L34: AMM Mechanics

Module E: DeFi Ecosystem

Blockchain & Cryptocurrency

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Learning Objectives

- Derive the constant product formula ($x \cdot y = k$) and its implications
- Analyze how liquidity provision works in AMMs
- Calculate impermanent loss and its implications
- Calculate slippage and price impact for trades
- Compare AMMs to traditional order book exchanges

The Problem: How can we trade without order books?

The Challenge

Traditional exchanges require continuous market makers to maintain order books. On blockchain, updating orders is expensive (gas costs) and slow (block times). We need a trading mechanism that works algorithmically without human market makers.

Why It Matters

- Without liquidity, decentralized exchanges can't compete with CEXs
- Order books are vulnerable to manipulation and front-running

What We Need

- System design principles
- Algorithmic pricing that adjusts automatically to supply and demand
- Passive liquidity provision without active management

The Cryptoeconomics Question

Creating systems with desired properties

Today's lesson: How AMM Mechanics addresses this challenge

Traditional Order Book Exchanges

How They Work:

- **Buyers** place bids, **sellers** place asks
- **Matching engine** pairs buy/sell orders
- Trade executes when bid meets ask

Example Order Book:

Bids (Buy)

Price	Size
$1,999	5 ETH
$1,998	10 ETH

Asks (Sell)

Price	Size
$2,000	8 ETH
$2,001	12 ETH

Challenges on Blockchain: Gas costs for order updates, slow block times, front-running.

Automated Market Makers (AMMs)

Key Idea: Replace order books with liquidity pools governed by mathematical formulas.

How It Works:

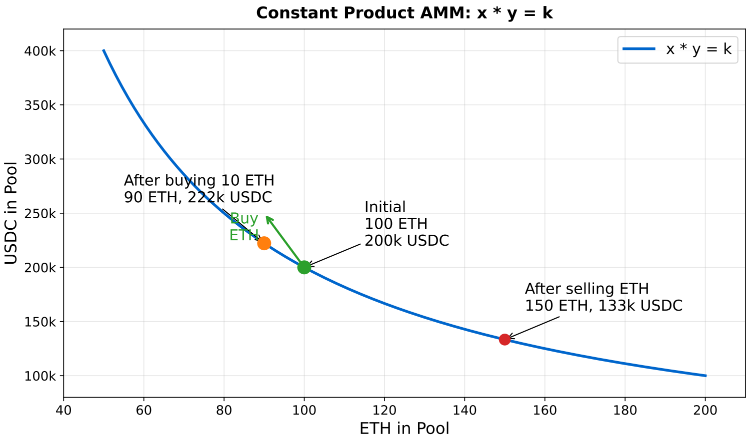
- Liquidity Providers (LPs) deposit token pairs into a pool
- Algorithm sets price based on pool ratio
- Users trade directly against the pool

Advantages:

- Always available liquidity (no need to wait for orders)
- Passive income for LPs (earn trading fees)
- Gas efficient (fewer transactions)

Trade-off: Price determined by formula, not market consensus.

The Constant Product Formula



Trades move along the curve; price is the slope at any point

Constant Product: Mathematics

Uniswap V2 Model:

$$x \cdot y = k$$

where x = token A quantity, y = token B quantity, k = constant.

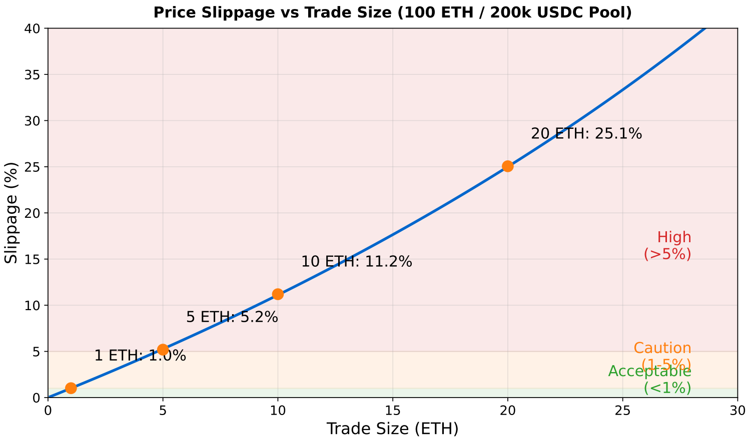
Example Pool:

