Merton default probability under climate shocks

Mathematics of green finance risk management

Joerg Osterrieder

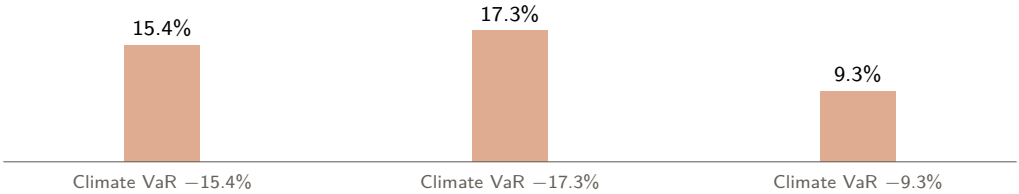
University of Twente

Asset value lost by Coal, Cement and Steel over five years

Coal, coal power plant
assets USD 2,600 m
debt USD 1,430 m
asset volatility 20%

Cement, cement works
assets USD 1,000 m
debt USD 350 m
asset volatility 35%

Steel, steel mill
assets USD 3,600 m
debt USD 2,340 m
asset volatility 20%



loss, per cent of asset value, 2026 to 2030, Orderly path

Coal, Cement and Steel belong to the six-firm book: six stylised firms (Coal, Cement, Steel, Port, Rice, Solar) with their inputs.

Rank the three firms by the rise in their one-year PD.

Stylised values.

Distance to default, equity value and the normal density turn asset shocks into PDs

1. Derive the Merton PD $\Phi(-DD)$ from lognormal assets and default below the debt
2. Compute a firm's distance to default and PD from V, D, σ, μ and T
3. Compute the fall in the distance to default and the new PD after an asset shock s
4. Compute asset value and asset volatility from equity value and equity volatility by one update
5. Estimate a PD rise from the normal density, and explain why it is largest near default
6. Convert a PD change between a relative change and percentage points

Assets, debt and volatility give a PD, and a climate shock moves it

1 How does the Merton model turn assets, debt and volatility into a PD?

2 How does a shock to asset value move the distance to default?

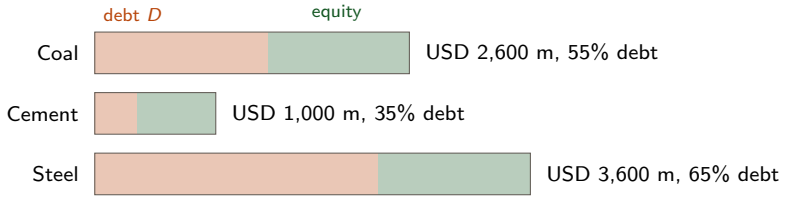
3 How do asset value and asset volatility follow from the equity price?

4 Why does the same kind of shock move three PDs so differently?

Coal, Cement and Steel owe 55, 35 and 65 per cent of their assets, and their five-year Climate VaR cuts those assets by 15.4, 17.3 and 9.3 per cent.

A firm defaults when its assets in one year fall below its debt

1 2 3 4 **How does the Merton model turn assets, debt and volatility into a PD?**

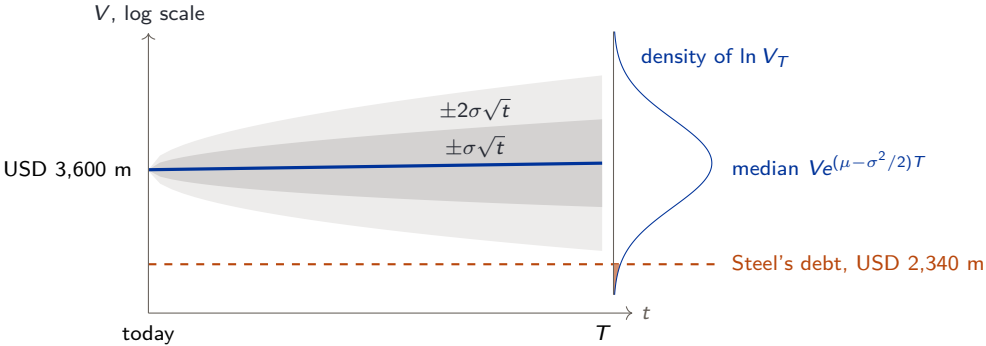


Merton (1974): assets V are financed by equity and by zero-coupon debt D due at T, here one year. The firm repays D if $V_T \geq D$. If $V_T < D$ it defaults, the lenders take V_T, and equity receives $\max(V_T - D, 0)$.

Stylised values.

Solar owes USD 280 m, and its assets end the year at USD 250 m. What do its lenders and its owners receive?

