Physical losses from frequency, severity and return period

Mathematics of green finance risk management

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Port, Solar and Rice and their largest losses on record



Rank the three firms by the loss each can expect in an average year today.

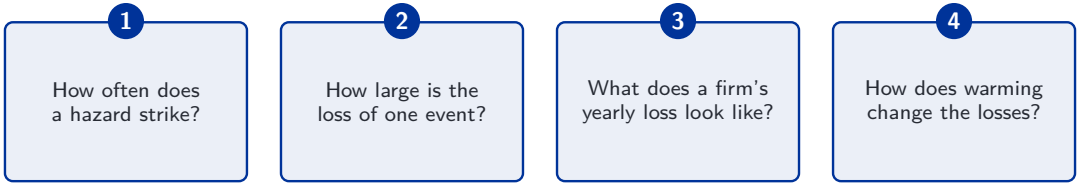
The three firms belong to the six-firm book: six firms (Coal, Cement, Steel, Port, Rice, Solar) with their inputs.

Stylised values.

Return periods, event losses and yearly losses of three firms

1. Compute the probability of at least one event during a loan from a return period
2. Estimate a Poisson rate from yearly counts and test it by the variance-to-mean ratio
3. Compute an event loss from exposure and a damage ratio, and the mean of a lognormal severity
4. Derive the mean and variance of a firm's yearly loss and compute its average annual loss
5. Compute the loss exceeded once in a given number of years from the exceedance curve
6. Compute the return period and average annual loss after warming multiplies the rate

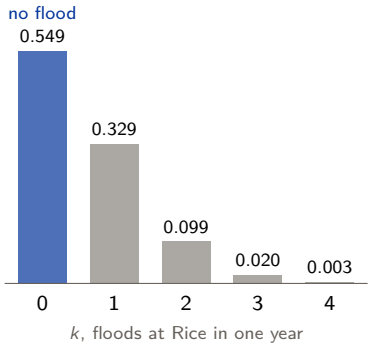
A hazard's rate and its loss per event give a firm's yearly loss



The rate counts the events of a year, the severity sets the loss of each event, and the yearly loss adds them up. The model lets warming multiply the rate and holds the severity fixed.

A Poisson count gives the probability of k hazard events in a year

1 2 3 4 **How often does a hazard strike?**



Events that arrive one at a time, independently, at a constant rate λ a year give a Poisson count N of events in a year:

$$P(N = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \qquad \mathrm{E}[N] = \mathrm{Var}[N] = \lambda.$$

Rice's river floods at $\lambda = 0.60$ a year. No flood comes in 0.549 of its years and one flood in 0.329.

Stylised values.

Solar's hail and typhoons arrive at $\lambda = 0.25$ a year. What is the probability of a year with no event at Solar?

The return period sets the probability of an event during a loan

Years are independent, so counts add: over a loan of n years the count N_n is Poisson with mean λn.

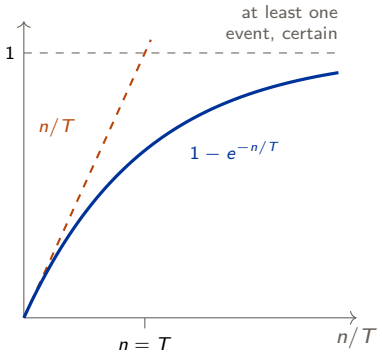
$$P(N_n = 0) = e^{-\lambda n} \qquad \text{the case } k = 0$$

$$P(N_n \geq 1) = 1 - e^{-\lambda n} \qquad \text{the complement}$$

$$= 1 - e^{-n/T} \qquad \text{return period } T = 1/\lambda$$

$$\approx n/T \qquad \text{for } n \text{ small against } T$$

The return period T is the mean wait between two events. The line n/T adds the yearly chances and counts a loan with two events twice, so it overstates the probability.



A loan runs exactly as many years as the event's return period. What is the probability of at least one event during the loan?

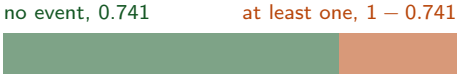
A 100-year event during a 30-year loan

$$\lambda = 1/100 = 0.01$$
$$\lambda n = 0.01 \times 30 = 0.30$$
$$P(N_n = 0) = \exp(-0.30) = 0.741$$
$$P(N_n \geq 1) = 1 - 0.741 = 25.9\%$$

The product $\lambda n = 0.30$ is the expected number of events during the loan. The probability of at least one is lower, 25.9 per cent, because some loans have two events. Stylised values.



