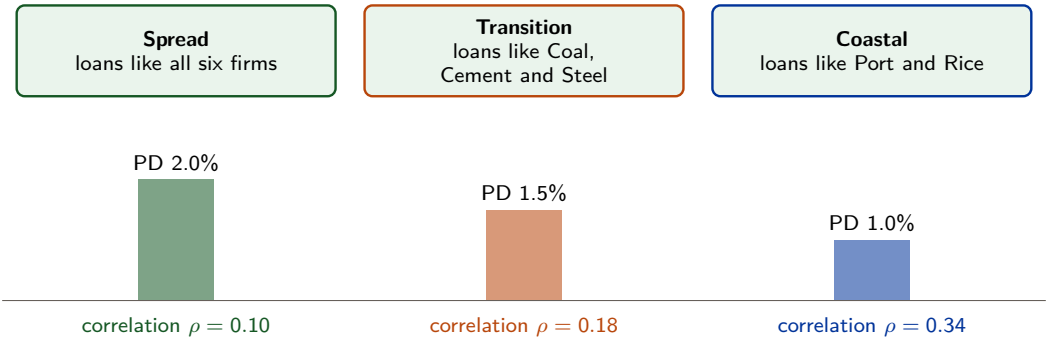
Vasicek portfolio loss and capital with a climate factor

Mathematics of green finance risk management

Joerg Osterrieder

University of Twente

Three loan books, each with a PD and a correlation



The six-firm book holds six stylised firms: Coal, Cement, Steel, Port, Rice and Solar. Each book holds 1,000 loans of USD 1 m for 1 year at a loss given default (LGD) of 40 per cent; two loans of one book have asset correlation ρ. Capital covers the book's loss above its expected loss in all but the worst year in a thousand.

Rank the three books by the capital each needs at 99.9 per cent.

Stylised values.

A threshold, a factor and a quantile turn a PD into a book's capital

1. Compute a loan's default threshold $\Phi^{-1}(\text{PD})$ and its PD given the economy factor, $p(z)$
2. Compute the 99.9 per cent quantile of a large book's default rate and the capital LGD $(q - \text{PD})$
3. Compute the Basel correlation R and the IRB capital of a book from its PD
4. Explain why the IRB formula gives two books of one PD the same capital
5. Compute a book's correlation $b^2 + c^2$ from its loadings on the economy and climate factors, and its capital
6. Compute a loan's PD under a climate stress $C = x$, and say what a supervisory stress test leaves out

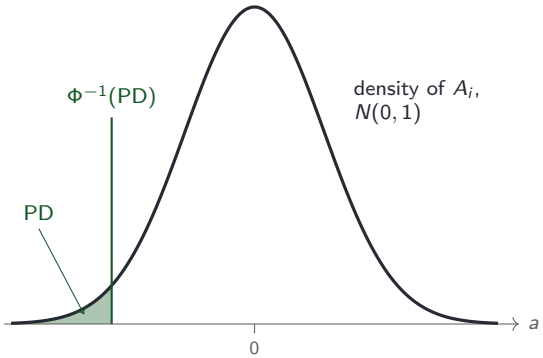
Default thresholds, two factors and a quantile set a loan book's capital

1. How does one common factor make defaults move together?
2. How is the loss of a large book distributed?
3. How does the Basel IRB formula set capital?
4. How does a climate factor raise a concentrated book's capital?

A loan defaults when its asset return falls below a threshold set by its PD. An economy factor and a climate factor move the loans of a book together, and capital covers the book's loss above its expected loss up to its 99.9 per cent quantile.

A loan defaults when its asset return falls below a PD threshold

1 2 3 4 **How does one common factor make defaults move together?**



a Spread loan

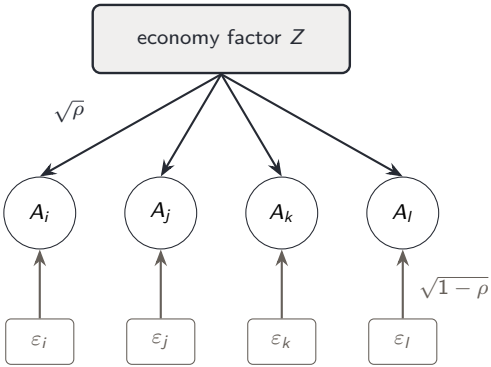
In the Merton (1974) model a firm defaults when its assets end below its debt; after its shock Coal's PD is 1.067 per cent.

Write the asset return as a standard normal A_i. The loan defaults when $A_i < \Phi^{-1}(\text{PD})$, which happens with probability PD. A Spread loan, PD 2.0 per cent, has the threshold $\Phi^{-1}(2.0\%) = -2.054$.

Stylised values.

What is the default threshold of a Coastal loan, PD 1.0 per cent, and how far below the Spread threshold does it lie?