The log of asset value in one year is normal with standard deviation $\sigma\sqrt{T}$



$\ln V_T = \ln V + (\mu - \sigma^2/2)\,T + \sigma\sqrt{T}\,Z$ with $Z \sim N(0,1)$: asset value drifts at $\mu = 5\%$, so its log is normal with mean $\ln V + (\mu - \sigma^2/2)T$ and standard deviation $\sigma\sqrt{T}$. Steel's debt lies just below the lower edge of its band of two standard deviations. Stylised values.

Where, in USD m, does the lower edge of Steel's band of two standard deviations end the year?

Lognormal assets put the probability of ending below the debt at $\Phi(-DD)$

$$\ln V_T = \ln V + (\mu - \tfrac{\sigma^2}{2})T + \sigma\sqrt{T}\,Z \qquad \text{lognormal assets}$$

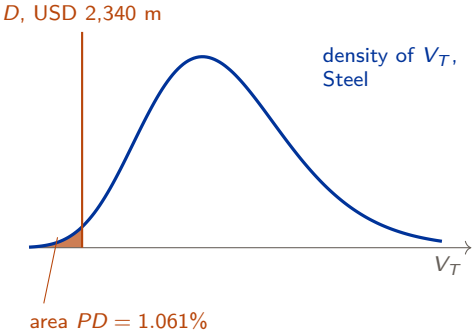
$$V_T < D \iff \ln V_T < \ln D \qquad \text{take logs}$$

$$\iff Z < -\frac{\ln(V/D) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{solve for } Z$$

$$\iff Z < -DD \qquad \text{name the distance}$$

$$PD = P(Z < -DD) = \Phi(-DD) \qquad Z \sim N(0,1)$$

DD counts the standard deviations of $\ln V_T$ between its mean and $\ln D$, and PD is the probability of ending below the debt.



Stylised values.

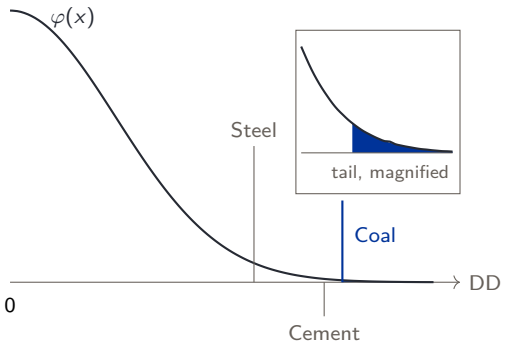
Which line changes, and by how much does Steel's DD rise, if default needs assets below 80 per cent of the debt?

Distance to default and PD of Coal

$$\ln(V/D) = \ln(2{,}600/1{,}430) = 0.5978$$
$$(\mu - \sigma^2/2)\, T = 0.05 - 0.02 = 0.03$$
$$DD = (0.5978 + 0.03)/0.20 = 3.139$$
$$PD = \Phi(-3.139) =$$

Coal has assets of USD 2,600 m, debt of USD 1,430 m, $\sigma = 20\%$, $\mu = 5\%$ and $T = 1$. For Cement the distance to default is 2.967 and PD 0.150%; for Steel DD is 2.304 and PD 1.061%. $\Phi(-DD)$ is the area beyond DD under the normal density $\varphi(x) = e^{-x^2/2}/\sqrt{2\pi}$.

Stylised values.



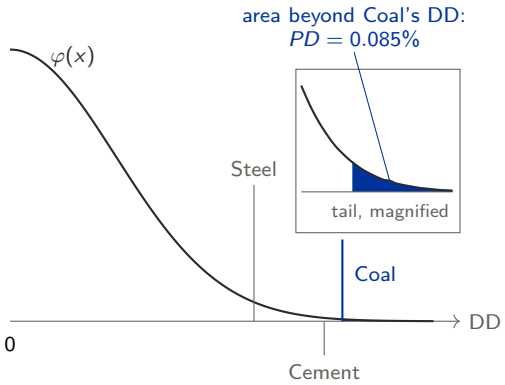
What are Steel's DD and PD if its asset volatility is 25 per cent, its assets and debt unchanged?

