Week 0d: Neural Networks

The Depth Challenge

Machine Learning for Smarter Innovation

BSc Course - Theory Foundation

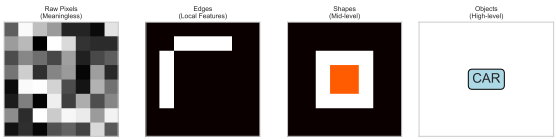
October 7, 2025

Course Outline

1. Part 1: The Challenge
2. Part 2: Shallow MLPs
3. Part 3: Modern Architectures
4. Part 4: Synthesis

1. Image Recognition Needs Hierarchical Features

- Raw pixels are meaningless noise
- Vision builds up complexity:
  - Edges from pixel gradients
  - Shapes from edge combinations
  - Objects from shape patterns
- **Example: Cat detection**
  - Pixels → edges → whiskers → face → cat
- Traditional ML: Manual feature engineering
- Deep learning: Automatic feature hierarchy
- **Problem**: Manual features require domain expertise and fail to generalize



Hierarchical feature learning mirrors biological vision - complexity emerges through layered abstraction

2. Single Perceptron: Linear Only

The Perceptron (Rosenblatt 1957)

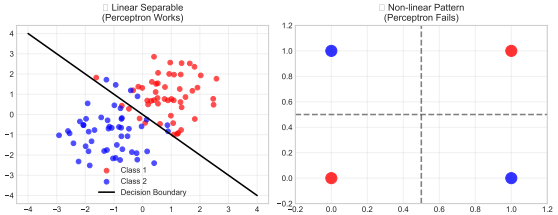
$$y = \text{sign}(w_1 x_1 + w_2 x_2 + b) \quad (1)$$
$$= \text{sign}(\mathbf{w}^T \mathbf{x} + b) \quad (2)$$

Learning Rule:

$$w_{new} = w_{old} + \eta(y_{true} - y_{pred})x$$

Geometric Interpretation:

- Creates a linear decision boundary
- Hyperplane in n-dimensional space
- Cannot separate non-linear patterns
- **Perceptron Convergence Theorem**: Guaranteed to converge if data is linearly separable



Single layer = single hyperplane = linear separation only - Rosenblatt's 1957 perceptron could only solve linearly separable problems

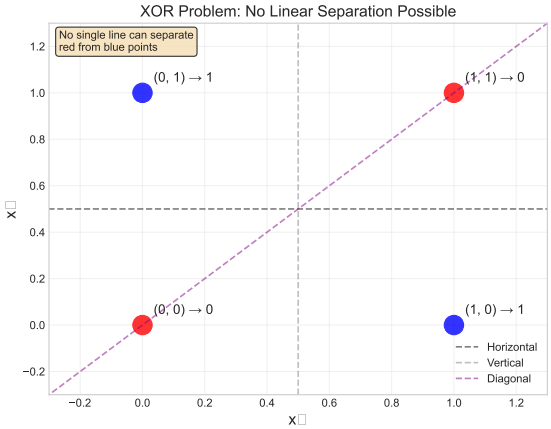
3. XOR Problem: Concrete Example

XOR Truth Table:

x_1	x_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0

The Problem:

- No single line separates the classes
- XOR is the simplest non-linear problem (2 inputs, 4 points)
- Requires non-linear decision boundary
- **Minsky & Papert (1969)**: Mathematical proof that perceptrons cannot solve XOR



XOR became the symbol of perceptron limitations - Minsky & Papert's 1969 proof triggered the first AI winter

4. Universal Approximation Theorem

Theoretical Foundation (Cybenko 1989, Hornik 1991):

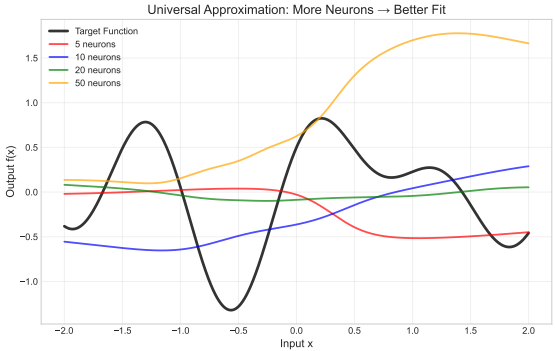
- Any continuous function can be approximated
- Single hidden layer with enough neurons
- Activation: sigmoid, tanh, ReLU
- Arbitrarily small error possible