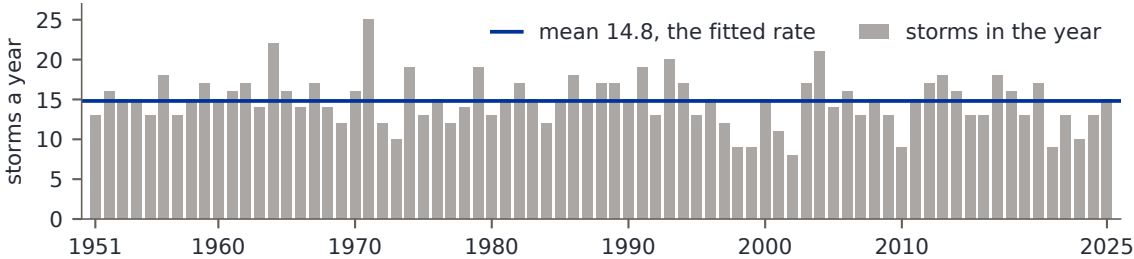
drawn to scale: the loan covers $n/T = 0.30$ of the return period



the two outcomes of the loan, to scale

Port's storm surge has a return period of 50 years. What is the probability it strikes at least once during the same loan?

Tropical storms in the Philippine area averaged 14.8 a year from 1951 to 2025

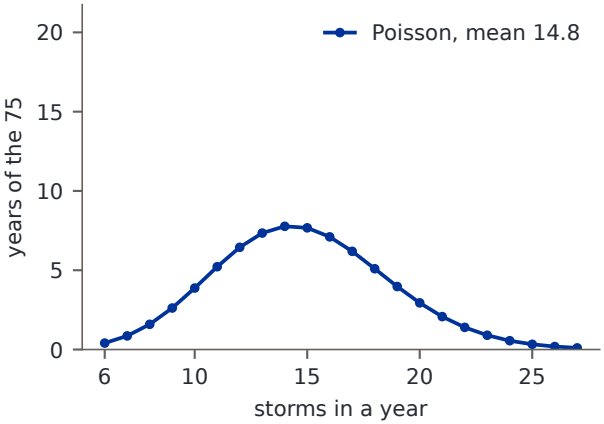


1,111 storms of tropical storm strength or more entered the Philippine Area of Responsibility in 75 years. The Poisson rate fitted by the sample mean is $\hat{\lambda} = 1{,}111/75 = 14.8$ a year. PAGASA's average of 20 a year counts every tropical cyclone that enters the area, a different count.

IBTrACS v04r01 (NOAA NCEI, doi:10.25921/82ty-9e16), WP file of 29 September 2026, RSMC Tokyo record; count by data/ph_cyclones/fetch.py, read 1 October 2026. PAGASA, Tropical Cyclone Information, Overview (web page), read 1 October 2026.

At the fitted rate, how many days pass between two storms on average?

Variance against mean of the Philippine storm counts

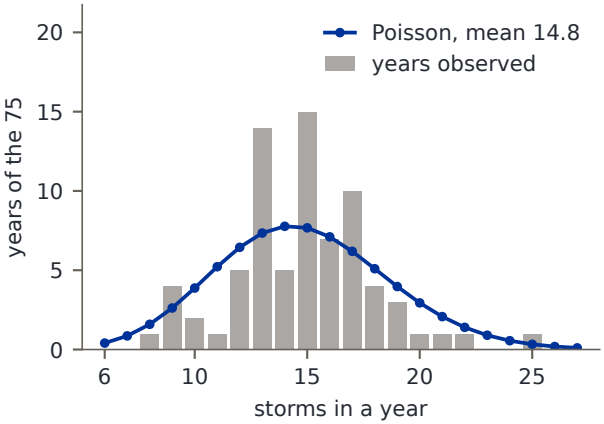


A Poisson count's variance equals its mean, 14.8. Guess the variance of the 75 Philippine counts.

IBTrACS v04r01 (NOAA NCEI, doi:10.25921/82ty-9e16), WP file of 29 September 2026, RSMC Tokyo record; count by data/ph_cyclones/fetch.py, read 1 October 2026.