Distance to default and PD of Coal

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$$(\mu - \sigma^2/2)\,T = 0.05 - 0.02 = 0.03$$
$$DD = (0.5978 + 0.03)/0.20 = 3.139$$
$$PD = \Phi(-3.139) = 0.085\%$$

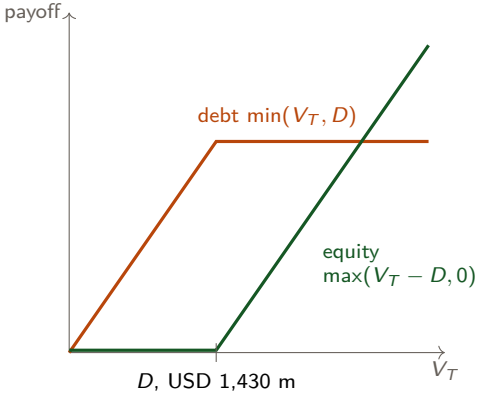
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Stylised values.



What are Steel's DD and PD if its asset volatility is 25 per cent, its assets and debt unchanged?

Equity receives the assets above the debt, the payoff of a call option



$$E_T = \max(V_T - D,\, 0)$$

$$\min(V_T, D) + \max(V_T - D,\, 0) = V_T$$

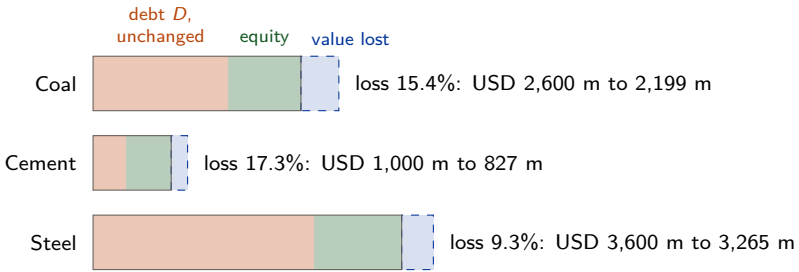
The owners hold a call option on the assets with strike D, and the lenders receive the rest. Below D the owners walk away.

Stylised values.

A put on the assets with strike D pays $\max(D - V_T, 0)$. *Write the lenders' payoff* $\min(V_T, D)$ *with it.*

A five-year Climate VaR cuts asset value and leaves the debt unchanged

1 2 3 4 **How does a shock to asset value move the distance to default?**



The shock s is the net carbon cost $N = (1 - \pi)C$ the firm keeps over 2026 to 2030, Orderly path, discounted to 2025, over asset value; the five-year Climate VaR is $-s$, Coal -15.4%. Assets fall to $V(1 - s)$; the debt D stays.

Stylised values.

What share of Steel's equity, $V - D$, does its shock remove?

A shock s moves the distance to default by $\ln(1-s)/(\sigma\sqrt{T})$

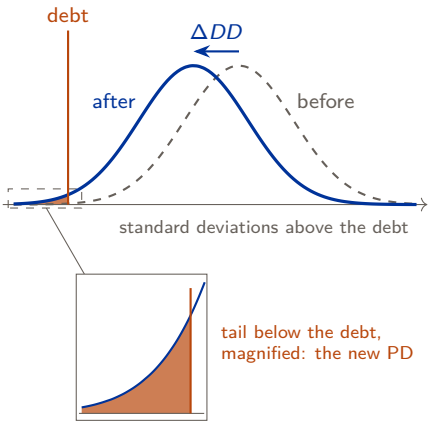
$$DD_1 = \frac{\ln\left(V(1-s)/D\right) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{assets } V(1-s)$$

$$= \frac{\ln(V/D) + \ln(1-s) + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{log of a product}$$

$$= DD_0 + \frac{\ln(1-s)}{\sigma\sqrt{T}} \qquad \text{split the fraction}$$

$$\Delta DD = \frac{\ln(1-s)}{\sigma\sqrt{T}} \approx -\frac{s}{\sigma\sqrt{T}} \qquad \ln(1-s) \approx -s$$

The debt line stays; the whole distribution moves towards it.



Coal before and after its shock.

What shock s would lower Coal's distance to default by a full 1.0?

Shock to Coal's distance to default and PD

$$
\begin{aligned}
s &= 15.42\% \text{ of } V = \text{USD } 2{,}600 \text{ m} \\
\ln(1-s) &= \ln(1-0.1542) = -0.16747 \\
\Delta DD &= -0.16747/0.20 = -0.837 \\
DD_1 &= 3.139 - 0.837 = 2.302 \\
PD_1 &= \Phi(-2.302) =
\end{aligned}
$$

Coal's shock lowers its distance to default from 3.139 to 2.302.

Stylised values.

