Implied Risk Premia

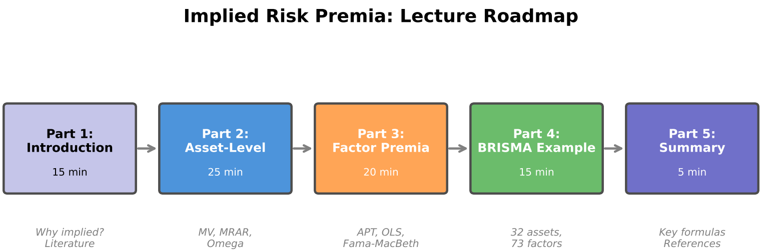
From Asset Weights to Expected Returns

BRISMA Project

Master/PhD Level

December 25, 2025

Lecture Roadmap



80 minutes: theoretical foundations through practical implementation

Part 1: Introduction

The Fundamental Question

Given:

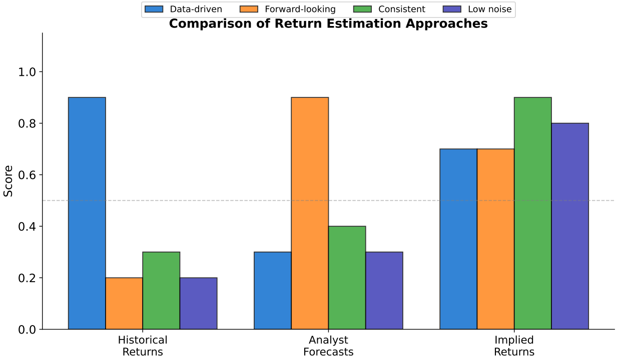
- Portfolio weights $\mathbf{w}$
- Covariance matrix $\mathbf{Q}$

Question:

What expected returns $\boldsymbol{\mu}$ would make this allocation optimal?

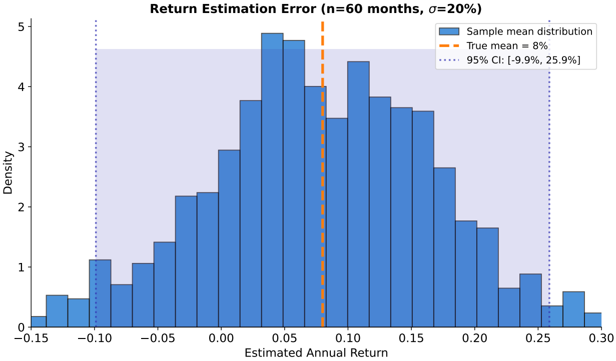
Key insight:

Extract the market's collective wisdom embedded in current allocations.



Implied returns avoid the pitfalls of historical estimation and subjective forecasts

The Estimation Challenge



Monte Carlo result:
Even with 5 years of data:

- True return: 8%
- Sample estimates vary widely
- 95% CI spans negative to 20%+

Implication:
Direct estimation of μ is unreliable. Inverse optimization provides an alternative.

Return estimation is the "Achilles' heel" of portfolio optimization (Merton, 1980)

Literature Review

Foundational Papers:

- Black & Litterman (1992) – Equilibrium implied returns
- Merton (1980) – Risk aversion estimation
- Ross (1976) – APT framework

Extensions:

- Fama & French (1993) – Multi-factor empirics
- He & Litterman (1999) – BL extensions
- Fama & MacBeth (1973) – Two-pass regression

Approach	Key Contribution
Historical	Data-driven but backward-looking
Analyst forecasts	Forward-looking but subjective
Implied returns	**Consistent, no forecasts needed**

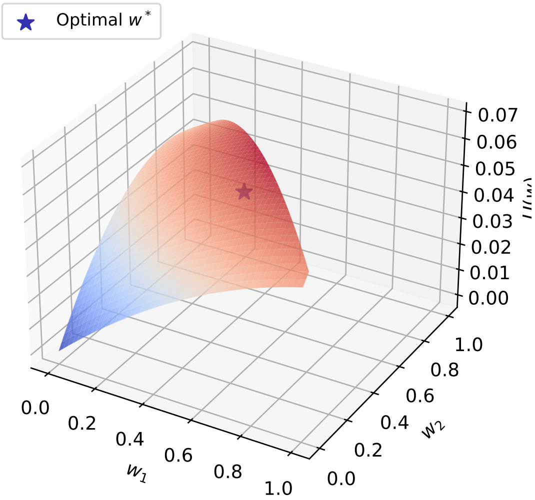
Implied returns: equilibrium theory + observed allocations