A lender prices cover that pays in years of 20 storms or more. Should it use the return period of the Poisson fit or of the record, and why?

Variance against mean of the Philippine storm counts



A Poisson count's variance equals its mean, 14.8. Guess the variance of the 75 Philippine counts.

Variance 9.5, ratio 0.64. The statistic

$$(n-1)\,\frac{\text{variance}}{\text{mean}} = 74 \times \frac{9.5}{14.8} = 47.5$$

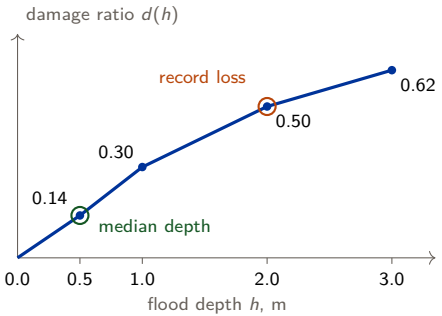
is chi-square on 74 degrees of freedom, lower tail 0.007: the counts vary less than Poisson counts.

IBTrACS v04r01 (NOAA NCEI, doi:10.25921/82ty-9e16), WP file of 29 September 2026, RSMC Tokyo record; count by data/ph_cyclones/fetch.py, read 1 October 2026.

A lender prices cover that pays in years of 20 storms or more. Should it use the return period of the Poisson fit or of the record, and why?

A flood loss is the exposure times the damage ratio at the flood depth

1 2 3 4 **How large is the loss of one event?**



A flood of depth h destroys the share $d(h)$ of the value exposed:

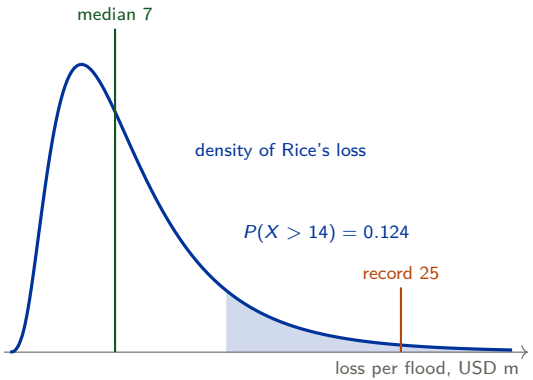
$$\text{loss} = \text{exposure} \times d(h).$$

Rice's mill exposes USD 50 m in buildings, machinery and stock. A flood at the median depth of 0.5 m costs 50×0.14, USD 7.0 m; one of 2.0 m costs USD 25.0 m, its record loss. Between points the curve is a straight line.

Stylised values.

A flood reaches 1.5 m at Rice's mill. What damage ratio does the curve give, and what loss in USD m?

A lognormal loss has a normal logarithm and a long right tail



A lognormal loss is $X = \exp(\mu + sZ)$ with Z standard normal. Its logarithm $\ln X$ is normal with mean μ and standard deviation s, the log-sd.

Half of all losses lie below the median e^{μ}, and the right tail extends far past it. Rice: $\mu = \ln 7 = 1.946$, $s = 0.6$.

Stylised values.

Which loss do 15.9 per cent of Rice's floods exceed, the loss one log-sd above the median?

Completing the square puts the lognormal mean at the median times $e^{s^2/2}$

$$\mathrm{E}[X] = \int \exp(\mu + sz)\,\varphi(z)\,dz \qquad X = \exp(\mu + sZ)$$

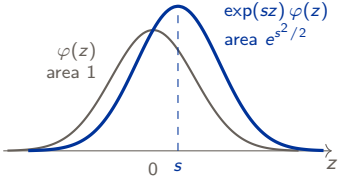
$$= e^{\mu} \int \tfrac{1}{\sqrt{2\pi}} \exp\big(-(z^2 - 2sz)/2\big)\,dz \qquad \varphi(z) = \tfrac{e^{-z^2/2}}{\sqrt{2\pi}}$$

$$= e^{\mu + s^2/2} \int \varphi(z - s)\,dz \qquad \text{complete the square}$$

$$= e^{\mu}\, e^{s^2/2} \qquad \text{a density integrates to 1}$$

$$\mathrm{E}[X^2] = e^{2\mu + 2s^2} \qquad \text{same steps, } 2\mu \text{ and } 2s$$

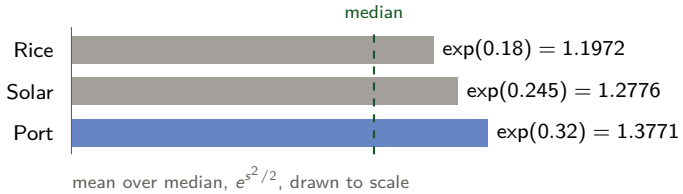
The lognormal mean lies above the median e^{μ} by the factor $e^{s^2/2}$, which grows with the log-sd s.