PD in per cent at each distance



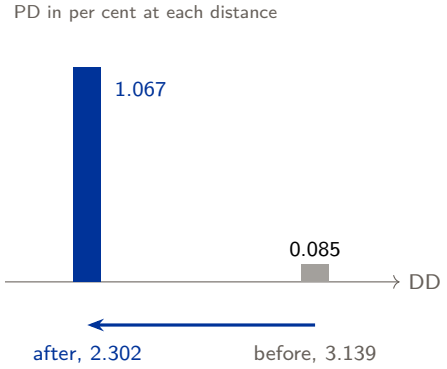
Cement loses 17.33 per cent at $\sigma = 0.35$. What are its ΔDD and its new DD?

Shock to Coal's distance to default and PD

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$$\ln(1-s) = \ln(1-0.1542) = -0.16747$$
$$\Delta DD = -0.16747/0.20 = -0.837$$
$$DD_1 = 3.139 - 0.837 = 2.302$$
$$PD_1 = \Phi(-2.302) = 1.067\%$$

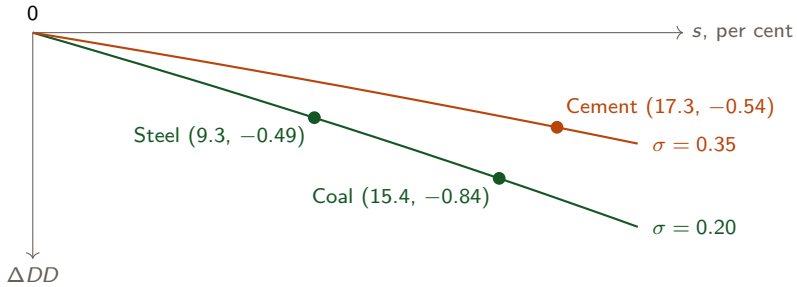
Coal's shock lowers its distance to default from 3.139 to 2.302. Its PD rises from 0.085% to 1.067%, 12.6 times as high.

Stylised values.



Cement loses 17.33 per cent at $\sigma = 0.35$. What are its ΔDD and its new DD?

Asset volatility divides the shock, so Cement's largest shock moves DD less than Coal's

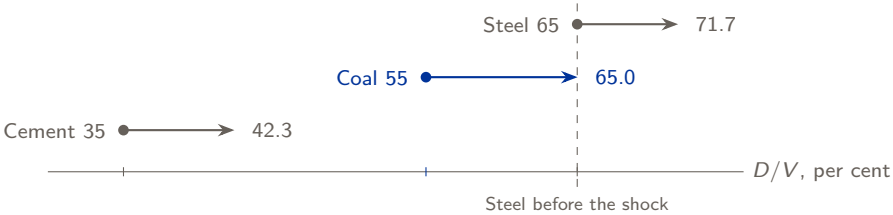


$\Delta DD = \ln(1 - s)/\sigma$ falls on a curve set by σ. Cement's 17.3% at $\sigma = 35\%$ moves its DD by -0.54, Coal's 15.4% at 20% by -0.84, and Steel's 9.3% by -0.49.

Stylised values.

At what asset volatility would Cement's shock move its DD exactly as far as Coal's?

After its shock Coal has almost Steel's leverage and distance to default



With leverage $L = D/V$, $DD = [-\ln L + (\mu - \sigma^2/2)T]/(\sigma\sqrt{T})$, and a shock raises L to $L/(1-s)$. Coal goes from 55% to 65.03%, almost Steel's 65%. Both have $\sigma = 20\%$, so Coal's distance to default after the shock, 2.302, is almost Steel's before it, 2.304.

Stylised values.

What shock would bring Cement to a leverage of 50 per cent, and what would its DD then be?

Shocks to the distance to default of a smelter and a refinery

aluminium smelter
assets USD 1,500 m, debt USD 600 m
asset volatility 30%
asset shock 15%

oil refinery
assets USD 2,000 m, debt USD 1,400 m
asset volatility 18%
asset shock 8%

In pairs, eight minutes, with $\mu = 5\%$ and $T = 1$ year, for each firm:

1. the distance to default DD_0 and the PD;
2. the change ΔDD and the distance DD_1 after the shock;
3. the PD after the shock and its rise in percentage points;
4. which PD rises more, and which input decides it.

$\Phi(-x)$ from the table on the sheet. Stylised values.