One economy factor in every loan's return gives two loans correlation ρ



$$A_i = \sqrt{\rho}\, Z + \sqrt{1-\rho}\, \varepsilon_i$$
$$\text{Var}\, A_i = \rho + (1-\rho) = 1$$
$$\text{Cov}(A_i, A_j) = \sqrt{\rho}\,\sqrt{\rho}\,\text{Var}\, Z = \rho$$

A_i is loan i's asset return; Z and every ε_i are independent and $N(0,1)$. Two loans move together through Z alone. The three books have $\rho = 0.10$, 0.18 and 0.34. Vasicek (2002) builds a book's loss on this model.

Stylised values.

A loan loads 0.6 on Z. What correlation does it share with an identical loan, and what weight does its own shock get?

Fixing the economy factor at z turns the default probability into $p(z)$

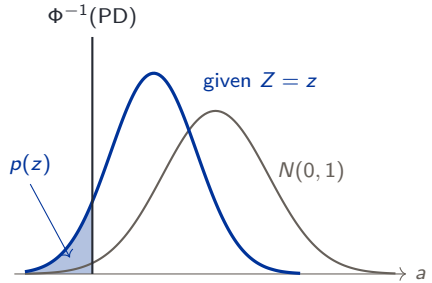
$$A_i < \Phi^{-1}(\mathrm{PD}) \qquad \text{default}$$

$$\sqrt{\rho}\,z + \sqrt{1-\rho}\,\varepsilon_i < \Phi^{-1}(\mathrm{PD}) \qquad \text{fix } Z = z$$

$$\varepsilon_i < \frac{\Phi^{-1}(\mathrm{PD}) - \sqrt{\rho}\,z}{\sqrt{1-\rho}} \qquad \text{subtract, divide}$$

$$p(z) = \Phi\left(\frac{\Phi^{-1}(\mathrm{PD}) - \sqrt{\rho}\,z}{\sqrt{1-\rho}}\right) \qquad \varepsilon_i \sim N(0,1)$$

With the economy factor fixed at z, every loan of a book defaults with the same probability $p(z)$, and the defaults are independent: each depends on its own shock ε_i alone.



A Coastal loan at $z = -2.0$: the density centres on $\sqrt{\rho}\,z$, with standard deviation $\sqrt{1-\rho}$. Stylised values.

In an average year, $z = 0$, what is $p(0)$ for a Coastal loan, and why does it lie below the loan's PD?

Default probability of a Coastal loan at $z = -2.0$

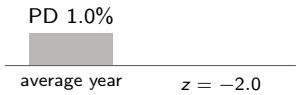
Coastal: PD 1.0 per cent, $\rho = 0.34$, $\sqrt{\rho} = 0.583$, $\sqrt{1-\rho} = 0.812$; the default probability in a year with $z = -2.0$:

$$\Phi^{-1}(\text{PD}) = -2.326$$
$$\sqrt{\rho}\, z = 0.583 \times (-2.0) = -1.166$$
$$\Phi^{-1}(\text{PD}) - \sqrt{\rho}\, z = -2.326 - (-1.166) = -1.160$$
$$\text{divided by } \sqrt{1-\rho} = -1.160/0.812 = -1.4286$$
$$p(-2.0) = \Phi(-1.4286) =$$

Guess first: how many times its PD does a Coastal loan default in this year?

Stylised values.

?

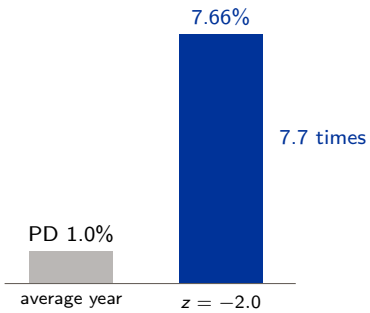


What is $p(-2.0)$ for a Spread loan, with PD 2.0 per cent and $\rho = 0.10$?

Default probability of a Coastal loan at $z = -2.0$

Coastal: PD 1.0 per cent, $\rho = 0.34$, $\sqrt{\rho} = 0.583$, $\sqrt{1-\rho} = 0.812$; the default probability in a year with $z = -2.0$:

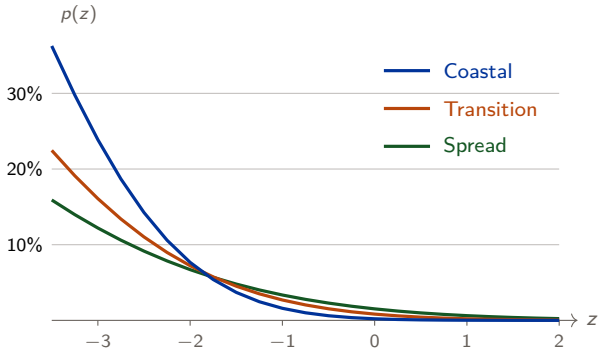
$$\Phi^{-1}(\text{PD}) = -2.326$$
$$\sqrt{\rho}\, z = 0.583 \times (-2.0) = -1.166$$
$$\Phi^{-1}(\text{PD}) - \sqrt{\rho}\, z = -2.326 - (-1.166) = -1.160$$
$$\text{divided by } \sqrt{1-\rho} = -1.160/0.812 = -1.4286$$
$$p(-2.0) = \Phi(-1.4286) = 7.66\%$$



Stylised values.