Drawn for Rice, $s = 0.6$: the weighted density is a normal density moved to s and scaled by $e^{s^2/2}$. Stylised values.

Rice's floods have median 7 and log-sd 0.6. What is $\mathrm{E}[X^2]$*, the mean of the squared loss?*

Mean loss per event of Port, Rice and Solar

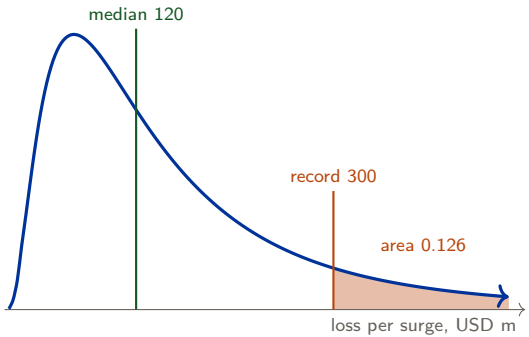


firm	median	s	$s^2/2$	$e^{s^2/2}$	mean E[X]
Rice	7	0.6	0.18	1.1972	8.38
Solar	12	0.7	0.245	1.2776	15.33
Port	120	0.8	0.32	1.3771	165.3

The larger the log-sd, the further the mean loss per event lies above the median; money in USD m. Stylised values.

A typhoon severity has median 12 and log-sd 0.9. What is the mean loss per typhoon?

A storm surge at Port exceeds the record loss with probability 0.126



$$P(X > x) = P\left(Z > \frac{\ln(x/e^{\mu})}{s}\right) \qquad \ln X = \mu + sZ$$

$$= 1 - \Phi\left(\frac{\ln(x/e^{\mu})}{s}\right) \qquad \Phi \text{, normal cdf}$$

$$\ln(300/120) = 0.916 \qquad x = 300$$

$$z = 0.916/0.8 = 1.145$$

$$P(X > 300) = 1 - 0.874 = 0.126$$

A storm surge at Port exceeds its record loss with probability 0.126: a lognormal tail is a normal tail at the log distance from the median, in units of s. Stylised values.

Rice's floods have median 7 and log-sd 0.6. With $\Phi(2.1) = 0.9821$ *and* $\Phi(2.2) = 0.9861$*, about what share of its floods exceed its record of 25?*

Flood losses of a cold store on Rice's river plain

flood rate	loan	exposure	median flood depth	log-sd of the loss
$\lambda = 0.05$ a year	$n = 15$ years	USD 60 m	1.0 m	$s = 0.5$

the damage ratio d at 1.0 m is read from Rice's depth-damage curve

In pairs, eight minutes, the flood losses of the cold store on Rice's river plain:

1. the return period, and the probability of at least one flood during the loan;
2. the loss of a flood at the median depth, which is the median loss;
3. the mean loss per flood, from the median and the log-sd.

Stylised values.

The cold store floods at least once during its loan with probability 52.8 per cent

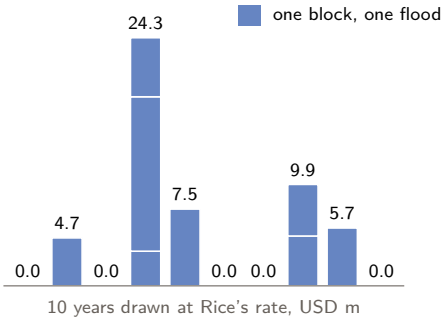
step	formula and inputs	result
return period	$1/0.05$	20 years
at least one flood	$1 - \exp(-0.05 \times 15) = 1 - \exp(-0.75)$	52.8%
median loss	60×0.30	18.0
mean loss per flood	$18.0 \times \exp(0.125) = 18.0 \times 1.1331$	20.4

The cold store floods during its loan with probability 52.8 per cent. Money in USD m. The two most common errors: adding the yearly chances, $15/20 = 75$ per cent, which counts a loan with two floods twice; and the factor e^{s^2} for the mean, which gives 23.1.