- 100 ETH and 200,000 USDC
- $k = 100 \times 200{,}000 = 20{,}000{,}000$
- Price: $\frac{y}{x} = \frac{200{,}000}{100} = 2{,}000$ USDC per ETH

After Buying 10 ETH:

- New ETH: 90, New USDC: $\frac{20{,}000{,}000}{90} = 222{,}222$
- Cost: 22,222 USDC for 10 ETH = \$2,222/ETH average

Slippage vs Trade Size



Slippage increases non-linearly; larger pools reduce slippage

Slippage Explained

Definition: The difference between expected price and executed price due to trade size.

Why Slippage Occurs:

- AMM formula moves price as reserves change
- Larger trades = larger price impact
- Smaller pools = more slippage

Slippage Formula:

$$\text{Slippage} = \frac{\text{Executed Price} - \text{Initial Price}}{\text{Initial Price}} \times 100\%$$

Slippage Tolerance: Users set maximum acceptable (e.g., 0.5%, 1%). Transaction reverts if exceeded.

Liquidity Provision

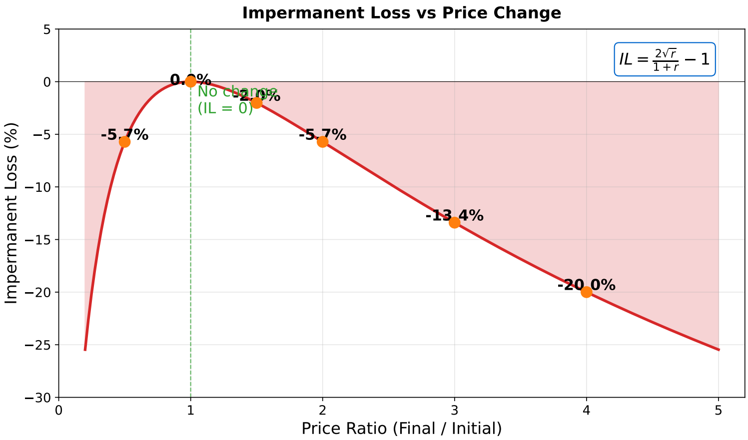
How to Become an LP:

1. Deposit equal value of both tokens (e.g., 1 ETH + 2,000 USDC)
2. Receive LP tokens representing pool share
3. Earn trading fees proportional to share
4. Withdraw anytime (burn LP tokens, receive reserves)

Example:

- Pool has 100 ETH + 200,000 USDC
- You deposit 10 ETH + 20,000 USDC
- Your share: $\frac{10}{110} = 9.09\%$ of pool

Fee Earnings: Uniswap charges 0.3% per trade; fees compound in pool reserves.



IL increases with price divergence; symmetric whether price rises or falls

Impermanent Loss: Concept

Definition: The opportunity cost of providing liquidity vs. simply holding tokens.

Occurs when:

- Token prices diverge from deposit ratio
- Arbitrageurs rebalance pool to match external prices
- LPs end up with more of the depreciated token

Example: ETH doubles to $4,000

- Initial: 1 ETH + 2,000 USDC = $4,000 total
- If held: 1 ETH @ $4,000 + 2,000 USDC = $6,000
- If LP: 0.707 ETH + 2,828 USDC = $5,656
- **IL: $344 (5.7%)**