What is $p(-2.0)$ for a Spread loan, with PD 2.0 per cent and $\rho = 0.10$?

The three books' default rates cross as the economy factor falls



The default rates $p(z)$ fall as the economy factor z rises. In an average year Spread defaults most: $p(0) = 1.52$ per cent against Coastal's 0.21. In a bad year the order turns: at $z = -2.0$ Coastal reaches 7.66, Transition 7.22 and Spread 6.70 per cent.

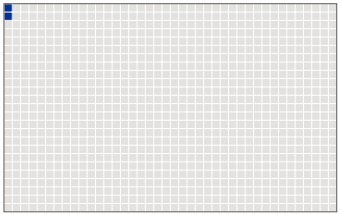
A larger ρ makes $p(z)$ steeper, so the curves cross: Coastal and Spread at $z = -1.82$, both at 6.0 per cent.

Stylised values.

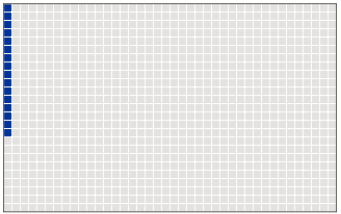
The chart leaves out a book with PD 3.0 per cent and $\rho = 0.10$. At which z does a Coastal loan default as often as one of its loans?

Given the economy factor, a book of 1,000 loans defaults at rate $p(z)$

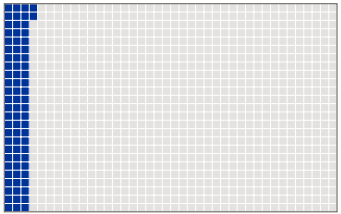
1 2 3 4 **How is the loss of a large book distributed?**



$z = 0$
2 defaults, $p(z) = 0.21\%$



$z = -1.0$
16 defaults, $p(z) = 1.59\%$



$z = -2.0$
77 defaults, $p(z) = 7.66\%$

Given the economy factor z, the 1,000 Coastal loans default independently, each with probability $p(z)$. The count is binomial with mean $n\,p(z)$, and the default rate has standard deviation $\sqrt{p(1-p)/n}$: 0.84 points at $z = -2.0$. As n grows the rate settles on $p(z)$, and the book's loss rate on LGD $p(Z)$.

Stylised values.

What is the standard deviation of the default rate at $z = -2.0$ in a book of 100,000 Coastal loans?

The default rate's α quantile is $p(z)$ at the economy factor's $1-\alpha$ quantile

$$L = \mathrm{LGD}\, p(Z) \qquad \text{large book}$$

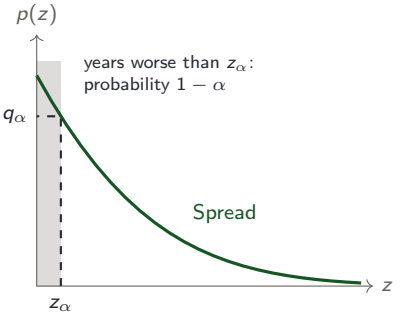
$$P[L \le \mathrm{LGD}\, p(z)] = P[Z \ge z] = \Phi(-z) \qquad p \text{ falls as } z \text{ rises}$$

$$\Phi(-z_\alpha) = \alpha \;\Rightarrow\; z_\alpha = -\Phi^{-1}(\alpha) \qquad \text{solve for } z_\alpha$$

$$q_\alpha = p(z_\alpha) = \Phi\left(\frac{\Phi^{-1}(\mathrm{PD}) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right) \qquad \text{substitute}$$

$$K = \mathrm{LGD}\,(q_\alpha - \mathrm{PD}) \qquad \text{less LGD} \cdot \mathrm{PD}$$

z_α is the economy factor's $1-\alpha$ quantile, q_α the default rate's α quantile and LGD q_α the loss rate's; K leaves out the expected loss. Vasicek (2002) states this distribution. Stylised values.



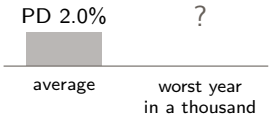
Vasicek (2002), Loan portfolio value, *Risk*, December 2002, pp. 160 to 162: BCBS (2005), An Explanatory Note on the Basel II IRB Risk Weight Functions, July 2005, References, p. 15, read 1 October 2026.

