Climate value-at-risk of firms and portfolios

Mathematics of green finance risk management

Formulas

net carbon cost $N_y = (1-\pi)\,C_y$; compliance cost $C_y = p_y E - p_y^2/(2k)$ while $p_y < \bar{p}$, and $\bar{p}E/2$ above it; abatement $a_y = p_y/k$, $k = \bar{p}/E$ (L1)

Climate VaR Climate VaR $= -\dfrac{1}{V}\displaystyle\sum_{y=2026}^{2040} \dfrac{\min(N_y, \text{EBITDA})}{(1+r)^t}$, $t = y - 2025$, V the EVIC, $r = 5$ per cent real, prices in constant US dollars of 2025; the loss is −Climate VaR

physical part $-\Delta AAL\left(1-(1+r)^{-T}\right)/(rV)$, with $\Delta AAL = (m-1)\,\lambda E[X]$ (L2); factor 10.38 for $T = 15$, 4.33 for $T = 5$

pass-through with no year capped, ∂ Climate VaR$/\partial\pi = -$Climate VaR$/(1-\pi)$; year y is capped when $\pi < 1 - \text{EBITDA}/C_y$; past a loss of 100 per cent the value is gone

portfolio Climate VaR$_P = \sum_i (h_i/H)$ Climate VaR$_i$ with $H = \sum_i h_i$; every claim loses the firm's share, and for the transition part every firm is on one path

over paths $E[L] = \sum_s P_s L_s$; the α quantile is the smallest L_s with $\sum_{L_{s'} \le L_s} P_{s'} \ge \alpha$

The six-firm book: six stylised firms

	E, Mt	π	$\bar{p}$, USD per t	EBITDA	EVIC	Climate VaR, 2026 to 2040, Orderly
Coal	4.0	0.80	400	320	2,600	−49.0 per cent
Cement	3.0	0.90	1,200	125	1,000	−62.4 per cent
Steel	6.0	0.90	800	450	3,600	−32.7 per cent
Port	0.15	0.50	300	240	1,500	transition −6.8, physical −4.6
Rice	0.03	0.50	300	30	250	transition −8.1, physical −10.4
Solar	0.004	0.50	300	60	400	transition −0.7, physical −2.0

Money in USD m. Physical part: Port $m = 3.0$, AAL USD 3.305 m; Rice $m = 1.5$, AAL USD 5.028 m; Solar $m = 1.2$, AAL USD 3.833 m. Orderly path: USD 100 per t in 2026, USD 300 in 2035, USD 333.33 in 2040, linear between. A holding of USD 100 m in each firm: portfolio -29.45 per cent, Orderly transition plus the warming scenario. Stylised values.

Exercise, in pairs, eight minutes

Glass, a glass works, emits $E = 2.0$ Mt a year and abates all its tonnes at $\bar{p} =$ USD 400 per t, so $k = 200$. It passes on $\pi = 0.6$ of its cost; EBITDA USD 100 m a year, EVIC USD 1,200 m, real rate 5 per cent, $t = y - 2025$.

year	2026	2027	2028
carbon price, USD per t	100	150	200

1. Each year's abatement $a = p/k$ and compliance cost $C = pE - p^2/(2k)$.
2. The net cost $N = (1-\pi)\,C$, and the loss, at most EBITDA.
3. Each loss discounted to 2025, their sum, and the sum over EVIC.
4. Then at $\pi = 0.7$: the capped years and the new Climate VaR.

Check values. The 2027 net cost is USD 97.5 m. The three present values add to USD 241.5 m. Stylised values.

Glossary

AAL average annual loss, the expected loss of one year, $\lambda E[X]$

EBITDA earnings before interest, taxes, depreciation and amortisation

EVIC enterprise value including cash: shares at market value plus debt and minorities at book value

VaR value at risk; the six-firm book's Climate VaR is one path's loss, with no quantile

MSCI index and data provider; its Climate VaR is a value change per scenario, with a 95th percentile for physical risk inside one warming scenario

NGFS Network for Greening the Financial System, whose scenario families the three paths follow

PCAF Partnership for Carbon Accounting Financials; it attributes a listed firm's emissions by the holding over EVIC

Mt million tonnes of CO_2

EVIC: PCAF (2025), Global GHG Accounting and Reporting Standard Part A, 3rd ed., section 5.2, p. 57, read 30 September 2026. MSCI's percentile: MSCI Climate Paris Aligned Filtered Indexes Methodology (May 2025), MSCI Climate Value-at-Risk, p. 18, read 30 September 2026.

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