Part 2: Asset-Level Implied Returns

Optimization problem:

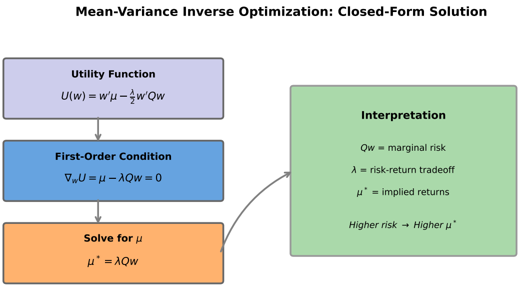
$$\max_{\mathbf{w}} \left\{ \mathbf{w}'\boldsymbol{\mu} - \frac{\lambda}{2}\mathbf{w}'\mathbf{Q}\mathbf{w} \right\}$$

$\mathbf{w}'\boldsymbol{\mu}$: Expected return
$\frac{\lambda}{2}\mathbf{w}'\mathbf{Q}\mathbf{w}$: Risk penalty



Higher λ = more risk-averse = lower risk exposure

The Closed-Form Solution



Derivation:

First-order condition:

$$\nabla_{\mathbf{w}} U = \boldsymbol{\mu} - \lambda \mathbf{Q}\mathbf{w} = \mathbf{0}$$

Implied returns:

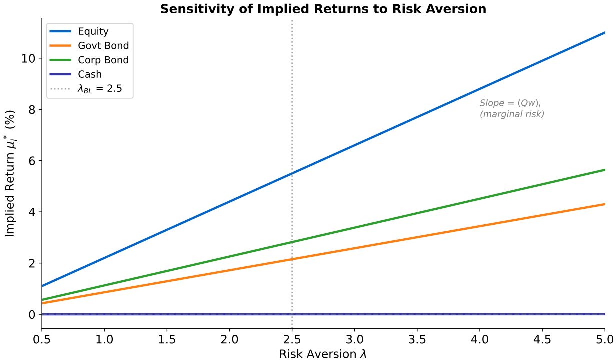
$$\boxed{\boldsymbol{\mu}^* = \lambda \mathbf{Q}\mathbf{w}}$$

Interpretation:

- $\mathbf{Qw}$ = marginal risk
- Higher risk $\rightarrow$ higher μ^*

Elegant closed-form solution: implied returns are linear in weights and risk

Risk Aversion Parameter λ



Estimation methods:

From observed returns:

$$\lambda^* = \frac{\mathbf{w}'\tilde{\boldsymbol{\mu}}}{\mathbf{w}'\mathbf{Q}\mathbf{w}}$$

Black-Litterman:

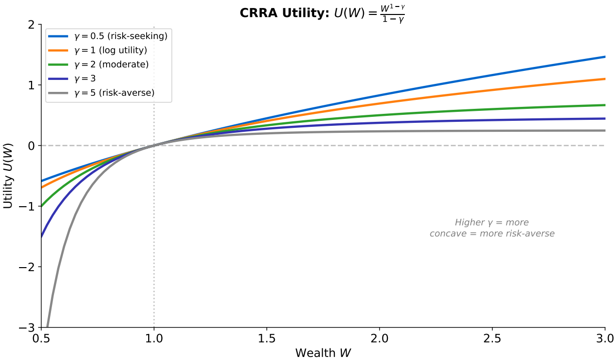
$$\lambda_{BL} = \frac{E[r_m] - r_f}{\sigma_m^2} \approx 2.5$$

Sensitivity:

$$\frac{\partial \mu_i^*}{\partial \lambda} = (\mathbf{Q}\mathbf{w})_i$$

Assets with higher marginal risk are more sensitive to λ changes

Alternative: CRRA Utility (MRAR)