What is z_α at $\alpha = 99$ per cent, and is the book's default rate there above or below its 99.9 per cent default rate?

The 99.9 per cent default rate and capital of the Spread book

Spread: PD 2.0 per cent, $\rho = 0.10$, $\sqrt{\rho} = 0.3162$, $\sqrt{1-\rho} = 0.9487$, LGD $= 40$ per cent.

$$
\begin{aligned}
z_\alpha = -\Phi^{-1}(99.9\%) &= -3.0902 \\
\sqrt{\rho}\, z_\alpha &= 0.3162 \times (-3.0902) = -0.9771 \\
\Phi^{-1}(\text{PD}) - \sqrt{\rho}\, z_\alpha &= -2.0537 - (-0.9771) = -1.0766 \\
\text{divided by } \sqrt{1-\rho} &= -1.0766/0.9487 = -1.1348 \\
q &= \Phi(-1.1348) = \\
K &= 40\% \times (q - 2.0\%) =
\end{aligned}
$$



Guess first: the default rate of the worst year in a thousand, for a PD of 2.0 per cent.

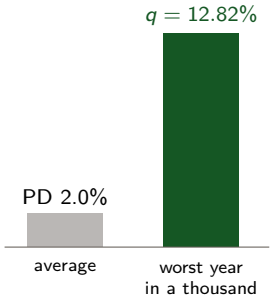
Stylised values.

What are the Spread book's 99 per cent default rate and capital, with $\Phi^{-1}(99\%) = 2.326$*?*

The 99.9 per cent default rate and capital of the Spread book

Spread: PD 2.0 per cent, $\rho = 0.10$, $\sqrt{\rho} = 0.3162$, $\sqrt{1-\rho} = 0.9487$, LGD $= 40$ per cent.

$$z_\alpha = -\Phi^{-1}(99.9\%) = -3.0902$$
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$$\Phi^{-1}(\text{PD}) - \sqrt{\rho}\, z_\alpha = -2.0537 - (-0.9771) = -1.0766$$
$$\text{divided by } \sqrt{1-\rho} = -1.0766/0.9487 = -1.1348$$
$$q = \Phi(-1.1348) = 12.82\%$$
$$K = 40\% \times (q - 2.0\%) = 4.33\%$$

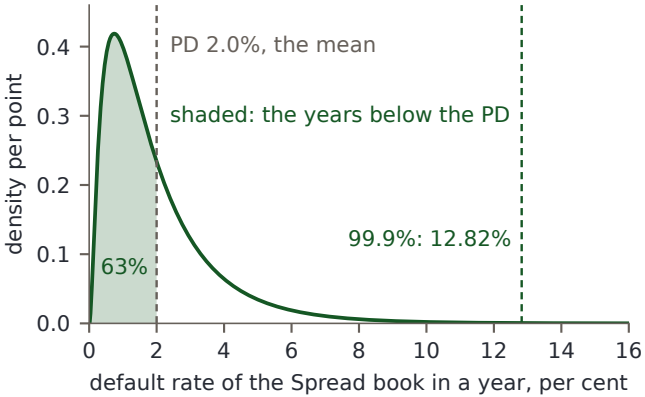


The book needs USD 43.3 m of capital on USD 1,000 m lent.

Stylised values.

What are the Spread book's 99 per cent default rate and capital, with $\Phi^{-1}(99\%) = 2.326$?

Spread's default rate stays below its PD in 63 per cent of years



The density of the Spread book's default rate is skewed: its mean is the PD, 2.0 per cent, its median $p(0) = 1.52$ per cent, its 99.9 per cent quantile 12.82 per cent.

$p(z)$ equals the PD at

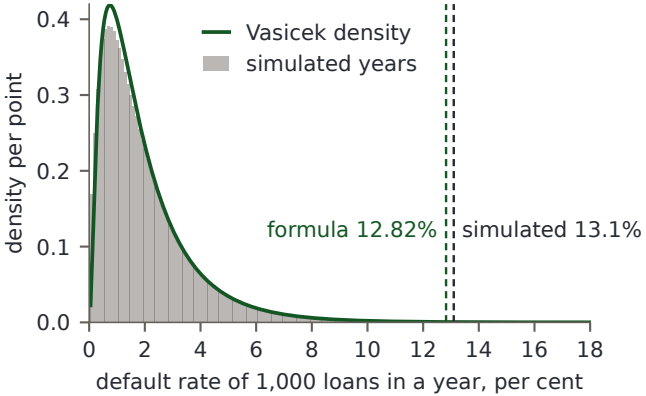
$$z^* = \Phi^{-1}(\text{PD})\,\frac{1-\sqrt{1-\rho}}{\sqrt{\rho}} = -0.33,$$

so the rate stays below the PD in the years with $z > z^*$: $\Phi(-z^*) = 63$ per cent of years.

Stylised values.