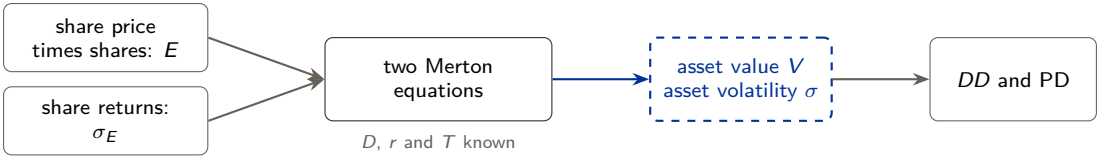
The refinery's smaller shock raises its PD 6.2 times as much as the smelter's

	smelter	refinery
$\ln(V/D)$	0.9163	0.3567
$(\mu - \sigma^2/2)T$	0.0050	0.0338
DD_0	3.071	2.169
PD, per cent	0.107	1.504
$\ln(1-s)$	−0.1625	−0.0834
$\Delta DD = \ln(1-s)/\sigma$	−0.542	−0.463
$DD_1 = DD_0 + \Delta DD$	2.529	1.706
PD after the shock, per cent	0.572	4.400
rise in percentage points	0.465	2.896

The refinery's low volatility turns its 8% into a fall of 0.463 in DD, close to the smelter's 0.542 from 15%, and it starts nearer default, at 2.169 against 3.071: debt of 70% of assets against 40% puts it there. Stylised values.

An equity value and its volatility give two equations in V and σ

1 2 **3** 4 **How do asset value and asset volatility follow from the equity price?**



$$E = V\,\Phi(d_1) - D\exp(-rT)\,\Phi(d_2), \qquad \sigma_E\,E = \Phi(d_1)\,\sigma V$$

$$d_1 = \frac{\ln(V/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}$$

Merton (1974) prices equity as a call on the assets with strike D at the risk-free rate $r = 3\%$: Coal's E is USD 1,212.4 m against $V - D$ of USD 1,170 m. Stylised values.

What do the two equations say for a firm with no debt, $D = 0$?

The equity delta and leverage V/E turn asset volatility into equity volatility

$$\frac{\partial E}{\partial V} = \Phi(d_1) \qquad \text{delta of the call}$$

$$\Delta E \approx \Phi(d_1)\,\Delta V \qquad \text{first order}$$

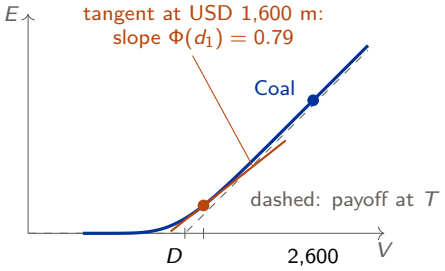
$$\frac{\Delta E}{E} \approx \Phi(d_1)\,\frac{V}{E}\,\frac{\Delta V}{V} \qquad \text{divide by } E$$

$$\sigma_E = \Phi(d_1)\,\frac{V}{E}\,\sigma \qquad \text{standard deviations}$$

$$\sigma_E\,E = \Phi(d_1)\,\sigma V \qquad \text{multiply by } E$$

A one per cent move in assets moves equity by $\Phi(d_1)\,V/E$ per cent: the delta times the leverage V/E. The delta is 0.9994 at Coal's assets and 0.79 near the debt, where the curve bends.

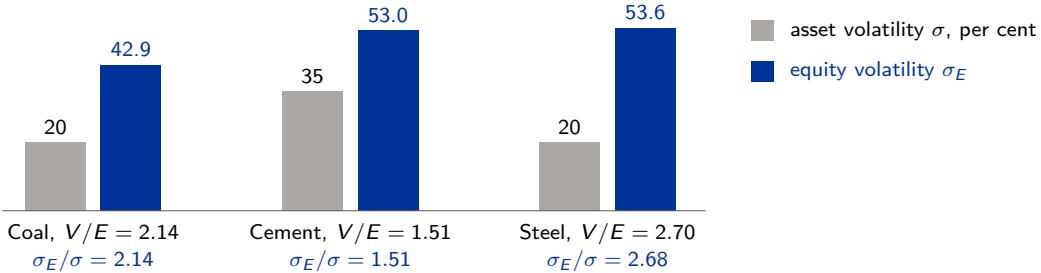
Stylised values.



Coal's equity value $E(V)$ in USD m, one year before T.

A firm has $\Phi(d_1) = 0.95$, assets of USD 1,000 m, equity of USD 300 m and $\sigma = 25\%$. What is its equity volatility?

Equity of Coal, Cement and Steel is 1.51 to 2.68 times as volatile as their assets



$\sigma_E/\sigma = \Phi(d_1)\, V/E$, and with $\Phi(d_1)$ near one the multiple is assets over equity. Of the three firms, Steel has the least equity per unit of assets and the most volatile shares, with assets as volatile as Coal's.

Stylised values.

What DD and PD does Coal get if its equity volatility, 42.9 per cent, is entered as σ?

