Merton default probability under climate shocks

Mathematics of green finance risk management

Formulas

lognormal assets	$\ln V_T = \ln V + (\mu - \sigma^2/2)\,T + \sigma\sqrt{T}\,Z$, with $Z \sim N(0,1)$
distance to default	$DD = [\ln(V/D) + (\mu - \sigma^2/2)\,T]/(\sigma\sqrt{T})$; $\quad PD = \Phi(-DD)$
asset shock s	$V \to V(1-s)$, $\quad \Delta DD = \ln(1-s)/(\sigma\sqrt{T})$, $\quad DD_1 = DD_0 + \Delta DD$
equity, a call on V	$E = V\,\Phi(d_1) - D\exp(-rT)\,\Phi(d_2)$, $\quad d_1 = [\ln(V/D) + (r + \sigma^2/2)T]/(\sigma\sqrt{T})$, $\quad d_2 = d_1 - \sigma\sqrt{T}$
equity volatility	$\sigma_E\,E = \Phi(d_1)\,\sigma V$
calibration	first guess $V_0 = E + D\exp(-rT)$, $\sigma_0 = \sigma_E E/V_0$; update $V_{k+1} = [E + D\exp(-rT)\,\Phi(d_2)]/\Phi(d_1)$ and $\sigma_{k+1} = \sigma_E E/[\Phi(d_1)V_{k+1}]$, with d_1, d_2 at V_k, σ_k
PD rise	$\Delta PD = \Phi(-DD_1) - \Phi(-DD_0) \approx \varphi(\bar{x})\,\lvert\Delta DD\rvert$, $\bar{x} = (DD_0 + DD_1)/2$; $\quad \varphi(x) = e^{-x^2/2}/\sqrt{2\pi}$

Coal, Cement and Steel

	V, USD m	D, USD m	D/V, %	σ, %	loss s, %	DD_0	PD, %	E, USD m	σ_E, %
Coal, coal power plant	2,600	1,430	55	20	15.4	3.139	0.085	1,212	42.9
Cement, cement works	1,000	350	35	35	17.3	2.967	0.150	660	53.0
Steel, steel mill	3,600	2,340	65	20	9.3	2.304	1.061	1,331	53.6

Three firms of the six-firm book: six stylised firms (Coal, Cement, Steel, Port, Rice, Solar) with their inputs. Drift $\mu = 5\%$, risk-free rate $r = 3\%$, $T = 1$ year. The loss s is minus the five-year Climate VaR, 2026 to 2030, Orderly path: Coal -15.4%, Cement -17.3%, Steel -9.3%. Stylised values.

Exercise, in pairs, eight minutes

For each firm: the distance to default DD_0 and the PD; ΔDD and DD_1 after the shock; the PD after the shock and its rise in percentage points. Which PD rises more, and which input decides it?

	V, USD m	D, USD m	σ, %	shock s, %
aluminium smelter	1,500	600	30	15
oil refinery	2,000	1,400	18	8

Check values. The smelter's DD_0 is 3.071. The refinery's ΔDD is -0.463. Stylised values.

Normal tail $\Phi(-x)$, per cent

x	$\Phi(-x)$	x	$\Phi(-x)$	x	$\Phi(-x)$	x	$\Phi(-x)$	x	$\Phi(-x)$
1.00	15.866	1.50	6.681	2.00	2.275	2.50	0.621	3.00	0.135
1.05	14.686	1.55	6.057	2.05	2.018	2.55	0.539	3.05	0.114
1.10	13.567	1.60	5.480	2.10	1.786	2.60	0.466	3.10	0.097
1.15	12.507	1.65	4.947	2.15	1.578	2.65	0.402	3.15	0.082
1.20	11.507	1.70	4.457	2.20	1.390	2.70	0.347	3.20	0.069
1.25	10.565	1.75	4.006	2.25	1.222	2.75	0.298	3.25	0.058
1.30	9.680	1.80	3.593	2.30	1.072	2.80	0.256	3.30	0.048
1.35	8.851	1.85	3.216	2.35	0.939	2.85	0.219	3.35	0.040
1.40	8.076	1.90	2.872	2.40	0.820	2.90	0.187	3.40	0.034
1.45	7.353	1.95	2.559	2.45	0.714	2.95	0.159	3.45	0.028

Between two rows, interpolate on a straight line. $\Phi(x) = 1 - \Phi(-x)$: for $d_1 = 2.00$, $\Phi(d_1) = 1 - 0.02275 = 0.97725$.

Glossary

PD	probability of default within the horizon, here one year	DD	distance to default: standard deviations of $\ln V_T$ between its mean and $\ln D$
call option	the right to buy at the strike; equity is a call on V with strike D	delta	$\partial E/\partial V = \Phi(d_1)$, the change in equity per unit of assets
fixed point	the V and σ that the update returns unchanged	VaR	value at risk; a Climate VaR is the change in value on one scenario path, negative for a loss
hot house world	a scenario with current policies only and the most warming	orderly transition	a scenario of early, gradual climate policy
ECB	European Central Bank	BCBS	Basel Committee on Banking Supervision

By 2050 the median firm's PD is 5.5% higher in a hot house world than in an orderly transition, and almost 25% higher for the median of the tenth of firms with the most physical damage; both are relative changes.

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