In what share of years does a large Coastal book, PD 1.0 per cent and $\rho = 0.34$, default below its PD?

A simulated book of 1,000 Spread loans follows the Vasicek density



In each of 1,000,000 simulated years, z is drawn from $N(0,1)$ and the number of defaults among 1,000 Spread loans from a binomial with probability $p(z)$.

The simulated 99.9 per cent default rate is 13.1 per cent against the formula's 12.82: at 1,000 loans the own shocks add little to the economy's.

Stylised values.

Would a book of 50 Spread loans have a 99.9 per cent default rate above or below 13.1 per cent, and why?

Capital of the River book of 500 loans

loans	PD	correlation	LGD	quantile	economy factor
500 of USD 1 m	2.5%	$\rho = 0.25$	40%	$\alpha = 99.9\%$	$z = -1.5$

$$\Phi^{-1}(2.5\%) = -1.960, \quad \Phi^{-1}(99.9\%) = 3.090, \quad \sqrt{\rho} = 0.500, \quad \sqrt{1-\rho} = 0.866$$

In pairs, eight minutes, for the River book:

1. the PD of one loan in a year with $z = -1.5$, $p(-1.5)$;
2. the 99.9 per cent quantile q of the book's default rate;
3. the capital $K = \text{LGD}\,(q - \text{PD})$, in per cent and in USD m.

$\Phi(x)$ is NORM.S.DIST(x, TRUE) in a spreadsheet.

Stylised values.

The River book needs USD 58.2 m of capital on its 500 loans

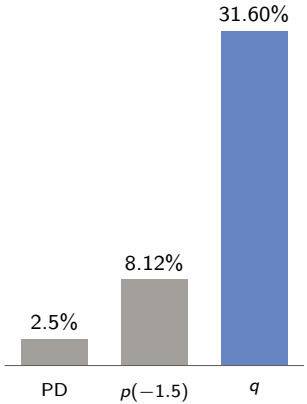
$$p(-1.5) = \Phi\left(\frac{-1.960 - (-0.75)}{0.866}\right) = \Phi(-1.397) = 8.12\%$$

$$q = \Phi\left(\frac{-1.960 - (-1.545)}{0.866}\right) = \Phi(-0.479) = 31.60\%$$

$$K = 40\% \times (31.60\% - 2.5\%) = 11.64\%$$

$$\text{capital} = 500 \times 11.64\% = \text{USD } 58.2 \text{ m}$$

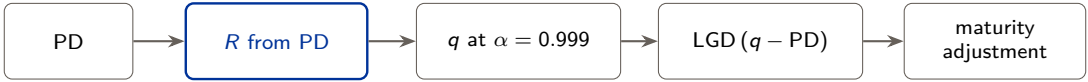
The product $\sqrt{\rho}\, z_\alpha$ is $0.500 \times (-3.090) = -1.545$. In the worst year in a thousand the River book's 500 loans default at 12.6 times their PD.



Stylised values.

The Basel IRB formula is the Vasicek capital at 99.9 per cent

1 2 **3** 4 **How does the Basel IRB formula set capital?**



$$K = \left[\mathrm{LGD}\,\Phi\left(\frac{\Phi^{-1}(\mathrm{PD}) + \sqrt{R}\,\Phi^{-1}(0.999)}{\sqrt{1-R}}\right) - \mathrm{PD}\cdot\mathrm{LGD}\right]\frac{1+(M-2.5)\,m}{1-1.5\,m},$$

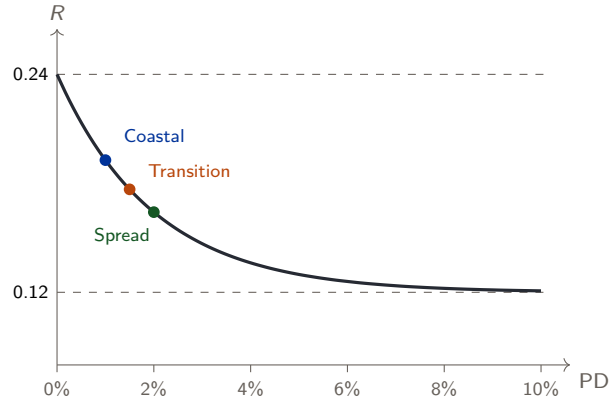
$$m = (0.11852 - 0.05478\ln \mathrm{PD})^2.$$

The internal ratings-based (IRB) formula is the Vasicek capital with ρ written R; M is the maturity in years and m its term, b in the Basel text. At $M = 1$ year the adjustment is 1: the books' capital is the IRB formula at $M = 1$ year.

BCBS (2006), International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Comprehensive Version, June 2006, paragraph 272, p. 64, read 1 October 2026.

What maturity adjustment does a loan with PD 1.0 per cent get at $M = 2.5$ years?