Mathematical Statement: For any $\epsilon > 0$ and continuous f on compact set K (bounded and closed), there exists network N such that:

$$\sup_{x \in K} |f(x) - N(x)| < \epsilon$$

Theory-Practice Gap:

- May need exponentially many neurons for single layer

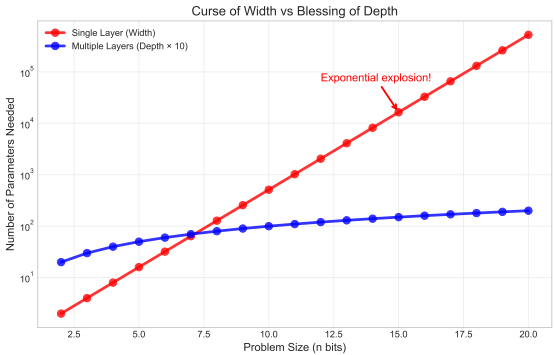


Theory says possible but doesn't tell us how many neurons - this gap motivates depth over width

5. Quantify: How Many Neurons/Layers Needed?

Practical Reality:

- Theory: Single layer sufficient
- Practice: Exponentially many neurons
- **Example 1:** Parity function on n bits
  - 1 layer: 2^{n-1} neurons needed
  - 2 layers: $O(n)$ neurons sufficient
- **Example 2:** Checkerboard pattern
  - 1 layer: $O(2^{\sqrt{n}})$ neurons
  - 2 layers: $O(n)$ neurons
- Curse of width vs. blessing of depth
- **General principle:** Depth allows exponentially more efficient representation



Depth provides exponential expressivity advantage - this is why 'deep' learning succeeded where shallow networks failed

6. Add Hidden Layer Approach

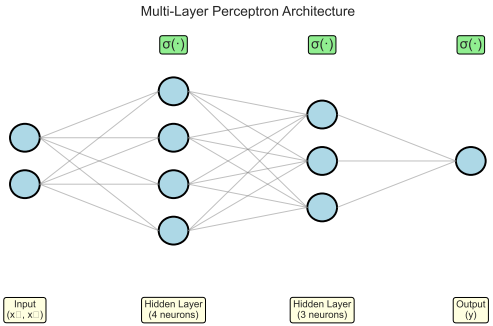
Multi-Layer Perceptron (MLP):

$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1) \quad (3)$$

$$y = \sigma(\mathbf{w}_2^T\mathbf{h} + b_2) \quad (4)$$

Key Innovation:

- Hidden layer creates feature combinations
- Hidden layer creates new representation where linear separation becomes possible
- Non-linear activation σ (sigmoid, tanh, ReLU)
- Each neuron = learned feature detector
- Output combines these features
- **Training**: Backpropagation (Rumelhart et al., 1986) computes gradients via chain rule



Hidden layer transforms input space to make linear separation possible - learned features replace manual engineering

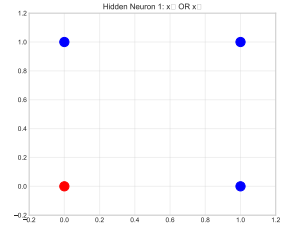
7. Worked Example: XOR Solved!

Network Architecture:

- Input: x_1, x_2
- Hidden: 2 neurons with sigmoid
- Output: 1 neuron with sigmoid

Solution Strategy:

- h_1: Detects x_1 OR x_2
- h_2: Detects x_1 AND x_2
- Output: h_1 AND NOT h_2



Actual Weights:

$$h_1 = \sigma(20x_1 + 20x_2 - 10) \quad (5)$$

$$h_2 = \sigma(20x_1 + 20x_2 - 30) \quad (6)$$

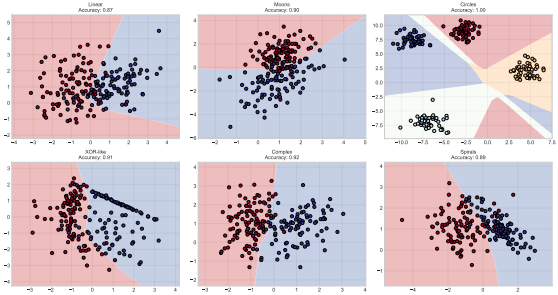
8. SUCCESS: Nonlinearity Achieved

What We Gained:

- Non-linear decision boundaries
- Universal approximation
- Automatic feature learning
- Backpropagation training

Applications Unlocked:

- Image classification (MNIST)
- Function approximation
- Pattern recognition
- Control systems



MLPs solved the non-linearity problem and enabled the neural network renaissance

9. FAILURE PATTERN: Vanishing Gradients in Deep Networks

The Problem with Depth:

Layers	Final Accuracy	Training Time
1	87.2%	10 min
2	91.5%	15 min
3	89.3%	25 min
4	82.1%	40 min
5	71.8%	60 min
6	58.3%	90 min

Observation: Deeper networks perform worse, not better!

