At period, we need fresh start for new sentence.

Step 2: INPUT Gate = 0.9

What it does: Filter new candidate info

Create candidate: [0.7, 0.5, 0.9]

Multiply by 0.9: [0.7, 0.5, 0.9] $\times$ 0.9 = [0.63, 0.45, 0.81]

Result: 90% of "dog" info ready to add.

Why? "dog" is new subject, very important!

Visual Flow (Steps 1-2):

Checkpoint: Key Point

Two INDEPENDENT operations:

- FORGET decides what OLD to keep
- INPUT decides what NEW to add
- They don't interfere!

From Separate Results to Final Output

Where We Left Off:

- Erased old: [0.08, 0.06, 0.04]
- Filtered new: [0.63, 0.45, 0.81]

Step 3: COMBINE (Addition!)

What it does: Add the two results together

$[0.08, 0.06, 0.04] + [0.63, 0.45, 0.81]$
$= [0.71, 0.51, 0.85]$

Result: Updated memory = "dog" info

Why addition? Preserves gradients! This is the key innovation that prevents vanishing.

Step 4: OUTPUT Gate = 0.9

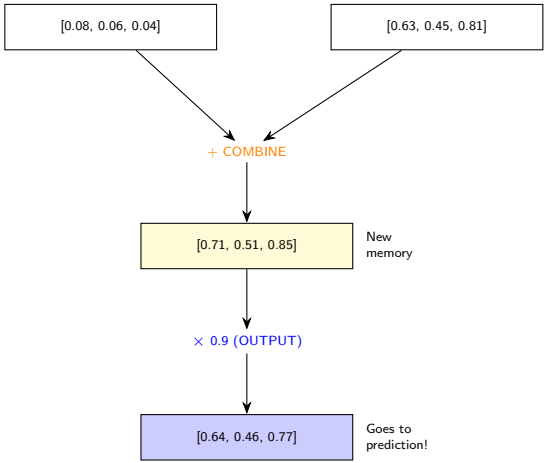
What it does: Filter what network sees

$[0.71, 0.51, 0.85] \times 0.9$
$= [0.64, 0.46, 0.77]$

Result: 90% revealed to next layer

Why? At "was", we NEED subject info for verb prediction!

Visual Flow (Steps 3-4):



From Hidden State to Prediction

Recap: Output Gate Result

From Step 4: [0.64, 0.46, 0.77]

This is the **hidden state** h_t

Where Does It Go?

1. **To Prediction Layer**
   $h_t \rightarrow$ Linear $\rightarrow$ Softmax $\rightarrow$ Probabilities
   Example at "was": Predict next word
2. **To Next Time Step**
   h_t feeds into next LSTM cell
   Used to compute next gates
3. **Optional: To Attention**
   In seq2seq models, decoder attends to these h_t values

Key Distinction:

- C_t = Long-term memory (protected)
- h_t = Working memory (filtered output)

But what does this look like mathematically?

Concrete Example:

At word "was" in "The dog was sleeping":

Memory C_t contains: [dog, context]
Output gate: 0.9 (reveal 90%)
Hidden state h_t: [0.64, 0.46, 0.77]

Prediction layer receives h_t:
$h_t \rightarrow$ Linear(512 $\rightarrow$ vocab) $\rightarrow$ Softmax

Top predictions:

- "sleeping": 0.35 (verb matches dog)
- "running": 0.18
- "eating": 0.12

The subject info ("dog") in h_t helps predict appropriate verb!

Intuition: Why Filter?

Not all memory is relevant NOW.

Examples:

- At "The": Output gate low (0.2) $\rightarrow$ hide memory, article doesn't need context
- At "was": Output gate high (0.9) $\rightarrow$ reveal memory, verb needs subject!

The OUTPUT gate is smart about WHEN to use memory.