Stylised values.

A year's loss adds up a Poisson number of independent event losses

1 2 **3** 4 **What does a firm's yearly loss look like?**



$$S = X_1 + X_2 + \cdots + X_N$$

with the number of events N Poisson at rate λ, and the losses X_i independent of each other and of N. A year with no event has $S = 0$.

Of the 10 years drawn for Rice, 5 have no flood and one has 3. Their average is 5.21.

Stylised values.

What is the probability that a year at Rice, at a rate of 0.60 floods a year, holds two floods or more?

Conditioning on the count gives the mean and variance of a year's loss

$$
\begin{aligned}
\mathrm{E}[S \mid N = n] &= n\,\mathrm{E}[X] && \text{condition on } N = n \\
\mathrm{Var}[S \mid N = n] &= n\,\mathrm{Var}[X] && \text{variances add} \\
\mathrm{E}[S] &= \mathrm{E}[N]\,\mathrm{E}[X] = \lambda\,\mathrm{E}[X] && \text{average over } N \\
\mathrm{Var}[S] &= \mathrm{E}[N]\,\mathrm{Var}[X] + \mathrm{Var}[N]\,\mathrm{E}[X]^2 && \text{total variance} \\
&= \lambda\big(\mathrm{Var}[X] + \mathrm{E}[X]^2\big) && \mathrm{Var}[N] = \mathrm{E}[N] = \lambda \\
&= \lambda\,\mathrm{E}[X^2]
\end{aligned}
$$

The mean $\mathrm{E}[S]$ is the average annual loss, AAL: the loss of an average year.

Rice, $\mathrm{Var}[S] = 60.4$



spread of the loss per flood

spread of the count

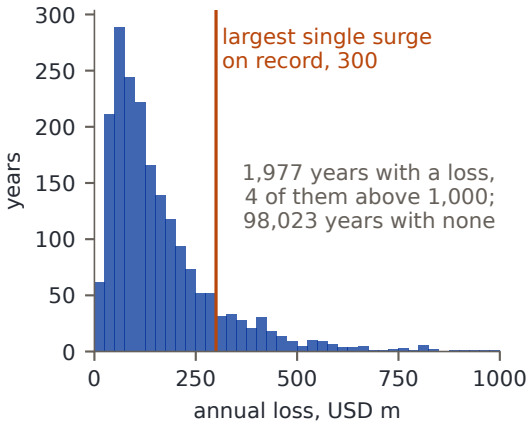
$\mathrm{E}[X] = 8.38$, $\mathrm{E}[X^2] = 100.7$. Stylised values.

A year holds exactly two events, never more or fewer. What are $\mathrm{E}[S]$ *and* $\mathrm{Var}[S]$*, and which term of the total variance vanishes?*

Average annual loss and standard deviation of Port

$$\mathrm{E}[X] = 120\ \exp(0.32) = 165.3$$
$$\mathrm{E}[S] = 0.02 \times 165.3 = 3.31$$
$$\mathrm{E}[X^2] = 120^2\ \exp(1.28) = 51{,}792$$
$$\mathrm{Var}[S] = 0.02 \times 51{,}792 = 1{,}035.8$$
$$\mathrm{sd}[S] = \sqrt{1{,}035.8} = 32.2$$

The standard deviation is 9.7 times the average annual loss: Port loses nothing in 98.0 per cent of years and a large sum in the rest. In 100,000 years drawn, the mean is 3.25 and the standard deviation 30.8. Money in USD m. Stylised values.



Show that the standard deviation over the AAL is $e^{s^2/2}/\sqrt{\lambda}$. What ratio does Rice have?