Real data from 1990s experiments - deeper meant worse performance

10. Diagnosis: Gradient Multiplication -¿ Exponential Decay

Chain Rule in Deep Networks:

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial h_n} \cdot \ldots \cdot \frac{\partial h_2}{\partial h_1} \cdot \frac{\partial h_1}{\partial W_1} \tag{8}$$

Sigmoid Derivative:

$$\sigma'(x) = \sigma(x)(1 - \sigma(x)) \tag{9}$$

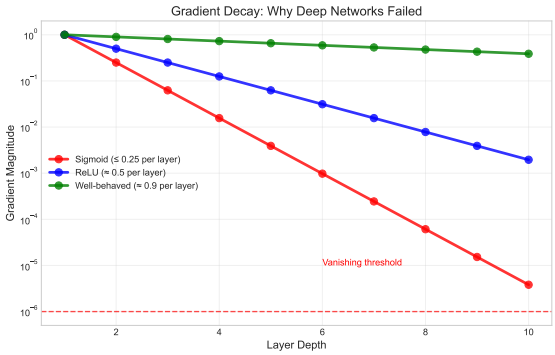
$$\leq 0.25 \tag{10}$$

The Problem: Each layer multiplies by ≤ 0.25

- 5 layers: $(0.25)^5 = 0.001$
- 10 layers: $(0.25)^{10} = 0.000001$

Comparison:

- Tanh: $\tanh'(x) = 1 - \tanh^2(x) \leq 1$, still problematic
- **Solution preview**: ReLU has constant gradient of 1 for positive inputs



Gradients vanish exponentially - early layers learn nothing

11. Gradient Flow Analysis

Mathematical Analysis:
For L-layer network with sigmoid activations:

$$\left| \frac{\partial L}{\partial W_1} \right| \leq C \cdot (0.25)^{L-1}$$

Consequences:

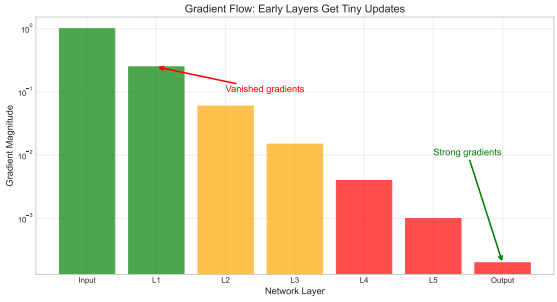
- Early layers: Tiny gradients
- Late layers: Large gradients
- Gradient mismatch problem
- Training becomes impossible

Historical Impact:

- 1990s: "Neural networks don't scale"
- SVMs and ensemble methods dominated
- Deep learning winter until 2006

Solutions (Part 3 Preview):

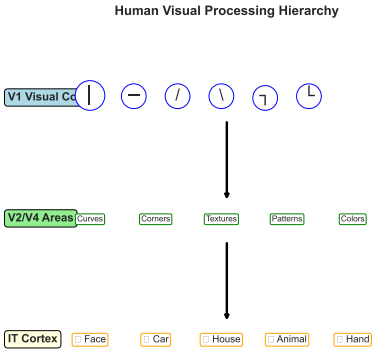
- ReLU activation (constant gradient)
- Batch normalization (stabilize distributions)
- Skip connections (ResNet shortcut paths)
- Better initialization (Xavier, He)



12. Human Introspection: Vision is Hierarchical

How Humans See (Hubel & Wiesel 1959):

- **Level 1**: Edge detection (V1 cortex)
  - Horizontal, vertical, diagonal lines
  - Local contrast detection
- **Level 2**: Texture & shape (V2, V4)
  - Curves, corners, textures
  - Spatial relationships
- **Level 3**: Objects (IT cortex)
  - Faces, cars, animals
  - Invariant recognition
- **Inspiration**: Fukushima's Neocognitron (1980) - precursor to modern CNNs