From Water Tank Analogy to Math

What We Just Learned

"Tap water"
"Gate = 0.9"
"Memory flows"

This was the INTUITION

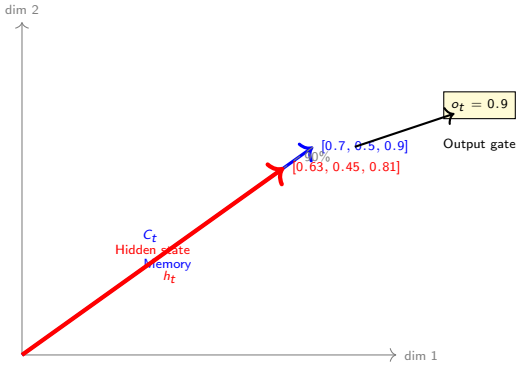
The Translation:

- **Water** $\longrightarrow$ Vector in space
- **Tank** $\longrightarrow$ Memory C_t
- **Tap setting 0.9** $\longrightarrow$ Multiply by 0.9
- **Flowing water** $\longrightarrow$ Scaled vector h_t

Checkpoint: Key Insight

The "tap" is just SCALAR MULTIPLICATION!
Gate value 0.9 = scale each dimension by 0.9

What's Actually Happening:



The Math:

Memory vector: $C_t = [0.7, 0.5, 0.9]$
Output gate: $o_t = 0.9$ (the "tap setting")
Element-wise multiplication:

Why Three Separate Gates? Real Scenarios

Reading: "The cat sat. The dog..."

Scenario 1: At "cat"

Gate Values:

- F = 0.8 (keep)
- I = 0.9 (STORE!)
- O = 0.8 (show)

What Happens:

- Keep previous context
- STORE subject strongly
- Show it to network

Goal:
Remember "cat" for rest of sentence

Memory:
→ [cat, context]

Scenario 2: At "."

Gate Values:

- F = 0.1 (ERASE!)
- I = 0.4 (small)
- O = 0.3 (HIDE)

What Happens:

- ERASE old sentence
- Small punctuation add
- HIDE memory

Goal:
Clean slate for new sentence

Memory:
→ [mostly empty]

Scenario 3: At "dog"

Gate Values:

- F = 0.7 (keep some)
- I = 0.9 (NEW!)
- O = 0.9 (REVEAL!)

What Happens:

- Keep some context
- STORE new subject
- REVEAL all info

Goal:
New focus, need it NOW

Memory:
→ [dog, some context]

Key Insight: Each situation needs DIFFERENT gate values!
That's why LSTM has three independent gates, not just one.
The network LEARNS which values to use for each word.

Now let's watch these gates in action on a real sentence...

Watch LSTM Process a Sentence

Sentence: "The cat was hungry. The dog was sleeping."

Word	Forget	Input	Output	Memory State
The	0.9	0.3	0.2	*article*
cat	0.8	**0.9**	0.8	*subject: cat*
was	0.9	0.7	0.9	*cat + verb*
hungry	0.8	0.8	0.7	*cat is hungry*
.	**0.1**	0.4	0.3	*sentence ends*
The	0.1	0.8	0.2	*new article*
dog	0.7	**0.9**	0.9	*subject: dog*
was	0.9	0.8	**0.9**	*using dog info*

0.1 = Forget

0.9 = Store/Use

Period → Reset

Notice the patterns? Let's explore what you observed...

What Did You Notice?

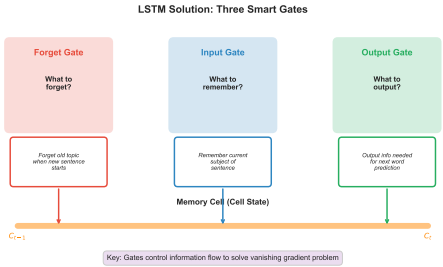
Common Observations:

Students usually notice:

- **"It drops to 0.1 at the period!"**
- **"It's 0.9 on important words (cat, dog)"**
- "The memory changes from cat to dog"
- "It resets between sentences"
- "Three different columns of numbers"

Key Questions:

1. HOW does it know to forget at period?
2. HOW does it know cat and dog are important?
3. HOW does it decide when to use memory?



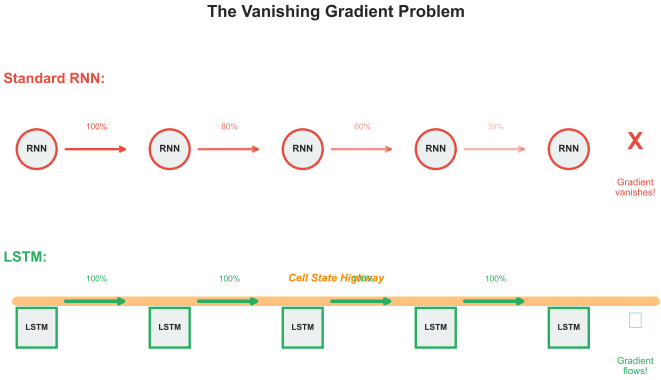
Checkpoint: The Big Reveal

Those three columns are called **GATES**:

- Forget Gate: Controls what to erase
- Input Gate: Controls what to store
- Output Gate: Controls what to use

But WHY do we need gates?