Losses above a threshold arrive as a Poisson count at a thinned rate

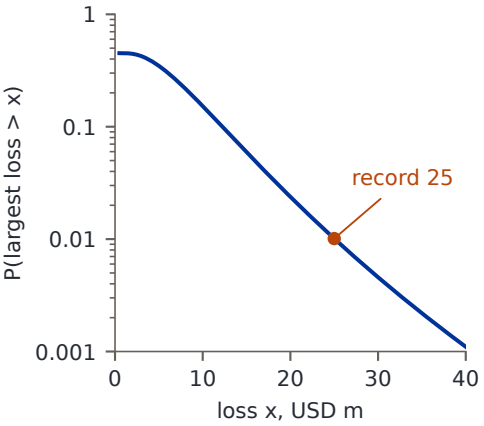
$$N_x = \text{events with } X_i > x \qquad \text{threshold } x$$

$$N_x \sim \text{Poisson}\big(\lambda\, P(X > x)\big) \qquad \text{thinning}$$

$$P(\text{no } X_i > x) = P(N_x = 0) = e^{-\lambda P(X>x)}$$

$$\text{OEP}(x) = 1 - e^{-\lambda P(X>x)} \qquad \text{occurrence exceedance}$$

Each event is kept with probability $P(X > x)$, independently: thinning leaves a Poisson count. The loss exceeded once in T years solves $\text{OEP}(x) = 1/T$, an annual probability; its thinned rate $-\ln(1 - 1/T)$ exceeds the rate $1/T$ by 0.5 per cent at $T = 100$. Rice's record of 25: $P(X > x) = 0.0169$, OEP $= 0.0101$, once in 99 years. Stylised values.

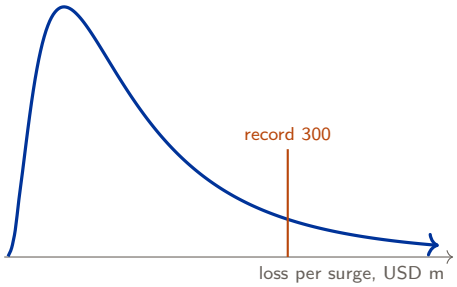


Solar's record of 40 has $P(X > x) = 0.0427$ per event, at rate 0.25. Once in how many years is it exceeded?

Port's loss exceeded once in 100 years, against its record loss

$$1 - e^{-\lambda P(X>x)} = 0.01$$
$$P(X > x) = -\ln(0.99)/0.02 =$$
$$x = 120 \exp\left(0.8\,\Phi^{-1}(\quad)\right)$$
$$=$$

Port's record loss is 300. Guess the loss Port exceeds once in 100 years. Stylised values.



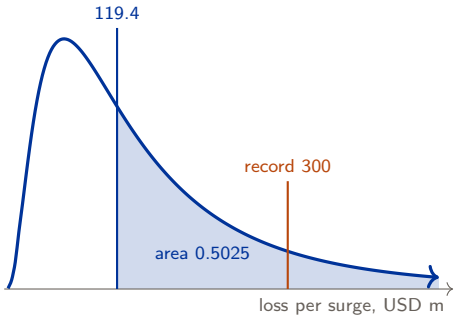
Port's record of 300 has $P(X > x) = 0.126$ per surge. Once in how many years is the record exceeded?

Port's loss exceeded once in 100 years, against its record loss

$$1 - e^{-\lambda P(X>x)} = 0.01$$
$$P(X > x) = -\ln(0.99)/0.02 = 0.5025$$
$$x = 120 \exp\left(0.8\,\Phi^{-1}(0.4975)\right)$$
$$= 120 \exp\left(0.8 \times (-0.0063)\right) = 119.4$$

Port's record loss is 300. Guess the loss Port exceeds once in 100 years. Stylised values.

Port's surge rate is 0.02 a year, so the loss exceeded in 0.01 of years is exceeded by half of all surges: it lies next to the median, far below the record.



Port's record of 300 has $P(X > x) = 0.126$ per surge. Once in how many years is the record exceeded?

Warming multiplies the event rate by m and divides the return period by m

1 2 3 **4** **How does warming change the losses?**

today: $T = 50$ years, $\lambda = 0.02$



after warming: $T/m = 16.7$ years, $m\lambda = 0.06$



Port's storm surge over 50 years, return periods to scale

$$\lambda' = m\lambda \qquad \text{the event rate}$$

$$T' = 1/(m\lambda) = T/m \qquad \text{the return period}$$

$$\mathrm{E}[S'] = m\lambda\ \mathrm{E}[X] = m\ \mathrm{E}[S] \qquad \text{severity unchanged}$$