Hubel & Wiesel (1959 Nobel Prize): Discovered hierarchical processing in cat visual cortex - inspired modern neural network architectures

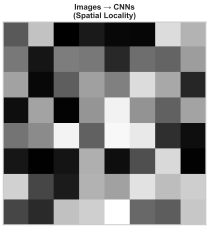
13. Hypothesis: Specialized Architectures Matching Data Structure

The Key Insight:

- Problem: Generic MLPs ignore data structure
- Solution: Architecture matches inductive bias
- **No Free Lunch Theorem**: No single architecture optimal for all problems
- Architecture choice encodes prior knowledge about problem structure

Examples:

- **Images**: Spatial locality -¿ CNNs
- **Sequences**: Temporal order -¿ RNNs
- **Graphs**: Node relationships -¿ GNNs
- **Language**: Long-range dependencies -¿ Transformers



Right architecture = built-in prior knowledge about the problem domain

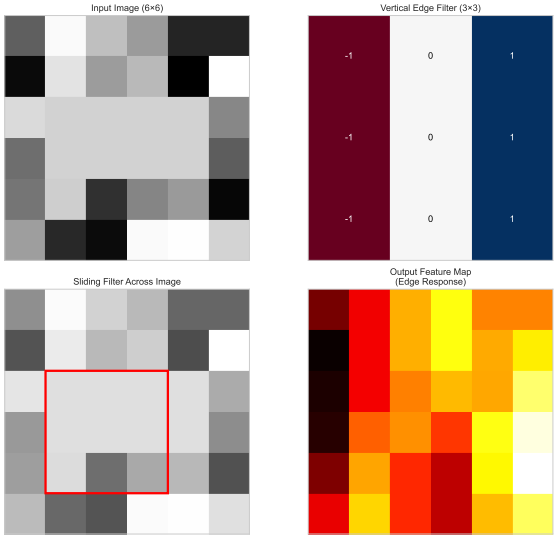
14. Zero-Jargon: Convolution as "Sliding Pattern Detector"

Convolution Intuition:

- Take a small "template" (3x3 filter)
- Slide it across the entire image
- At each position: compute similarity
- Result: "Where is this pattern?"

Example Filters:

- Edge detector: $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
- Blur: $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$



Convolution = template matching with learnable templates

15. Geometric Intuition: Filters Detect Edges/Textures

What Filters Learn:

Layer 1: Low-level features

- Edges, corners, blobs
- Oriented lines at different angles
- Color gradients

Layer 2: Mid-level features

- Textures, patterns
- Simple shapes
- Motifs and repeating elements

Layer 3+: High-level features

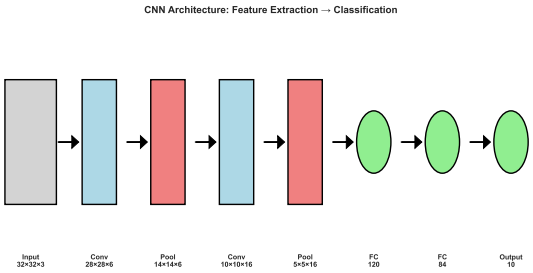
- Object parts (eyes, wheels)
- Complex patterns

Each layer builds more complex features from simpler ones

16. CNN Architecture Details

Key Components:

1. **Convolutional Layers**
   - Multiple filters per layer
   - Shared weights across spatial locations
   - Parameter sharing reduces overfitting
2. **Pooling Layers**
   - Downsampling (max, average)
   - Translation invariance
   - Computational efficiency
3. **Fully Connected**
   - Final classification
   - Combines all learned features



CNN = Feature extraction (conv+pool) + Classification (FC)

17. Full Walkthrough: Convolve Filter with Actual Numbers

Example Calculation:

Input (3x3): $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Filter (3x3): $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

Convolution (element-wise multiply + sum):

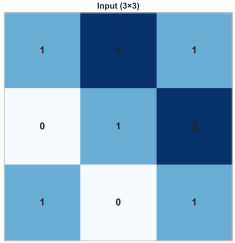
$$= (-1)(1) + (0)(2) + (1)(1) + \quad (11)$$