One update of Coal's asset value and volatility from its equity

$$V_0 = E + D\exp(-rT) = 1{,}212.4 + 1{,}430 \times 0.9704 = 2{,}600.1$$

$$\sigma_0 = \sigma_E E / V_0 = 0.4287 \times 1{,}212.4/2{,}600.1 = 0.1999$$

$$d_1 = \frac{\ln(2{,}600.1/1{,}430) + 0.03 + 0.1999^2/2}{0.1999} = 3.24$$

$$d_2 = d_1 - \sigma_0 = 3.04$$

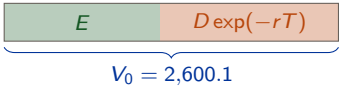
$$V_1 = \frac{E + D\exp(-rT)\,\Phi(d_2)}{\Phi(d_1)} = \frac{1{,}212.4 + 1{,}387.7 \times 0.9988}{0.9994} = 2{,}600.0$$

$$\sigma_1 = \frac{\sigma_E E}{\Phi(d_1)\,V_1} = 0.2000$$

Coal's equity is worth USD 1,212.4 m with $\sigma_E = 42.87\%$, at $r = 3\%$ and $T = 1$. The first guess sets $\Phi(d_1) = \Phi(d_2) = 1$; the update solves the equity equation for V and the volatility equation for σ. Repeated, it settles at asset value USD 2,600.0 m and asset volatility 20.0%.

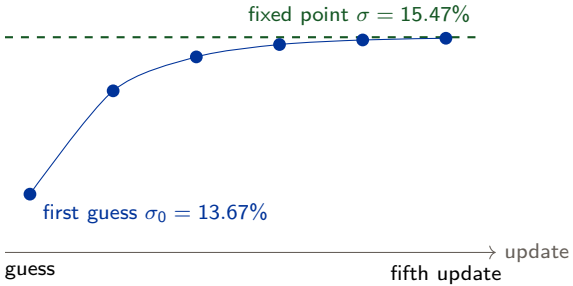
Stylised values.

deep in the money:
$\Phi(d_1) = 0.9994$



Start Steel the same way: equity of USD 1,331 m, $\sigma_E = 53.6\%$ and debt of USD 2,340 m. What are V_0 and σ_0?

Near default the first guess understates asset volatility and the PD



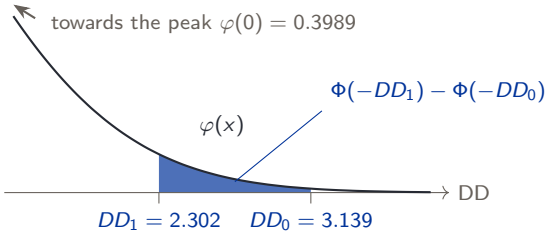
A shipyard's equity is worth USD 200 m with $\sigma_E = 80\%$, and its debt of USD 1,000 m is 86% of its assets. The first guess gives V_0 of USD 1,170.4 m and a PD of 7.4%. The fixed point, asset value USD 1,160.4 m and asset volatility $\sigma = 15.47\%$, gives DD 1.207 and PD 11.4%.

Stylised values.

Why does the first guess $\sigma_0 = \sigma_E E/V_0$ come out low for a firm close to default?

A PD rise is the area under the normal density between two distances

1 2 3 **4** **Why does the same kind of shock move three PDs so differently?**



$\Phi(-DD)$ is the area of the density beyond DD, so a fall between two distances to default, from DD_0 to DD_1, adds the strip between them. The density peaks at zero and is 9.7 times as high at Coal's DD_1 as at its DD_0.

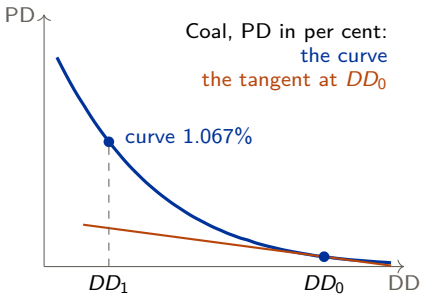
Stylised values.

A firm's DD falls from 1.50 to 1.00. By how many percentage points does its PD rise?