Why Do We Need Controlled Memory?



Key: LSTM uses addition (cell state) instead of multiplication (RNN hidden state)

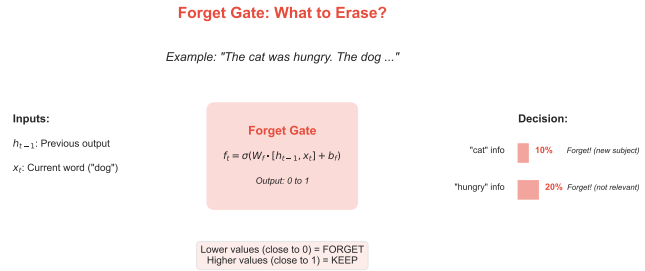
RNN Problem:

- Gradients vanish ($0.5^{50} \approx 0$)
- Forgets early information
- Can't handle long dependencies
- Would lose "cat" by "dog"

LSTM Solution:

- Cell state highway (addition not multiplication)
- Three gates for CONTROL
- Can preserve info for 100+ steps
- Then ERASE when sentence ends

Forget Gate: How We Get That 0.1



Back to Our Table - Row 5:

Word	Forget
"."	**0.1**

What This 0.1 Means:

- 0.0 = forget everything
- 1.0 = keep everything
- 0.1 = forget 90% (keep only 10%)

Why at period?

The Formula That Produces 0.1:

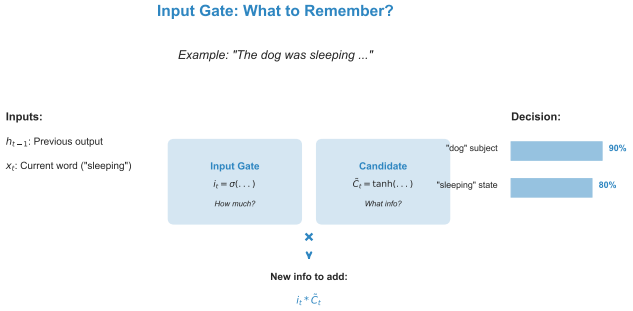
$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

How It Decides:

1. Look at current word (".")
2. Look at previous hidden state
3. Compute weighted sum
4. Apply sigmoid $\longrightarrow$ output 0 to 1

Cell State Update:

Input Gate: How We Get That 0.9



Back to Our Table - Row 7:

Word	Input
"dog"	**0.9**

What This 0.9 Means:

- 0.0 = add nothing
- 1.0 = add everything
- 0.9 = add 90% of candidate

Why at "dog"?

The Formulas (Two Parts):

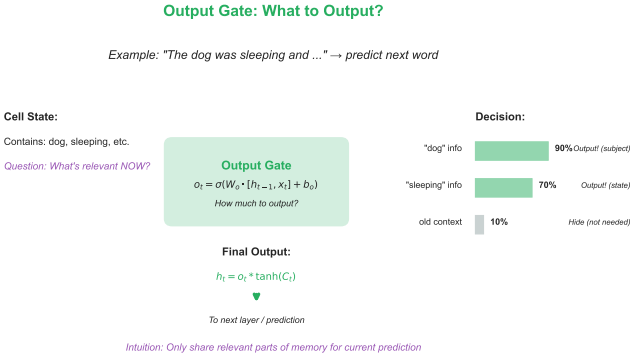
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

How It Works:

1. Create candidate info ($\tilde{C}_t$) with tanh
2. Decide how much to use ($i_t = 0.9$)
3. Multiply: 0.9 × candidate
4. Add to cell state

Output Gate: When to USE Memory



Back to Our Table - Row 8:

Word	Output
"was"	**0.9**

What This 0.9 Means:

- 0.0 = hide everything
- 1.0 = reveal everything
- 0.9 = output 90% of memory

Why at "was"?