The Basel correlation falls from 0.24 toward 0.12 as PD rises



Stylised values.

$$w = \frac{1 - \exp(-50\,\mathrm{PD})}{1 - \exp(-50)}$$

$$R = 0.12\,w + 0.24\,(1 - w)$$

In the Basel formula the weight w rises from zero toward one as PD rises, so R falls from 0.24 toward 0.12. The three books get $R = 0.164$, 0.177 and 0.193: PD alone sets them.

BCBS (2006), International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Comprehensive Version, June 2006, paragraph 272, p. 64, read 1 October 2026.

At what PD does the formula give a correlation R of 0.15?

IRB capital of the Spread book at $M = 1$ year

$$w = \frac{1 - e^{-50 \times 2.0\%}}{1 - e^{-50}} = 0.63212$$

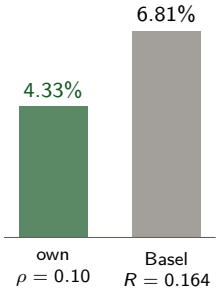
$$R = 0.12 \times 0.63212 + 0.24 \times 0.36788 = 0.16415$$

$$\sqrt{R}\, z_\alpha = 0.4052 \times (-3.0902) = -1.2521$$

$$\frac{\Phi^{-1}(\mathrm{PD}) - \sqrt{R}\, z_\alpha}{\sqrt{1 - R}} = \frac{-2.0537 - (-1.2521)}{0.9142} = -0.8768$$

$$q = \Phi(-0.8768) = 19.03\%$$

$$K = 40\% \times (19.03\% - 2.0\%) = 6.81\%$$

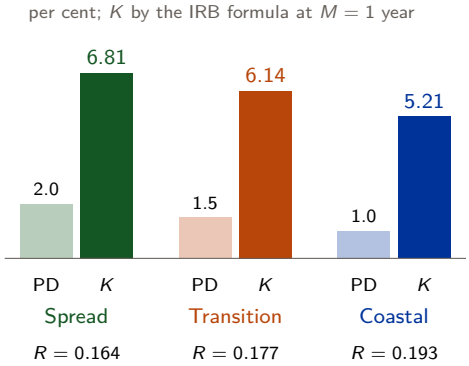


Basel's R lies above the book's own ρ, so the IRB formula asks the Spread book for more capital than its own model.

Stylised values.

What Basel correlation and IRB capital at $M = 1$ year does the Coastal book get with its PD of 1.0 per cent?

The IRB formula ranks the three books by PD, Spread first



Basel's R moves by 0.029 from Spread to Coastal, and against the PD: a lower PD gets a higher R. $\Phi^{-1}(\text{PD})$ in the numerator sets the order, and the IRB capital at $M = 1$ year follows the PD: 6.81, 6.14 and 5.21 per cent.

Stylised values.

A book with Coastal's PD of 1.0 per cent has own correlation 0.10. Compare its IRB capital with its capital at that correlation, and explain the gap.

The formula reproduces the Basel Committee's illustrative risk weights within 0.01 points

PD	Annex 5	formula	three decimals
1.0%	92.32%	92.32%	92.317%
2.0%	114.86%	114.85%	114.854%

Annex 5 of the Basel Committee's framework tabulates risk weights $12.5\,K$, the risk-weighted assets per unit of exposure at default, at $M = 2.5$ years and LGD 45%, within 0.01 points of the formula.

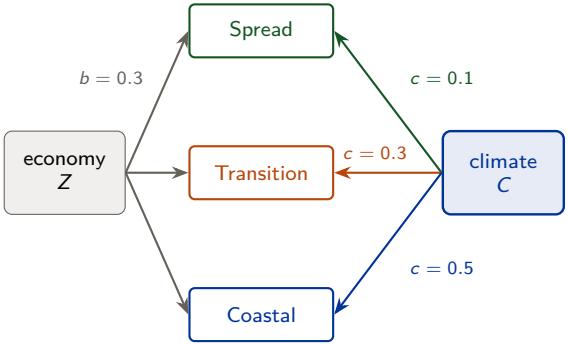
In the foundation approach the bank estimates PD and the rules set LGD 40% on non-financial corporates, 45% on sovereigns, banks and other financial institutions, and $M = 2.5$ years: the books need 8.17%, 7.51%, 6.56%.

BCBS (2006), International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Comprehensive Version, June 2006, Annex 5, Illustrative IRB Risk Weights for UL, corporate exposures, LGD 45%, M 2.5 years, turnover EUR 50 million, p. 279, read 1 October 2026. BCBS, Basel Framework, CRE32 (version in force from 1 January 2023, published 27 March 2020), paragraph 32.6, read 1 October 2026; *M*: paragraph 32.44.

The Coastal book's IRB capital at $M = 1$ year is 5.21 per cent. What risk weight is that?