CRRA utility:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

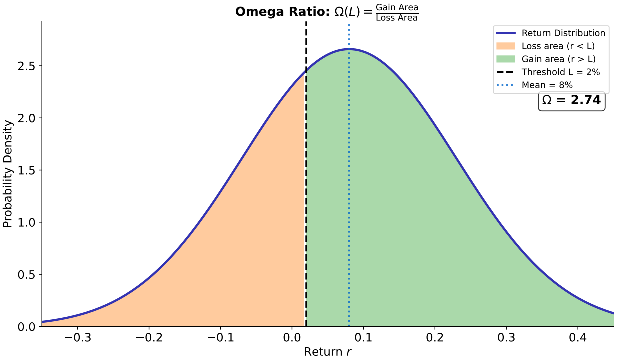
MRAR (Morningstar):

$$MRAR_\gamma = \left[\frac{1}{T}\sum_t \left(\frac{1+r_{p,t}}{1+r_{f,t}}\right)^{-\gamma}\right]^{-12/\gamma} - 1$$

Inverse optimization:
Requires bilevel numerical optimization (no closed-form).

MRAR captures asymmetric risk preferences - penalizes losses more than gains

Alternative: Omega Ratio



Definition:

$$\Omega(L) = \frac{E[\max(r - L, 0)]}{E[\max(L - r, 0)]}$$

Interpretation:

- Ratio of gains to losses
- L = threshold return
- $\Omega > 1$ is desirable

Inverse optimization:
Also requires numerical methods.

Omega focuses on downside risk – no distributional assumptions required

Utility Function Comparison

Utility	Closed-Form?	Risk Measure	Parameter
Mean-Variance	Yes	Variance (symmetric)	λ
MRAR (CRRA)	No	Power utility (asymmetric)	γ
Omega	No	LPM (downside only)	L

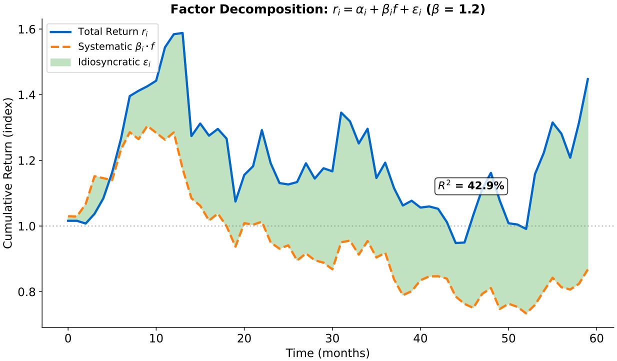
When to use each:

- **Mean-Variance**: Default choice – analytical tractability
- **MRAR**: When asymmetric risk preferences matter
- **Omega**: When downside risk is the primary concern

Mean-Variance remains dominant due to closed-form solution and interpretability

Part 3: Factor-Based Risk Premia

Why Factor Decomposition?



Curse of dimensionality:

$n = 89$ assets $\rightarrow$ 4,005 covariances

Factor model:

$$r_i = \alpha_i + \sum_j \beta_{ij} f_j + \epsilon_i$$

Benefits:

- Fewer parameters
- Economic interpretability
- New asset extension

Factor models reduce 4,005 parameters to 460 with 5 factors (89% reduction)

The APT Framework

Arbitrage Pricing Theory (Ross, 1976):

$$\mathbf{r} = \boldsymbol{\alpha} + \mathbf{B}\mathbf{f} + \boldsymbol{\epsilon}$$

Key assumptions:

- Returns driven by k common factors
- Uncorrelated idiosyncratic errors
- $E[\epsilon_i f_j] = 0$ for all i, j

Implied covariance:

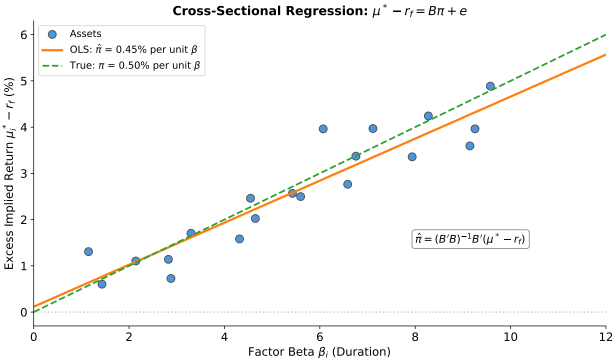
$$\mathbf{Q} = \mathbf{B}\boldsymbol{\Sigma}_f\mathbf{B}' + \mathbf{D}$$

where $\mathbf{D}$ = diagonal residual variances.