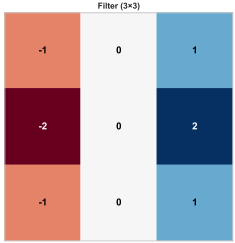
$$(-2)(0) + (0)(1) + (2)(2) + \quad (12)$$

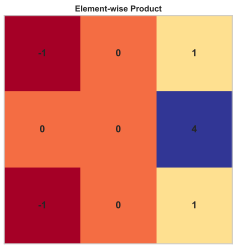
$$(-1)(1) + (0)(0) + (1)(1) \quad (13)$$

$$= -1 + 0 + 1 - 0 + 0 + 4 - 1 + 0 + 1 \quad (14)$$

$$= 4 \quad (15)$$







Output Value

Sum = 4

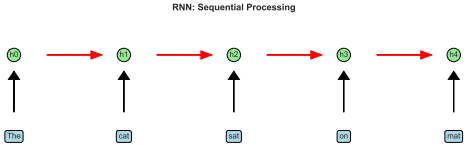
High response (4) means vertical edge detected at this location

Recurrent Neural Networks:

$$h_t = \tanh(W_{hh} h_{t-1} + W_{xh} x_t + b_h) \quad (16)$$

$$y_t = W_{hy} h_t + b_y \quad (17)$$

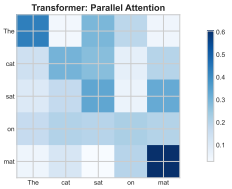
- Hidden state carries memory
- Sequential processing ($O(n)$ path length)
- Good for: Time series, NLP
- Problem: Vanishing gradients over time



Transformers ("Attention Is All You Need" 2017):
Attention Mechanism:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right) V$$

- Query: What I'm looking for
- Key: What I have to offer
- Value: What I'll return if matched



Key Advantages:

- Self-attention: $O(1)$ path length
- Parallel processing (no sequential bottleneck)
- Long-range dependencies without vanishing gradients
- State-of-the-art for language (GPT, BERT)

Transformers enable O(1) path length vs O(n) for RNNs - revolutionized NLP and beyond

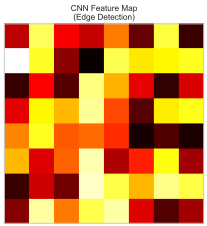
19. Visualization: Feature Maps, Attention Heatmaps

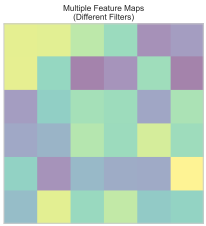
CNN Feature Maps:

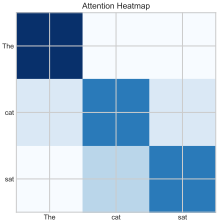
- Each filter produces a feature map
- Bright areas = high activation
- Shows what the network "sees"
- Layer 1: Edges and textures
- Layer N: Complex patterns

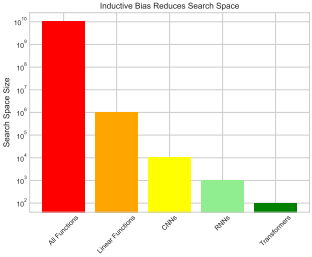
Transformer Attention:

- Attention weights as heatmaps
- Shows which words influence others
- Different heads learn different patterns
- Interpretable relationships
- **Technique**: Grad-CAM visualizes which regions influenced decision
- Attention weights reveal learned linguistic structure (syntax trees emerge naturally)









Visualization reveals the internal representations learned by neural networks

20. Why It Works: Inductive Biases Reduce Search Space

The Core Principle:
Without Structure:

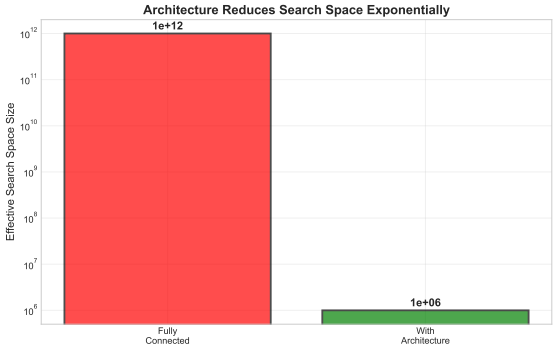
- Search space: All possible functions
- Size: Exponential in parameters
- Sample complexity: Intractable

With Architecture:

- Built-in assumptions about data
- Drastically reduced search space
- Faster learning, better generalization

