Vasicek portfolio loss and capital with a climate factor

Mathematics of green finance risk management

Formulas

one factor	$A_i = \sqrt{\rho}\,Z + \sqrt{1-\rho}\,\varepsilon_i$, with Z and every ε_i independent and $N(0,1)$; loan i defaults when $A_i < \Phi^{-1}(\text{PD})$
PD given $Z = z$	$p(z) = \Phi\left[(\Phi^{-1}(\text{PD}) - \sqrt{\rho}\,z)/\sqrt{1-\rho}\right]$
large book	loss rate $L = \text{LGD}\,p(Z)$; its α quantile is $\text{LGD}\,q_\alpha$, with $q_\alpha = p(z_\alpha) = \Phi\left[(\Phi^{-1}(\text{PD}) + \sqrt{\rho}\,\Phi^{-1}(\alpha))/\sqrt{1-\rho}\right]$ and $z_\alpha = -\Phi^{-1}(\alpha)$
capital	$K = \text{LGD}\,(q_\alpha - \text{PD})$ per unit lent; expected loss $\text{EL} = \text{LGD} \cdot \text{PD}$
Basel correlation	$w = [1 - \exp(-50\,\text{PD})]/[1 - \exp(-50)]$, $\quad R = 0.12\,w + 0.24\,(1-w)$
IRB capital	$K = \left[\text{LGD}\,\Phi\left((\Phi^{-1}(\text{PD}) + \sqrt{R}\,\Phi^{-1}(0.999))/\sqrt{1-R}\right) - \text{PD} \cdot \text{LGD}\right] \times \text{MA}$, $\quad \text{RWA} = 12.5\,K \cdot \text{EAD}$
maturity adjustment	$\text{MA} = [1 + (M-2.5)\,m]/(1-1.5\,m)$, $\quad m = (0.11852 - 0.05478 \ln \text{PD})^2$; $\quad \text{MA} = 1$ at $M = 1$ year
two factors	$A_i = b\,Z + c\,C + \sqrt{1-b^2-c^2}\,\varepsilon_i$, one C for all three books; two loans of one book have correlation $\rho = b^2 + c^2$
climate stress	with the climate factor fixed at $C = x$: $\text{PD}(x) = \Phi\left[(\Phi^{-1}(\text{PD}) - c\,x)/\sqrt{1-c^2}\right]$

BCBS (2006), International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Comprehensive Version, June 2006, paragraph 272, p. 64, read 1 October 2026. Vasicek (2002), Loan portfolio value, *Risk*, December 2002, pp. 160 to 162.

Spread, Transition and Coastal

book	PD, %	loading b on Z	loading c on C	own $\rho = b^2 + c^2$	Basel R
Spread, loans like all six firms	2.0	0.3	0.1	0.10	0.164
Transition, loans like Coal, Cement and Steel	1.5	0.3	0.3	0.18	0.177
Coastal, loans like Port and Rice	1.0	0.3	0.5	0.34	0.193

Each book holds 1,000 loans of USD 1 m for 1 year, at LGD 40 per cent; capital at $\alpha = 99.9$ per cent, by the IRB formula at $M = 1$ year and at the book's own ρ. The climate stress fixes $C = -2.0$. The six-firm book holds six stylised firms: Coal, Cement, Steel, Port, Rice and Solar.

Stylised values.

Exercise, in pairs, eight minutes

The River book holds 500 loans of USD 1 m, each with PD 2.5 per cent, asset correlation $\rho = 0.25$ and LGD 40 per cent. Find

1. the PD of one loan in a year with $z = -1.5$, $p(-1.5)$;
2. the 99.9 per cent quantile q of the book's default rate;
3. the capital $K = \text{LGD}\,(q - \text{PD})$, in per cent and in USD m.

$\Phi^{-1}(2.5\%) = -1.960 \quad \Phi^{-1}(99.9\%) = 3.090 \quad \sqrt{\rho} = 0.500 \quad \sqrt{1-\rho} = 0.866$

In a spreadsheet, $\Phi(x)$ is NORM.S.DIST(x, TRUE) and $\Phi^{-1}(p)$ is NORM.S.INV(p).

Check values. The argument of Φ in $p(-1.5)$ is -1.397. The product $\sqrt{\rho}\,z_\alpha$ in q is -1.545. Stylised values.

Glossary

ASRF	asymptotic single risk factor: a book so large and so spread that one factor alone drives its loss rate
BCBS	Basel Committee on Banking Supervision
CRE	the credit risk chapters of the Basel Framework: CRE31 holds the IRB formula, CRE32 its inputs
ECB	European Central Bank
EAD	exposure at default: the amount owed when the loan defaults
EL	expected loss, $\text{LGD} \cdot \text{PD}$ per unit lent
F-IRB	foundation IRB: the bank estimates PD, the rules set LGD, 40% on non-financial corporates and 45% on sovereigns, banks and other financial institutions, and $M = 2.5$ years (Basel Framework from 1 January 2023, CRE32, paragraphs 32.6 and 32.44)
IRB	internal ratings-based approach of the Basel capital rules; every IRB corporate PD is floored at 0.05% (paragraph 32.4)
LGD	loss given default: the share of the amount owed that is lost
PD	probability of default within one year
RWA	risk-weighted assets, $12.5\,K \cdot \text{EAD}$
m	maturity term of the Basel formula; the Basel text writes it b, the letter used here for the loading on Z
ρ, R	a book's own asset correlation; the Basel correlation, set by PD
Z, C	the economy factor and the climate factor, each $N(0,1)$

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