# Portfolio decarbonisation at least tracking error

Mathematics of green finance risk management

## The six-firm book

| firm | EVIC, USD m | emissions, Mt a year | revenue, USD m | $c_i$, t per USD m | $\sigma_i$, % | financed, kt |
|---|---|---|---|---|---|---|
| Coal | 2,600 | 4.0 | 800 | 5,000 | 42.9 | 153.8 |
| Cement | 1,000 | 3.0 | 500 | 6,000 | 53.0 | 300.0 |
| Steel | 3,600 | 6.0 | 3,000 | 2,000 | 53.6 | 166.7 |
| Port | 1,500 | 0.15 | 600 | 250 | 29.1 | 10.0 |
| Rice | 250 | 0.03 | 200 | 150 | 32.7 | 12.0 |
| Solar | 400 | 0.004 | 200 | 20 | 37.4 | 1.0 |

USD 100 m held in each firm; benchmark weights 16.7 per cent each, WACI 2,236.7; $\sigma_i$ the equity volatility of the Merton model, returns uncorrelated. Stylised values.

## Exercise, in pairs, eight minutes

A benchmark holds four equities, USD 50 m in each, weights 25 per cent each, returns uncorrelated. Compute (1) the financed emissions of each holding and of the four together; (2) the benchmark's WACI; (3) the WACI and the cut $R$ without Gas, the other three held equally; (4) the tracking error of that portfolio.

| | activity | EVIC, USD m | emissions, Mt a year | revenue, USD m | equity volatility, % |
|---|---|---|---|---|---|
| Gas | gas power plant | 1,200 | 2.4 | 600 | 30 |
| Glass | glass works | 400 | 0.6 | 200 | 40 |
| Paper | paper mill | 750 | 0.3 | 300 | 25 |
| Wind | wind farm | 600 | 0.012 | 150 | 35 |

**Check values.** The four holdings finance 196.0 kt. The benchmark's WACI is 2,020 t per USD m. Stylised values.

## Formulas

| | |
|---|---|
| financed emissions | $FE_i = (h_i / EVIC_i)\, E_i$: the holding over enterprise value including cash, times the firm's emissions (PCAF) |
| carbon intensity, WACI | $c_i = E_i / Rev_i$; $\mathrm{WACI}(w) = \sum_i w_i c_i = c^\top w$, weights by portfolio value (TCFD) |
| target cut $R$ | $c^\top w \le (1 - R)\, c^\top b$ against the benchmark $b$ |
| tracking error | $TE = \sqrt{(w-b)^\top \Sigma\, (w-b)}$; with uncorrelated returns $TE^2 = \sum_i \sigma_i^2 (w_i - b_i)^2$ |
| least tracking error | $\Delta w_i = -\kappa\,(c_i - m)/\sigma_i^2$, $m = \sum_j (c_j/\sigma_j^2) / \sum_j (1/\sigma_j^2)$, $\kappa = R\, c^\top b / S$, $S = \sum_i (c_i - m)^2/\sigma_i^2$, $TE = R\, c^\top b / \sqrt{S}$ |
| cut of stock $i$ | $-\Delta w_i = TE\,[(c_i - m)/\sigma_i^2] / \sqrt{S}$; the weight reaches zero at $R_0 = R\, b_i / \mathrm{cut}_i$; check every $w_i \ge 0$ |
| Paris-aligned path | $c_t = (1 - 50\%)(1 - 7\%)^t\, c_0$, cut $R_t = 1 - c_t / c_0$; the regulation divides by EVIC, the book by revenue |

Paris-aligned cut: Commission Delegated Regulation (EU) 2020/1818 of 17 July 2020 (EU Climate Transition and Paris-aligned Benchmarks), OJ L 406, 3.12.2020, p. 17, Article 11, first subparagraph, read 1 October 2026; the climate transition cut, the yearly reduction, GHG intensity and EVIC: Articles 9, 7, 1(c) and 1(d) of the same regulation.

## Glossary

| | |
|---|---|
| ASEAN | Association of Southeast Asian Nations, 11 members |
| CTB | EU climate transition benchmark: GHG intensity at least 30% under its universe's |
| EVIC | enterprise value including cash: the market capitalisation of ordinary and preferred shares plus the book value of total debt and non-controlling interests, without deducting cash |
| GDP | gross domestic product, the sovereign version's divisor |
| GHG | greenhouse gas, $CO_2$ and the others in tonnes of $CO_2$ equivalent; GHG intensity: absolute GHG emissions divided by millions of euros in enterprise value including cash |
| PAB | EU Paris-aligned benchmark: GHG intensity at least 50% under its universe's |
| PCAF | Partnership for Carbon Accounting Financials, the standard for financed emissions |
| TCFD | Task Force on Climate-related Financial Disclosures |
| TE | tracking error, the volatility of $r_w - r_b$ |
| WACI | weighted average carbon intensity, t $CO_2$ per USD m of revenue |

ASEAN members: Agence Kampuchea Presse (AKP), Phnom Penh, 26 October 2025, read 1 October 2026. PCAF: PCAF (2025), Global GHG Accounting and Reporting Standard Part A, 3rd ed., section 5.2, p. 58, read 30 September 2026. TCFD: TCFD (2021), Implementing the Recommendations, Table 3, p. 52, read 30 September 2026.

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