Physical losses from frequency, severity and return period

Mathematics of green finance risk management

Three firms and their hazards

firm	activity	hazard	rate λ	return period T	median loss	log-sd s	record loss	multiplier m
Port	container port on a delta	storm surge	0.02	50.0	120	0.8	300	3.0
Rice	rice mill on a river plain	river flood	0.60	1.7	7	0.6	25	1.5
Solar	solar farm	hail and typhoon	0.25	4.0	12	0.7	40	1.2

The three firms belong to the six-firm book: six firms (Coal, Cement, Steel, Port, Rice, Solar) with their inputs. Rates in events a year, return periods in years, losses in USD m per event. Rice's mill exposes USD 50 m; its depth-damage curve gives the damage ratio $d(h)$ at flood depth h: $d(0.0) = 0.00$, $d(0.5) = 0.14$, $d(1.0) = 0.30$, $d(2.0) = 0.50$, $d(3.0) = 0.62$, read as a straight line between two points. Stylised values.

Exercise, in pairs, eight minutes

A cold store on Rice's river plain floods at rate $\lambda = 0.05$ a year. A bank lends to it for 15 years. The store exposes USD 60 m; a flood at the median depth of 1.0 m destroys the share d of it read from Rice's curve at that depth; the loss per flood is lognormal with log-sd $s = 0.5$. Compute:

1. the return period, and the probability of at least one flood during the loan;
2. the loss of a flood at the median depth, which is the median loss;
3. the mean loss per flood.

Check values. The probability of at least one flood during the loan is 52.8 per cent. The median loss is USD 18.0 m.

Stylised values.

Formulas

Poisson count	$P(N = k) = e^{-\lambda}\lambda^k/k!$, $\quad \mathrm{E}[N] = \mathrm{Var}[N] = \lambda$
return period	$T = 1/\lambda$; over n years $P(N_n \geq 1) = 1 - e^{-\lambda n} = 1 - e^{-n/T}$
event loss	exposure $\times\, d(h)$, the damage ratio at the hazard's intensity
lognormal loss	$X = \exp(\mu + sZ)$, Z standard normal; median e^{μ}; $\mathrm{E}[X] = e^{\mu}\, e^{s^2/2}$; $\mathrm{E}[X^2] = e^{2\mu+2s^2}$
tail of a loss	$P(X > x) = 1 - \Phi\big(\ln(x/e^{\mu})/s\big)$
year's loss	$S = X_1 + \cdots + X_N$; $\mathrm{E}[S] = \lambda\, \mathrm{E}[X]$, the average annual loss (AAL); $\mathrm{Var}[S] = \lambda\, \mathrm{E}[X^2]$
exceedance	$\mathrm{OEP}(x) = 1 - e^{-\lambda P(X>x)}$; the loss exceeded once in T years solves $\mathrm{OEP}(x) = 1/T$: $x = e^{\mu} \exp\big(s\, \Phi^{-1}(1-q)\big)$ with $q = -\ln(1 - 1/T)/\lambda$
warming	rate $m\lambda$; return period T/m; AAL times m; the severity unchanged

Standard normal distribution $\Phi(z)$:

z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
$\Phi(z)$	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849
z	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5
$\Phi(z)$	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938

Measured anchors

Tropical storms in the Philippine Area of Responsibility, 1951 to 2025: mean 14.8 a year, variance 9.5, ratio 0.64. A heavy 1-day precipitation event over land of once in 10 years now likely occurs 1.3 times as often at 1 °C of global warming, and will likely occur 1.5 times as often at 1.5, 1.7 at 2 and 2.7 at 4 °C. Local sea levels of once per century are projected to occur at least annually at most locations by 2100 under all RCP scenarios, and at least once per year at many locations by 2050.

IBTrACS v04r01 (NOAA NCEI, doi:10.25921/82ty-9e16), WP file of 29 September 2026, RSMC Tokyo record; count by data/ph_cyclones/fetch.py, read 1 October 2026. IPCC (2021), AR6 WG I, Summary for Policymakers, Figure SPM.6, heavy precipitation over land, p. 18, column 4°C, read 1 October 2026. IPCC (2019), SROCC, Summary for Policymakers, B.3.4, p. 20, read 1 October 2026. IPCC (2019), SROCC, Summary for Policymakers, B.3, p. 20, read 1 October 2026.

Glossary

AAL	average annual loss, the mean of a year's loss
AR6	the Sixth Assessment Report of the IPCC
CMIP6	Coupled Model Intercomparison Project, phase 6, the climate models behind the IPCC medians
IBTrACS	International Best Track Archive for Climate Stewardship, of NOAA
IPCC	Intergovernmental Panel on Climate Change
log-sd	standard deviation of the logarithm of a loss
SROCC	IPCC Special Report on the Ocean and Cryosphere in a Changing Climate
OEP	occurrence exceedance probability, of the largest event loss of a year
PAGASA	the Philippine weather service, Philippine Atmospheric, Geophysical and Astronomical Services Administration
PAR	Philippine Area of Responsibility, the area PAGASA watches
RCP	Representative Concentration Pathway, a scenario of greenhouse gas concentrations
RSMC Tokyo	the regional centre of the Japan Meteorological Agency that tracks western Pacific storms
SPM	Summary for Policymakers
USD m	millions of US dollars

Co-funded by the European Union

Licence and attribution

Physical losses from frequency, severity and return period, formula and exercise sheet - Mathematics of Green Finance Risk Management, Lecture 2

Co-funded by the European Union. Published by the GREEN FINANCE project, an Erasmus+ Capacity Building in Higher Education action, grant agreement 101237817.

Free to use, copy and adapt under Creative Commons Attribution 4.0 International (CC BY 4.0), creativecommons.org/licenses/by/4.0/. Attribute it to the GREEN FINANCE consortium, Erasmus+ grant 101237817.

This copy was published at https://digital-ai-finance.github.io/Green-Finance/teaching/module4-mathematics/M4_Mathematics_L2_Sheet.pdf

The project site is https://digital-ai-finance.github.io/Green-Finance. A question about this file goes to joerg.osterrieder@utwente.nl.

Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Education and Culture Executive Agency (EACEA). Neither the European Union nor EACEA can be held responsible for them.

Licence page added 2 October 2026.