To first order the PD rise is the density times the fall in DD

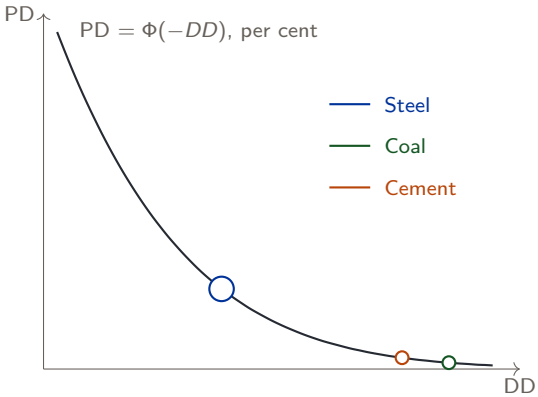
$$
\begin{aligned}
PD &= \Phi(-DD) && \text{Merton PD}\\
\frac{dPD}{dDD} &= -\varphi(-DD) = -\varphi(DD) && \text{chain rule, symmetry}\\
\Delta PD &\approx \varphi(DD_0)\,|\Delta DD| && \text{tangent at } DD_0\\
\Delta PD &= \Phi(-DD_1) - \Phi(-DD_0) && \text{the strip}\\
&\approx \varphi(\bar{x})\,|\Delta DD| && \text{midpoint } \bar{x}\\
\frac{\varphi(x_1)}{\varphi(x_0)} &= e^{(x_0^2 - x_1^2)/2} && \text{ratio of two densities}
\end{aligned}
$$

To first order the PD rise is $\varphi(DD_0)$ times the fall. The density grows as DD falls, so the tangent understates a large move; the density at the midpoint $\bar{x} = (DD_0 + DD_1)/2$ corrects most of it.
Stylised values.



By what factor is the density at a DD of 2.0 larger than at 3.0?

PD rises of Coal, Cement and Steel after their shocks



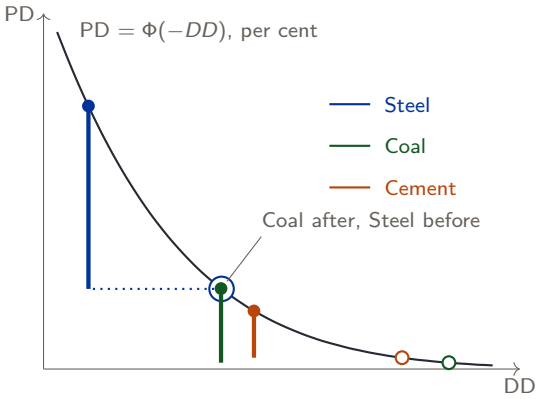
	DD_0	DD_1	PD rise, points
Coal	3.139	2.302	
Cement	2.967	2.423	
Steel	2.304	1.815	

Rank the three PD rises from the two distances.

Stylised values.

What is Steel's PD rise in percentage points if its shock doubles to 18.6 per cent?

PD rises of Coal, Cement and Steel after their shocks



	DD_0	DD_1	PD rise, points
Coal	3.139	2.302	0.982
Cement	2.967	2.423	0.618
Steel	2.304	1.815	2.412

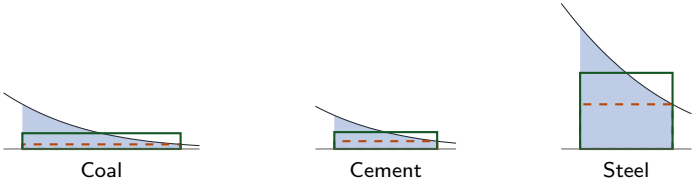
Rank the three PD rises from the two distances.

Steel moves least in DD and most in PD: it starts where the curve is steepest. Coal falls 0.837 in DD, to 2.302, past Cement's 2.423.

Stylised values.

What is Steel's PD rise in percentage points if its shock doubles to 18.6 per cent?

The density at the start ranks Cement above Coal, and the midpoint corrects it



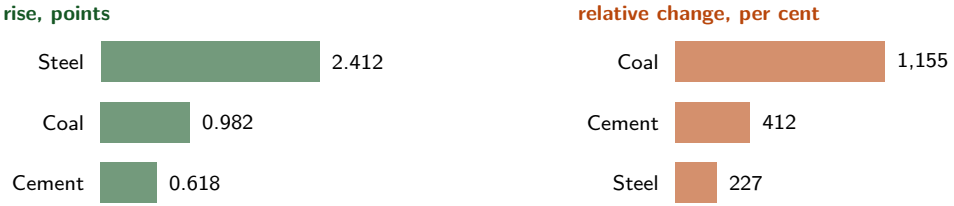
shaded: the exact rise
dashed: density at DD_0
solid: density at the midpoint

	$\lvert\Delta DD\rvert$	$\varphi(DD_0)$	start estimate	midpoint	φ at midpoint	midpoint estimate	exact
Coal	0.837	0.0029	0.2	2.720	0.0099	0.8	0.982
Cement	0.544	0.0049	0.3	2.695	0.0106	0.6	0.618
Steel	0.489	0.0281	1.4	2.059	0.0479	2.3	2.412

Estimates are $\varphi \times |\Delta DD|$ as a percentage: Coal's $0.0029 \times 0.837 = 0.0024$ is 0.2 percentage points. At its start Coal's density is the lowest, and across its long fall it grows 9.7 times. Stylised values.