Impermanent Loss: Key Values

Formula:

$$\text{IL} = \frac{2\sqrt{r}}{1+r} - 1$$

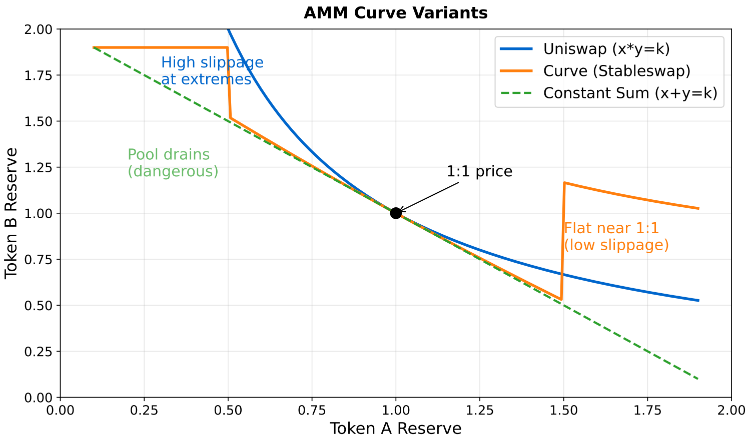
where r is the price ratio (final/initial).

Common Scenarios:

- 1.25x price change: -0.6% IL
- 1.5x price change: -2.0% IL
- 2x price change: -5.7% IL
- 3x price change: -13.4% IL
- 4x price change: -20.0% IL

Why "Impermanent"? Loss only realized on withdrawal; if prices return, loss disappears.

AMM Curve Variants



Different curves optimize for different use cases

AMM Variants Explained

Uniswap (Constant Product): $x \cdot y = k$

- General purpose, works for any pair
- Higher slippage near extremes

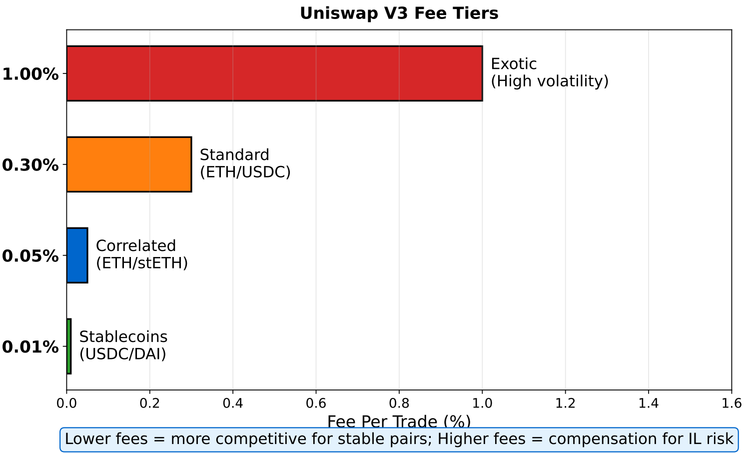
Curve (Stableswap): Hybrid formula

- Flat curve near 1:1 (low slippage for stables)
- Dominates stablecoin swaps (USDC/DAI)

Balancer (Weighted): $\prod x_i^{w_i} = k$

- Custom weights (e.g., 80/20 instead of 50/50)
- Index fund functionality

Fee Tiers in AMMs



Higher fees compensate LPs for impermanent loss risk in volatile pairs

Arbitrage in AMMs

How Arbitrage Works:

1. External market price deviates from AMM price
2. Arbitrageur buys cheaper asset, sells expensive one
3. AMM pool rebalances to match external price

Example:

- CEX: 1 ETH = $2,100
- Uniswap pool: 1 ETH = $2,000
- Arbitrageur: Buy on Uniswap, sell on CEX, profit $100/ETH

Benefit: Keeps AMM prices aligned with global markets.
Cost: LPs experience impermanent loss from adjustments.

MEV and Front-Running

Maximal Extractable Value (MEV):

- Profit from reordering/inserting transactions
- Particularly prevalent in AMM trades

Common MEV Strategies:

1. **Front-Running:** See large buy, buy first, sell after
2. **Sandwich Attacks:** Buy before user, sell after
3. **Arbitrage:** Exploit price differences

Mitigation:

- Private mempools (Flashbots Protect)
- Batch auctions (CoW Swap)

Comparison: AMM vs Order Book

AMM (Uniswap)

- Always available liquidity
- Passive LP income
- Slippage on large trades
- Impermanent loss risk

Order Book (Binance)

- Liquidity depends on makers
- Active market making
- Better for large trades
- No impermanent loss

Trend: Hybrid models emerging (e.g., dYdX order book on Cosmos).