$$\mathrm{OEP}'(x) = 1 - e^{-m\lambda P(X>x)}$$

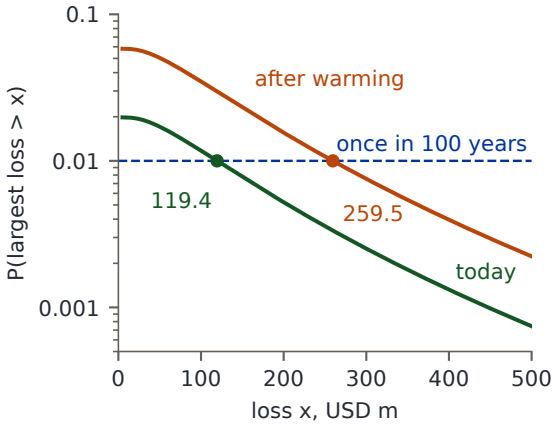
Port's surge rate rises by the factor $m = 3.0$, and its 50-year surge returns every 16.7 years. Stylised values.

Rice's flood rate of 0.60 a year rises by $m = 1.5$. What return period does a flood at Rice have then?

Port's loss exceeded once in 100 years, today and after warming

$$m\lambda = 3.0 \times 0.02 = 0.06$$
$$P(X > x) = -\ln(0.99)/0.06 = 0.1675$$
$$x = 120 \exp\left(0.8\,\Phi^{-1}(0.8325)\right)$$
$$= 120 \exp(0.8 \times 0.9641) = 259.5$$

Today the loss is 119.4. The rate 3.0 times higher multiplies it by 2.17: the curve rises at every loss, and the loss read at 0.01 moves out along the tail of the severity. Stylised values.



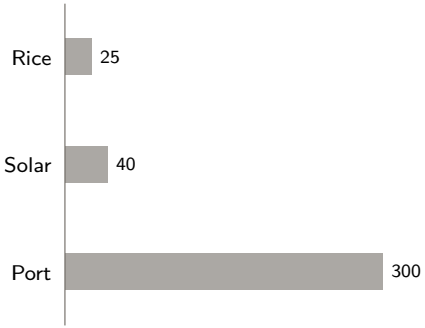
After warming, once in how many years is Port's record of 300 exceeded, with $P(X > x) = 0.126$ per surge?

Average annual loss of Port, Rice and Solar, today and after warming

firm	λ	E[X]	λ E[X]	$m\lambda$	$m\lambda$ E[X]
Rice	0.60	8.38		0.90	
Solar	0.25	15.33		0.30	
Port	0.02	165.3		0.06	

Multipliers m: Rice 1.5, Solar 1.2, Port 3.0. Rank the three firms by the average annual loss (AAL) today and after warming.

Stylised values.



Solar's multiplier is uncertain. Above which value of m would Solar's warmed AAL pass Rice's warmed AAL?

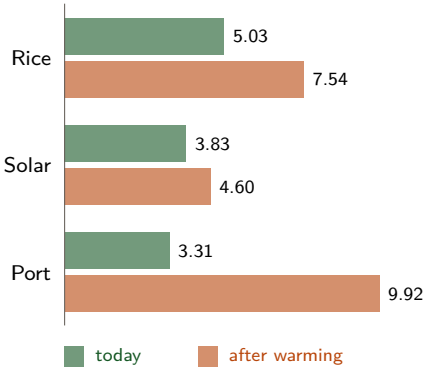
Average annual loss of Port, Rice and Solar, today and after warming

firm	λ	E[X]	λ E[X]	$m\lambda$	$m\lambda$ E[X]
Rice	0.60	8.38	5.03	0.90	7.54
Solar	0.25	15.33	3.83	0.30	4.60
Port	0.02	165.3	3.31	0.06	9.92

Multipliers m: Rice 1.5, Solar 1.2, Port 3.0. Rank the three firms by the average annual loss (AAL) today and after warming.

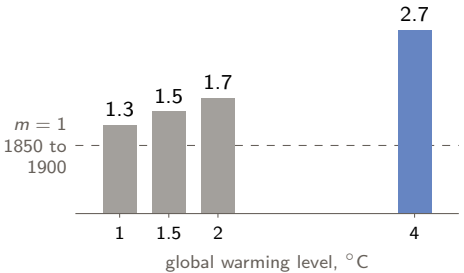
Today Rice loses most in an average year and Port least, the reverse of the record order. After warming Port loses most.

Stylised values.



Solar's multiplier is uncertain. Above which value of m would Solar's warmed AAL pass Rice's warmed AAL?