Factor	**Economic Meaning**	**Example Proxy**
Market	Aggregate equity risk	MSCI World
Duration	Interest rate sensitivity	10Y bond return
Credit	Default risk premium	IG spread

CAPM is a special case with $k = 1$ (market factor only)

Cross-Sectional Regression



Extract factor premia:

$$\boldsymbol{\mu}^* - r_f\mathbf{1} = \mathbf{B}\boldsymbol{\pi} + \mathbf{e}$$

OLS solution:

$$\boxed{\hat{\boldsymbol{\pi}} = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'(\boldsymbol{\mu}^* - r_f\mathbf{1})}$$

Standard errors:

$$\text{Var}(\hat{\boldsymbol{\pi}}) = \sigma_e^2(\mathbf{B}'\mathbf{B})^{-1}$$

Cross-sectional regression extracts factor premia from asset-level implied returns

Fama-MacBeth (1973) Two-Pass Procedure

PASS 1: Time-Series

For each asset i:

$r_{i,t} = \alpha_i + \beta_i f_t + \varepsilon_{i,t}$

Output: $\hat{B}$ *matrix*

PASS 2: Cross-Sectional

For each time t:

$r_{i,t} - r_f = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_i + \eta_{i,t}$

Output: $\{\hat{\gamma}_t\}_{t=1}^{T}$

FINAL ESTIMATE

$\hat{\pi} = \frac{1}{T}\sum_t \hat{\gamma}_t$, SE $= \frac{\sigma_\gamma}{\sqrt{T}}$

Shanken Correction:

Adjusts SE for errors-in-variables

Final estimate: $\hat{\pi}_j = \frac{1}{T}\sum_t \hat{\gamma}_{j,t} \qquad SE(\hat{\pi}_j) = \frac{\sigma_{\gamma_j}}{\sqrt{T}}$

Two-pass procedure allows time-varying premia and proper standard errors

THE BRIDGE: From Existing Portfolio to New Asset Pricing

STEP 1
Portfolio Weights w

STEP 2
Implied Returns μ^*

$\mu^* = \lambda Q w$

STEP 3
Factor Premia π

$\pi = (B'B)^{-1}B'(\mu^* - r_f)$

STEP 4
New Asset

$\mu_{new} = r_f + B'_{new}\pi$

KEY INSIGHT

New assets can be priced using only their factor betas

No historical return data required!

Price new assets using only factor betas – no historical returns needed

The formula:

$$\boxed{\mu_{new} = r_f + \mathbf{B}'_{new}\hat{\boldsymbol{\pi}}}$$

Example: BANTLEON Fund

Factor	Beta	Premium	Contribution
Equity	0.8	5.0%	4.0%
Duration	5.0	0.5%	2.5%
Credit	0.3	2.0%	0.6%
Total excess return			**7.1%**

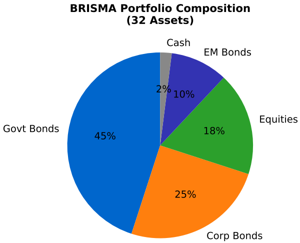
Applications:

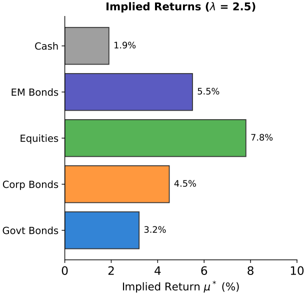
- Newly launched funds
- Private assets with no track record
- Hypothetical portfolio scenarios

This "bridge" enables pricing of any asset with known factor exposures

Part 4: BRISMA Worked Example

BRISMA Portfolio Overview





Dataset:

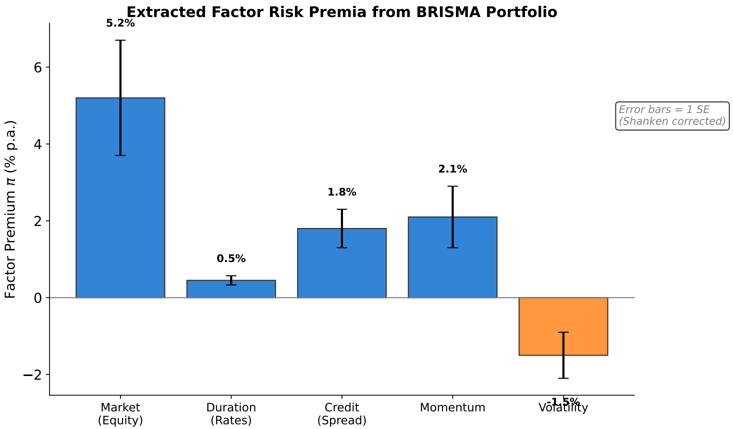
- 32 assets
- 73 risk factors
- 10 years history
- 3 covariance matrices