A climate loading c adds c^2 to a book's correlation

1 2 3 **4** **How does a climate factor raise a concentrated book's capital?**



$$A_i = b\,Z + c\,C + \sqrt{1 - b^2 - c^2}\,\varepsilon_i$$

$$\text{Var}\,A_i = b^2 + c^2 + (1 - b^2 - c^2) = 1$$

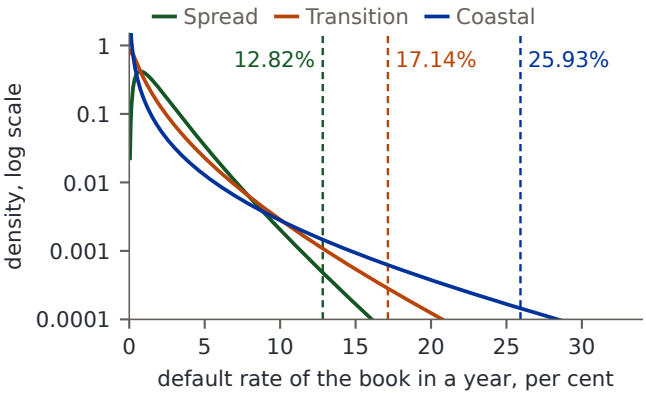
$$\text{Corr}(A_i, A_j) = b^2 + c^2 \quad \text{one book}$$

One climate factor C drives all three books; Z, C and the ε_i are independent $N(0,1)$. Every book loads 0.3 on Z; the loading c separates them: Spread $0.09 + 0.01 = 0.10$, Transition $0.09 + 0.09 = 0.18$, Coastal $0.09 + 0.25 = 0.34$.

Stylised values.

A loan loads 0.3 on Z and 0.6 on C. What weight does its own shock get, and what correlation do two such loans share?

At their own correlations Coastal's default rate has the longest tail



Each curve is the density of a large book's default rate at its own correlation $b^2 + c^2$: 0.10, 0.18 and 0.34. A dashed line marks the 99.9 per cent quantile: Spread 12.82, Transition 17.14, Coastal 25.93 per cent.

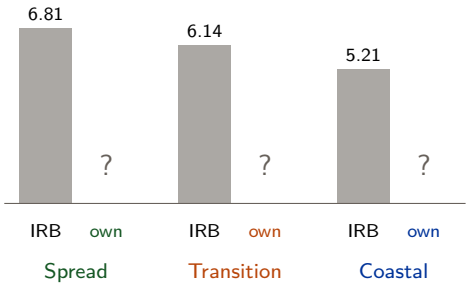
Basel's R, 0.164 to 0.193, comes from PD alone: Spread's own correlation lies below it and Coastal's above it. In the tail Coastal defaults most.

Stylised values.

To what climate loading would Coastal's c have to fall for its own correlation to equal its Basel R of 0.193?

Capital of the three books under their own correlations and under Basel

capital K, per cent
Basel IRB at $M = 1$ year; own at the book's correlation

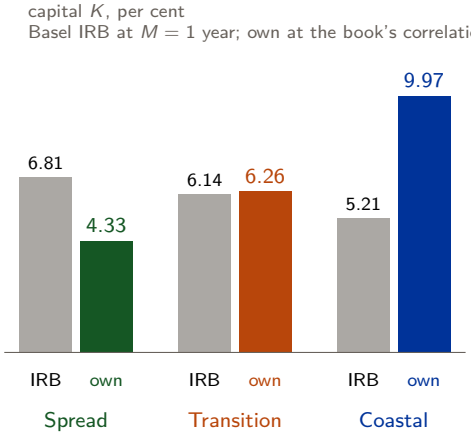


The IRB formula gives Coastal 5.21 per cent. Guess its capital at its own correlation of 0.34.

Stylised values.

What capital would the Coastal book need if its climate loading fell to Transition's 0.3?

Capital of the three books under their own correlations and under Basel



$K = \mathrm{LGD}\,(q - \mathrm{PD})$ with the default rates 25.93, 17.14 and 12.82 per cent: Coastal $40\% \times (25.93 - 1.0) = 9.97$, Transition 6.26, Spread 4.33 per cent. The order reverses. Coastal needs 1.9 times its IRB capital, USD 99.7 m against USD 52.1 m, a gap of USD 47.6 m.

Stylised values.

What capital would the Coastal book need if its climate loading fell to Transition's 0.3?