Which firm's start estimate comes closest to its exact rise, and why that firm?

A relative PD change and a change in points rank the three firms differently



By 2050 the median firm's PD is 5.5% higher in a hot house world than in an orderly transition, and almost 25% higher for the median firm of the tenth with the most physical damage: relative changes. On Steel's 1.061%, 5.5% higher is 1.119%, up 0.058 points, arithmetic with no horizon. Stylised values.

Alogoskoufis, S., Dunz, N., Emambakhsh, T., Hennig, T., Kaijser, M., Kouratzoglou, C., Muñoz, M. A., Parisi, L. and Salleo, C. (2021), ECB economy-wide climate stress test: Methodology and results, ECB Occasional Paper No 281, September 2021, section 5.3, p. 44, read 1 October 2026. Alogoskoufis, S., Dunz, N., Emambakhsh, T., Hennig, T., Kaijser, M., Kouratzoglou, C., Muñoz, M. A., Parisi, L. and Salleo, C. (2021), ECB economy-wide climate stress test: Methodology and results, ECB Occasional Paper No 281, September 2021, section 5.5, p. 49, read 1 October 2026.

What PD does almost 25% higher give a firm of that tenth with Steel's 1.061%, and what rise in points?

Leverage and volatility set the distance, and the density turns its fall into PD

1	How does the Merton model turn assets, debt and volatility into a PD?	$PD = \Phi(-DD)$, $DD = [\ln(V/D) + (\mu - \sigma^2/2)T]/(\sigma\sqrt{T})$; equity is a call. Coal: PD 0.085%.
2	How does a shock to asset value move the distance to default?	$\Delta DD = \ln(1-s)/(\sigma\sqrt{T})$: Coal −0.837, Cement −0.544, Steel −0.489. Volatility divides the shock; leverage rises to $D/[V(1-s)]$.
3	How do asset value and asset volatility follow from the equity price?	Update V and σ in $E = V\Phi(d_1) - D\exp(-rT)\Phi(d_2)$ and $\sigma_E E = \Phi(d_1)\sigma V$: Coal's equity returns USD 2,600 m and 20%.
4	Why does the same kind of shock move three PDs so differently?	The area under φ between DD_1 and DD_0, about φ at the midpoint times $\lvert\Delta DD\rvert$; φ at Steel's DD_0 is 9.7 times φ at Coal's. Steel 2.412 points, Coal 0.982, Cement 0.618.

Stylised values.

Sources on the Merton model and climate stress tests

- Alogoskoufis, S., Dunz, N., Emambakhsh, T., Hennig, T., Kaijser, M., Kouratzoglou, C., Muñoz, M. A., Parisi, L. and Salleo, C. (2021), ECB economy-wide climate stress test: Methodology and results, ECB Occasional Paper No 281, September 2021, section 5.3, p. 44, read 1 October 2026, and section 5.5, p. 49; the views of its authors.
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- Merton, R. C. (1974), On the pricing of corporate debt: the risk structure of interest rates, Journal of Finance 29(2), 449 to 470, Crossref record of DOI 10.1111/j.1540-6261.1974.tb03058.x, read 1 October 2026.

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Checkpoint: Merton PD under a climate shock

1. $\ln V_T \sim N(8.10, 0.25^2)$, $\ln D = 7.50$: give DD and PD. Which derivation line changes if default needs assets below 90% of the debt?
2. V USD 800 m, D USD 480 m, $\sigma = 25\%$, $\mu = 5\%$, $T = 1$: give DD and PD.
3. A 10% asset shock: give ΔDD, DD, PD and the rise in points.
4. E USD 100 m, $\sigma_E = 60\%$, D USD 500 m, $r = 3\%$, $T = 1$: give V_0, σ_0, V_1 and σ_1.
5. DD falls from 2.00 to 1.60 for one firm, from 3.20 to 2.80 for another: estimate both PD rises at the midpoint; explain the gap.
6. A PD goes from 0.40% to 0.50%: give the change in points and the relative change, and raise 2.0% by 5.5%.

Stylised values.

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Merton default probability under climate shocks - Mathematics of Green Finance Risk Management, Lecture 4

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The project site is https://digital-ai-finance.github.io/Green-Finance. A question about this file goes to joerg.osterrieder@utwente.nl.

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