10-year heavy precipitation events are projected to occur 2.7 times as often at 4 °C



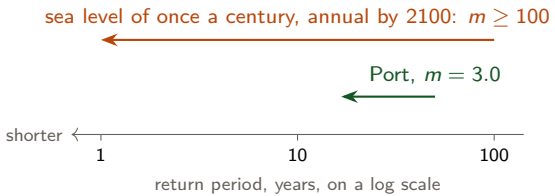
A heavy 1-day precipitation event over land, once in 10 years without human influence, now likely occurs 1.3 times as often and will likely occur m times as often at higher warming. The bars are CMIP6 medians; the figure gives 5 to 95% ranges.

At 4 °C the return period is $10/2.7 = 3.7$ years.

IPCC (2021), AR6 WG I, Summary for Policymakers, Figure SPM.6, heavy precipitation over land, p. 18, column 4°C, read 1 October 2026.
IPCC (2021), AR6 WG I, Summary for Policymakers, Figure SPM.6 caption, p. 19, read 1 October 2026.

What return period does the 10-year event have at 2 °C, and at the present 1 °C?

Centennial sea levels are projected to recur at least annually at most locations by 2100



Sea levels of once per century, centennial events, are projected to occur at least annually at most locations by 2100 under all RCP scenarios, and at least once per year at many locations by 2050. A 100-year event turned annual has $m \geq 100$, above Port's 3.0. Stylised values.

IPCC (2019), SROCC, Summary for Policymakers, B.3.4, p. 20, read 1 October 2026. IPCC (2019), SROCC, Summary for Policymakers, B.3, p. 20, read 1 October 2026.

If Port's 50-year surge struck every year, what multiplier m would that be, and what AAL would Port have?

A firm's yearly loss follows from its event rate, its severity and warming

1	How often does a hazard strike?	Poisson counts at rate λ, return period $T = 1/\lambda$: a 100-year event strikes a 30-year loan with probability 25.9%. Philippine storms: mean 14.8, variance 9.5.
2	How large is the loss of one event?	Exposure times the damage ratio. A lognormal severity has mean median times $e^{s^2/2}$: Port 165.3 against 120.
3	What does a firm's yearly loss look like?	$\mathrm{E}[S] = \lambda\ \mathrm{E}[X]$, $\mathrm{Var}[S] = \lambda\ \mathrm{E}[X^2]$, $\mathrm{OEP}(x) = 1 - e^{-\lambda P(X>x)}$. Port's loss once in 100 years is 119.4, below its record of 300.
4	How does warming change the losses?	Return period T/m, AAL times m: today Rice 5.03, Solar 3.83, Port 3.31; after warming Port is first at 9.92. IPCC: the 10-year heavy precipitation event, projected 2.7 times as often at 4 °C.

Stylised values.
IBTrACS v04r01, RSMC Tokyo record; IPCC (2021), Figure SPM.6; read 1 October 2026.

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All sources read on 1 October 2026.

Declarations

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Checkpoint: return periods, severities and yearly losses

1. A flood has a return period of 200 years. What is the probability of at least one during a 20-year loan?
2. A site counted 1, 7, 2, 0, 9, 3, 1 and 9 events in 8 years. Fit λ, compute the variance-to-mean ratio and say what it means for a Poisson model.
3. A warehouse exposes USD 80 m; a flood at the median depth destroys 0.25 of it, and the loss has log-sd 1.0. Give the median and mean loss per flood.
4. A firm has 0.4 events a year with a lognormal loss of median 5 and log-sd 0.5, in USD m. Compute its AAL and the standard deviation of its yearly loss.
5. With $\Phi(1.9) = 0.9713$ and $\Phi(2.0) = 0.9772$, compute that firm's loss exceeded once in 100 years.
6. Firm A: AAL 3.0, $m = 1.2$. Firm B: AAL 2.5, return period 25 years, $m = 1.6$. Rank them after warming and give B's new return period.

Stylised values.

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Physical losses from frequency, severity and return period - Mathematics of Green Finance Risk Management, Lecture 2

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This copy was published at https://digital-ai-finance.github.io/Green-Finance/teaching/module4-mathematics/M4_Mathematics_L2_Physical_Losses.pdf

The project site is https://digital-ai-finance.github.io/Green-Finance. A question about this file goes to joerg.osterrieder@utwente.nl.

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