A climate stress at $C = -2.0$ raises Coastal's PD above Spread's

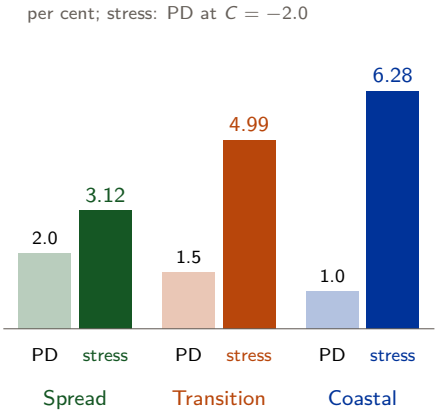
$$A_i = c\,C + \sqrt{1 - c^2}\,U_i, \quad U_i \sim N(0,1)$$

$$\mathrm{PD}(x) = \Phi\left(\frac{\Phi^{-1}(\mathrm{PD}) - c\,x}{\sqrt{1 - c^2}}\right)$$

$$\text{Coastal: } \frac{-2.3263 - (-1.0)}{0.8660} = -1.5315$$

$$\mathrm{PD}(-2.0) = \Phi(-1.5315) = 6.28\%$$

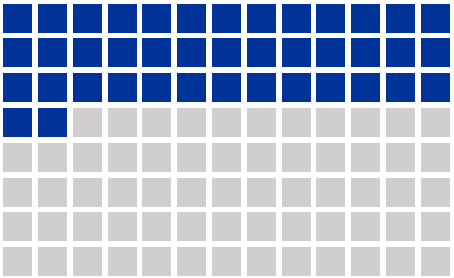
U_i collects $b\,Z$ and the own shock, divided by $\sqrt{1 - c^2}$. The model fixes $C = x$ and averages over Z; a supervisory climate scenario also fixes the economy's path.



Stylised values.

What is a Coastal loan's PD under a milder stress, $C = -1.0$?

The ECB's 2022 stress test projected 41 banks' losses at around EUR 70 billion



41 gave projections

63 gave no projections

104 banks took part, all cells

All 104 banks reported their stress-testing capabilities and reliance on carbon-emitting sectors; 41 directly supervised banks also projected losses.

Under a three-year disorderly transition scenario and two physical scenarios, their credit and market losses would amount to around EUR 70 billion; the ECB adds that "this estimate significantly understates the actual risk".

ECB Banking Supervision (2022), 2022 climate risk stress test, July 2022, Executive summary, p. 5, read 1 October 2026. Participation: ECB Banking Supervision, press release, 8 July 2022.

Which input of the climate-stress formula plays the role of the ECB's fixed scenario, and what does the result leave out?

Correlation inside a book decides its capital, and Basel sets R from PD alone

1	How does one common factor make defaults move together?	A loan defaults when $\sqrt{\rho}\,Z + \sqrt{1-\rho}\,\varepsilon_i < \Phi^{-1}(\text{PD})$; given $Z = z$ all default with one $p(z)$: Coastal's 1.0 per cent becomes 7.66 at $z = -2.0$.
2	How is the loss of a large book distributed?	Its loss rate is LGD $p(Z)$, with α quantile LGD $p(-\Phi^{-1}(\alpha))$ and capital LGD $(q - \text{PD})$: Spread's q is 12.82 per cent.
3	How does the Basel IRB formula set capital?	The IRB formula is that capital at 0.999 with R set by PD alone; at $M = 1$ year Spread 6.81, Transition 6.14, Coastal 5.21 per cent, the PD order.
4	How does a climate factor raise a concentrated book's capital?	A climate loading c adds c^2 to a book's correlation: at their own 0.10, 0.18 and 0.34 the books need 4.33, 6.26 and 9.97 per cent, the reverse order.

Stylised values.

Sources on loan portfolio value, Basel capital and climate stress tests

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- ECB Banking Supervision (2022). 2022 climate risk stress test, July 2022; press release of 8 July 2022.
- R. C. Merton (1974). On the pricing of corporate debt: the risk structure of interest rates. *Journal of Finance* 29(2), 449 to 470.
- O. Vasicek (2002). Loan portfolio value. *Risk*, December 2002, pp. 160 to 162.

Read on 1 October 2026, the press release on 30 September 2026.

Declarations

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Checkpoint: default rate, Basel capital and the climate factor

1. A loan has PD 3.0 per cent and $\rho = 0.20$. What is $p(-2.0)$?
2. Find the 99.9 per cent default rate and capital of a large book of such loans, LGD 40 per cent.
3. Find the Basel R and IRB capital at $M = 1$ year for PD 3.0 per cent.
4. Spread loans, PD 2.0 per cent and $b = 0.3$, move to Coastal's region at $c = 0.45$. How do their IRB and own capital change?
5. A book with PD 1.5 per cent has $b = c = 0.4$. What are its ρ and capital?
6. A loan with PD 2.0 per cent loads 0.4 on the climate factor. Find its PD at $C = -1.5$ and what that PD leaves out.

Stylised values.



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Vasicek portfolio loss and capital with a climate factor - Mathematics of Green Finance Risk Management, Lecture 5

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The project site is https://digital-ai-finance.github.io/Green-Finance. A question about this file goes to joerg.osterrieder@utwente.nl.

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