Examples:

- CNNs assume translation invariance
- RNNs assume sequential dependence
- Transformers assume attention patterns

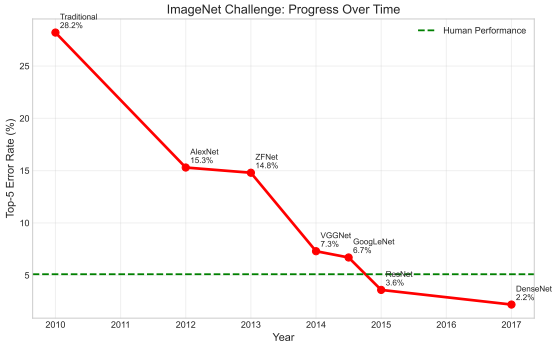


Architecture = built-in prior knowledge that guides learning

21. Experimental Validation: ImageNet Accuracy Over Time

ImageNet Challenge Results:

Year	Model	Top-5 Error
2010	Traditional CV	28.2%
2012	AlexNet (CNN)	15.3%
2013	ZFNet	14.8%
2014	VGGNet	7.3%
2014	GoogLeNet	6.7%
2015	ResNet	3.6%
2017	DenseNet	2.2%
-	Human Performance	5.1%



CNNs achieved superhuman performance in just 5 years

22. Solutions to Vanishing Gradients

1. ReLU Activation:

$$\text{ReLU}(x) = \max(0, x) \tag{18}$$

$$\frac{d}{dx}\text{ReLU}(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases} \tag{19}$$

Comparison:

- Sigmoid: $\sigma'(x) \leq 0.25$ (vanishes)
- ReLU: gradient $= 1$ (constant for $x > 0$)
- Enables training 100+ layer networks

2. Other Solutions:

- Batch Normalization (2015): Stabilize distributions
- Skip connections (ResNet 2015): Shortcut paths
- Xavier/He initialization: Preserve variance

Gradient Flow Comparison:
Sigmoid Network (10 layers):

- Layer 10 gradient: 1.0
- Layer 5 gradient: 0.001
- Layer 1 gradient: 0.000001

ReLU Network (10 layers):

- Layer 10 gradient: 1.0
- Layer 5 gradient: 1.0
- Layer 1 gradient: 1.0

Result:

- ReLU: All layers learn at similar rates
- Deep networks become trainable
- 2012 breakthrough: AlexNet (8 layers with ReLU)

ReLU's constant gradient solved the vanishing gradient problem - enabling the deep learning revolution

23. Deep Learning Evolution Timeline

Key Milestones:

2012 - AlexNet:

- CNNs + ImageNet breakthrough
- 8-layer network, ReLU activation
- GPU acceleration

2014 - Sequence-to-Sequence:

- RNNs for machine translation
- Encoder-decoder architecture

2015 - ResNet:

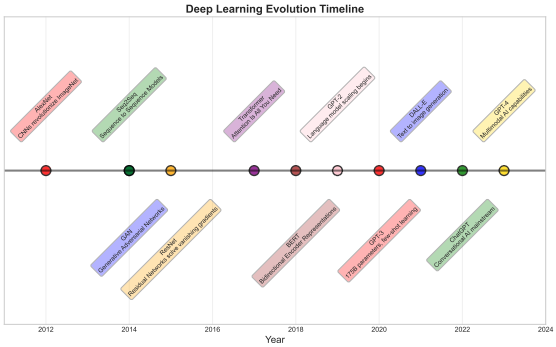
- 152 layers with skip connections
- First to surpass human performance on ImageNet
- Solved vanishing gradient problem

2017 - Transformers:

- "Attention Is All You Need"
- Self-attention mechanism
- Foundation for GPT, BERT

2020 - GPT-3:

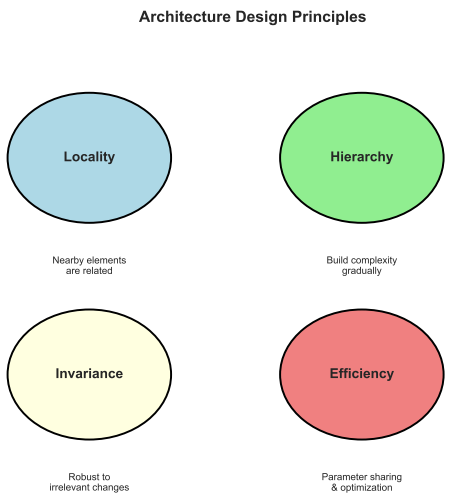
- 175 billion parameters



24. Architecture Design Principles

Universal Design Principles:

1. **Locality:**
   - Nearby elements are related
   - CNNs: Spatial locality
   - RNNs: Temporal locality
2. **Hierarchy:**
   - Build complexity gradually
   - Low-level -¿ High-level features
   - Mirrors human cognition
3. **Invariance:**
   - Robust to irrelevant changes
   - Translation, rotation, scale
   - Attention: Permutation invariance
4. **Efficiency:**
   - Parameter sharing
   - Computational optimization
   - Memory constraints



Good architectures encode the right inductive biases for the domain

24.5 When to Use Each Architecture

Decision Criteria:

Use CNNs when:

- Data has spatial structure (images, video)
- Translation invariance needed
- Local patterns matter
- Examples: Vision, medical imaging

Use RNNs when:

- Sequential dependencies
- Variable-length sequences
- Real-time processing needed
- Examples: Speech, time series

Use Transformers when:

- Long-range dependencies critical
- Parallel processing available
- Sufficient compute budget
- Examples: NLP, multimodal AI

Use Generic MLPs when:

- Tabular data (no structure)
- Small datasets ($<$10k examples)
- Interpretability required
- Examples: Finance, healthcare

Key Rule:

- Match architecture to data structure
- Start simple, add complexity if needed
- More parameters $\neq$ better performance

Architecture choice depends on data structure, computational budget, and domain requirements

25. Modern Applications: Computer Vision, NLP, Multimodal

Computer Vision:

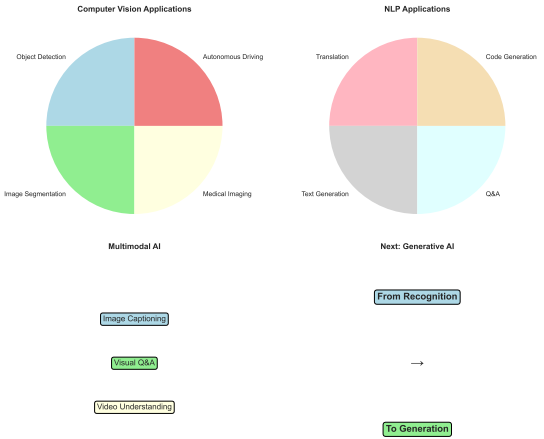
- Object detection (YOLO, R-CNN)
- Image segmentation
- Medical imaging diagnosis
- Autonomous driving

Natural Language Processing:

- Machine translation approaching human quality
- Text generation (GPT family)
- Question answering
- Code generation (GitHub Copilot)

Multimodal AI:

- Image captioning
- Visual question answering
- Video understanding
- Robotics integration



Neural networks now match or exceed human performance in many domains

25.5 Common Pitfalls in Neural Network Design

Pitfall 1: Wrong Architecture

- Using MLP for images (ignores structure)
- Solution: Match architecture to data

Pitfall 2: Too Deep Too Soon

- 100 layers without skip connections
- Solution: Start shallow, add depth incrementally

Pitfall 3: Poor Initialization

- All weights $= 0$ (symmetry breaking fails)
- Solution: Xavier/He initialization

Pitfall 4: Ignoring Overfitting

- Training accuracy 99%, test 60%
- Solution: Dropout, regularization, augmentation

Pitfall 5: Wrong Learning Rate

- Too high: Divergence, too low: No learning
- Solution: LR schedules, adaptive optimizers

Pitfall 6: Insufficient Data

- Deep networks need 1000s of examples
- Solution: Transfer learning, data augmentation

Most neural network failures stem from architecture mismatch, poor initialization, or insufficient data

26. Summary & Preview: Generative AI

What We Learned:

- Perceptrons: Linear limitations
- MLPs: Non-linear but shallow
- Deep networks: Vanishing gradients
- Modern architectures: Structured solutions

Key Insights:

- Architecture matters more than size
- Three breakthroughs: ReLU activation, specialized architectures, skip connections
- Depth provides exponential expressivity advantage

Next: Generative AI

- From classification (is this a cat?) to generation (draw me a cat)
- VAEs, GANs, Diffusion models
- Large language models
- Applications in innovation

Neural networks: From solving XOR to generating Shakespeare