Parameters:

- $\lambda = 2.5$ (BL default)
- $r_f = 1.93\%$ (ESTR)

BRISMA: Bantleon Risk Model Analysis – real institutional portfolio data

Extracted Factor Premia



Market premium dominates; volatility shows negative premium (risk hedging value)

Step-by-Step Calculation

1. **Load data:**
$$\mathbf{w} \in \mathbb{R}^{32}, \quad \mathbf{Q} \in \mathbb{R}^{32\times 32}, \quad \lambda = 2.5$$
2. **Compute implied returns:**
$$\boldsymbol{\mu}^* = \lambda \mathbf{Q}\mathbf{w}$$
3. **Extract factor premia:**
$$\hat{\boldsymbol{\pi}} = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'(\boldsymbol{\mu}^* - r_f\mathbf{1})$$
4. **Price new asset:**
$$\mu_{new} = r_f + \mathbf{B}'_{new}\hat{\boldsymbol{\pi}}$$

```
from brisma.inverse_optimization import inverse_optimize_mv
mu_star = inverse_optimize_mv(weights, Q, lambda_param)
```

Complete Python implementation available in python/brisma/

Sensitivity Analysis

	Risk Aversion λ			
Asset Class	1.5	2.5	3.5	5.0
Govt Bonds	1.9%	3.2%	4.5%	6.4%
Corp Bonds	2.7%	4.5%	6.3%	9.0%
Equities	4.7%	7.8%	10.9%	15.6%
EM Bonds	3.3%	5.5%	7.7%	11.0%
Cash	1.1%	1.9%	2.7%	3.8%

Key observations:

- All returns scale linearly with λ
- Equity premium remains highest across all λ
- Cash has lowest implied return (lowest risk contribution)

Sensitivity analysis essential for robustness – results depend on λ choice

Part 5: Summary

Key Formulas

Concept	Formula
MV implied returns	$\boldsymbol{\mu}^* = \lambda \mathbf{Q}\mathbf{w}$
Constrained case	$\boldsymbol{\mu}^* = \lambda \mathbf{Q}\mathbf{w} + \gamma^* \mathbf{1}$
Lambda estimation	$\lambda = \mathbf{w}'\boldsymbol{\mu}/\mathbf{w}'\mathbf{Q}\mathbf{w}$
Factor premia (OLS)	$\hat{\boldsymbol{\pi}} = (\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'(\boldsymbol{\mu}^* - r_f \mathbf{1})$
New asset pricing	$\mu_{new} = r_f + \mathbf{B}_{new}'\hat{\boldsymbol{\pi}}$
Shanken correction	$c = 1 + \hat{\boldsymbol{\pi}}'\hat{\boldsymbol{\Sigma}}_f^{-1}\hat{\boldsymbol{\pi}}$

These six formulas form the complete toolkit for implied risk premia analysis

Key Takeaways

1. **Implied returns** avoid noisy historical estimation
2. **Mean-Variance** provides closed-form solution: $\mu^* = \lambda Q w$
3. **Factor decomposition** reduces dimensionality and enables new asset pricing
4. **Cross-sectional regression** extracts factor premia from asset returns
5. **Fama-MacBeth** procedure provides proper standard errors
6. **The Bridge**: Weights $\rightarrow$ Implied Returns $\rightarrow$ Factor Premia $\rightarrow$ New Assets

BRISMA Implementation:

- 32 assets, 73 factors, 3 covariance matrices
- Full Python library: `python/brisma/`
- Documentation: `docs/implied_risk_premia.md`

Theory + Practice = Actionable risk premia for portfolio management

References

- Black, F. & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*.
- Fama, E. & MacBeth, J. (1973). Risk, Return, and Equilibrium. *Journal of Political Economy*.
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- Ross, S. (1976). The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory*.
- Shanken, J. (1992). On the Estimation of Beta-Pricing Models. *Review of Financial Studies*.

Full bibliography in docs/implied_risk_premia.md

Questions?

Documentation: `docs/implied_risk_premia.md`
Code: `python/brisma/inverse_optimization.py`
Tutorial: `notebooks/implied_premia_tutorial.ipynb`