The Formulas:

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$

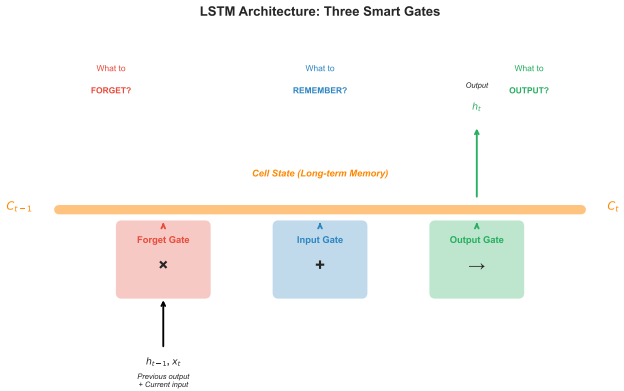
$$h_t = o_t \odot \tanh(C_t)$$

How It Works:

1. Look at cell state (has "dog" info)
2. Decide what's relevant NOW
3. Filter memory through gate (0.9)
4. Send h_t to prediction layer

Key Insight:

The Big Picture: Three Gates Working Together



The Cell State Highway:

- Protected memory channel
- Information flows easily
- Gates control entry/exit
- Gradients don't vanish!

At Each Time Step:

1. **Forget:** Erase old (0.1 → erase "cat")
2. **Input:** Add new (0.9 → add "dog")

Intuition: Visual Analogy

Think of LSTM like a notebook:

- Forget Gate = Eraser
  (Clear old notes at period)
- Input Gate = Pen

Now Look Again - You Understand EVERYTHING!

Sentence: "The cat was hungry. The dog was sleeping."

Word	Forget	Input	Output	What LSTM "Thinks"
The	0.9 (keep)	0.3 (weak)	0.2 (hide)	Article seen, nothing special yet
cat	0.8 (keep)	**0.9** (STORE!)	0.8 (show)	Subject! Important noun!
was	0.9 (keep)	0.7 (add)	0.9 (need!)	Verb connects to cat
hungry	0.8 (keep)	0.8 (add)	0.7 (show)	Describes the cat's state
.	**0.1** (ERASE!)	0.4 (end)	0.3 (hide)	Sentence over! Clear memory!
The	0.1 (clear)	0.8 (new!)	0.2 (hide)	NEW sentence starts fresh
dog	0.7 (keep)	**0.9** (NEW!)	0.9 (use!)	NEW subject! (forgot cat)
was	0.9 (keep)	0.8 (add)	**0.9** (USE!)	Using DOG info for prediction

Checkpoint: The Magic Transition

Watch rows 4→5→6→7: **hungry** → **.** → **The** → **dog**
Memory Evolution: [cat, hungry] → FORGET (0.1) → [end] → ADD (0.9) → [dog]
This intelligent memory control is what RNNs cannot do! LSTM uses gates to:

- Preserve important info (0.9 on subject nouns)
- Erase when context changes (0.1 at sentence boundaries)
- Reveal info when needed (0.9 output for predictions)

Summary: From Table to Understanding

Your Learning Journey:

1. **Intuition:** Water tank analogy
   (Three valves on one tank)
2. **Concepts:** What gates are
   (Volume knobs from 0 to 1)
3. **Mechanics:** How they work
   (4-step process with real numbers)
4. **Usage:** What happens to output
   (Hidden state $\rightarrow$ prediction layer)
5. **Mastery:** Complete understanding
   (Table makes perfect sense now!)

Key Equations:

$$f_t = \sigma(W_f[h_{t-1}, x_t] + b_f)$$
$$i_t = \sigma(W_i[h_{t-1}, x_t] + b_i)$$
$$o_t = \sigma(W_o[h_{t-1}, x_t] + b_o)$$
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$
$$h_t = o_t \odot \tanh(C_t)$$

Real World: Where LSTMs Excel

Applications (2015-2020):

- Machine Translation (Google Translate)
- Speech Recognition (Siri, Alexa)
- Text Generation (early GPT)
- Video Analysis
- Music Generation
- Handwriting Recognition

Modern Context (2024):
Transformers now dominate NLP, but LSTMs:

- Still used in time series
- Efficient for streaming data
- Foundation for understanding attention

The Core Insight:
That table showed you *exactly* how gates work. Every 0.1 and 0.9 has a purpose. That's the real magic of LSTMs!

Questions?