Carbon price paths and the present value of carbon costs

Mathematics of green finance risk management

The three paths, USD per tonne of CO_2 in constant US dollars of 2025

	2026	2030	2031	2035	2040	2050	PV, 2026 to 2050
Orderly	100	188.89	211.11	300	333.33	400	3,689
Delayed	10	10	100	200	366.67	700	3,326
Current	10	15	16.25	21.25	27.5	40	309

Linear between the points of each path; PV per Mt emitted a year at the real rate of 5 per cent, in USD m. Coal, a coal power plant: $E = 4.0$ Mt a year, $\bar{p} = 400$, $\pi = 0.80$. Cement: $E = 3.0$, $\bar{p} = 1{,}200$, $\pi = 0.90$. Stylised values.

Formulas

marginal abatement cost	$\text{MAC}(a) = k\,a$, with $k = \bar{p}/E$
abatement	$a^* = \min(p/k,\ E)$
compliance cost	$C(p) = p\,(E - a^*) + k\,a^{*2}/2 = pE - p^2/(2k)$ while $p < \bar{p}$; $\bar{p}E/2$ once $p \geq \bar{p}$
net cost	$N = (1 - \pi)\,C$, with π the share passed on to customers; C is the compliance cost before pass-through
scenario price	$p_t = p_a + (p_b - p_a)(t - a)/(b - a)$ between two points $a < t < b$
Hotelling path	$p_y = p_{2026}\,(1 + r)^{y - 2026}$
real rate	$1 + i = (1 + r)(1 + j)$: nominal rate i, inflation j; constant-dollar costs are discounted at r
present value	$PV = \sum_y N_y/(1 + r)^t$, $t = y - 2025$, a net cost paid at the end of year y, valued at the end of 2025
growing annuity	T yearly costs from N_1 in 2026, each $1 + g$ times the one before: $S = N_1\,(1 - q^T)/(r - g)$, $q = (1 + g)/(1 + r)$; at $g = r$, $S = TN_1/(1 + r)$
break-even rate	the real rate at which two paths have the same present value; per Mt without abatement 2.1 per cent

Exercise, in pairs, eight minutes

A paper mill emits E a year, abates fully at $\bar{p}$ and passes on the share π. Its carbon price is p_{2026} in 2026 and p_{2027} in 2027; the real rate is r.

	E, Mt	$\bar{p}$, USD per t	π	p_{2026}, USD per t	p_{2027}, USD per t	r, per cent
paper mill	2.0	500	0.60	150	200	5

1. the slope k and the abatement a^* in each year;
2. the compliance cost C and the net cost N in each year;
3. the value of the two net costs at the end of 2025, each discounted with $t = y - 2025$.

Check values. In 2026 the mill abates $a^* = 0.60$ Mt, and its net cost is USD 102 m. Stylised values.

Glossary

CO_2	carbon dioxide; Mt is a million tonnes	MAC	marginal abatement cost, the cost of abating one more Mt
NGFS	Network for Greening the Financial System, central banks and supervisors; its scenarios set carbon price paths	IAM	integrated assessment model, such as REMIND-MAgPIE
US$2010	constant US dollars of 2010, the unit of NGFS carbon prices	NDCs	Nationally Determined Contributions, the pledges of the Paris Agreement
pass-through	the share π of a carbon cost a firm passes on in its prices	full-abatement price	$\bar{p}$, the price at which the firm has abated all its emissions
PV	present value, the sum of discounted costs	real rate	the interest rate net of inflation, for costs in constant dollars
six-firm book	six stylised firms, Coal, Cement, Steel, Port, Rice and Solar, and the three paths	constant dollars	prices of one base year; the book's base year is 2025

The NGFS unit: NGFS (2024), long-term scenarios for central banks and supervisors, Phase V, p. 21, figure Shadow Carbon Price, REMIND-MAgPIE, axis label, read 1 October 2026.

Co-funded by the European Union

Licence and attribution

Carbon price paths and the present value of carbon costs, formula and exercise sheet - Mathematics of Green Finance Risk Management, Lecture 1

Co-funded by the European Union. Published by the GREEN FINANCE project, an Erasmus+ Capacity Building in Higher Education action, grant agreement 101237817.

Free to use, copy and adapt under Creative Commons Attribution 4.0 International (CC BY 4.0), creativecommons.org/licenses/by/4.0/. Attribute it to the GREEN FINANCE consortium, Erasmus+ grant 101237817.

This copy was published at https://digital-ai-finance.github.io/Green-Finance/teaching/module4-mathematics/M4_Mathematics_L1_Sheet.pdf

The project site is https://digital-ai-finance.github.io/Green-Finance. A question about this file goes to joerg.osterrieder@utwente.nl.

Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Education and Culture Executive Agency (EACEA). Neither the European Union nor EACEA can be held responsible for them.

Licence page added 2 October 2026.