Problem Solved? AMM Mechanics

The Original Problem

How can we trade without order books?

How AMM Mechanics Solves It

- The constant product formula $(x \cdot y = k)$ provides algorithmic pricing without manual order placement
- Liquidity providers deposit token pairs and earn passive trading fees (0.3% per trade)
- Pool always has liquidity available 24/7, no need to wait for matching orders

Remaining Limitations

- Impermanent loss: LPs face opportunity cost when token prices diverge (up to 20%+ for 4x price moves)
- Capital inefficiency: Liquidity spread across entire price curve, not concentrated where trades occur
- MEV extraction: Sandwich attacks and front-running impose hidden costs on traders

Open Questions

- Can AMMs ever achieve the capital efficiency of centralized exchange order books?
- Risk: Are trading fees sufficient to compensate LPs for impermanent loss in volatile pairs?

AMM Mechanics solves the order book problem but introduces new trade-offs between accessibility and efficiency

Summary

Key Takeaways:

- AMMs use $x \cdot y = k$ to provide algorithmic liquidity
- Price determined by reserve ratio; trades move price
- Slippage increases non-linearly with trade size
- LPs earn fees but face impermanent loss when prices diverge
- IL formula: $\frac{2\sqrt{r}}{1+r} - 1$ (up to 20%+ for 4x moves)
- Curve optimizes for stables; Balancer for weighted pools
- MEV is a hidden cost for AMM traders

Next Lecture: Uniswap Deep Dive - V1 to V4 evolution, concentrated liquidity.

Questions for Reflection

1. Calculate the cost to buy 5 ETH from a pool with 100 ETH and 200,000 USDC.
2. Why does slippage increase non-linearly with trade size?
3. How do trading fees help offset impermanent loss for LPs?
4. Why is Curve more suitable for stablecoin trading than Uniswap V2?
5. What are the trade-offs of concentrated liquidity in Uniswap V3?

Quiz Questions (1–5)

Q1. In the constant product formula $x \cdot y = k$, what happens to k when a trade occurs?
A) Increases B) Decreases C) Remains constant D) Becomes zero
Answer: C – The constant k remains the same; only x and y change inversely.

Q2. A pool has 100 ETH and 200,000 USDC. What is the current price per ETH?
A) $1,000 B) $2,000 C) $100 D) $20,000
Answer: B – Price $= y/x = 200{,}000/100 = 2{,}000$ USDC per ETH.

Q3. Which factor does NOT contribute to slippage in an AMM?
A) Trade size B) Pool depth C) Block time D) Reserve ratio
Answer: C – Block time affects confirmation speed, not price slippage from AMM mechanics.

Q4. What is the primary benefit of providing liquidity to an AMM pool?
A) Guaranteed returns B) Earning trading fees C) Price speculation D) Voting rights
Answer: B – LPs earn a share of trading fees (e.g., 0.3% in Uniswap V2).

Q5. If you set a 1% slippage tolerance and actual slippage is 1.5%, what happens?
A) Trade executes normally B) Transaction reverts C) Fees double D) Pool pauses
Answer: B – Transaction reverts to protect user from excessive slippage.

Quiz Questions (6–10)

Q6. Impermanent loss occurs when:
A) Fees are too low B) Token prices diverge C) Pool is empty D) Gas costs rise
Answer: B – IL happens when token prices change relative to deposit ratio.

Q7. If ETH price doubles (2x), what is the approximate impermanent loss?
A) 2.0% B) 5.7% C) 13.4% D) 20.0%
Answer: B – 2x price change results in approximately 5.7% IL.

Q8. Why is impermanent loss called "impermanent"?
A) It's always small B) Fees offset it C) Loss disappears if prices return D) It's paid by traders
Answer: C – If token prices return to original ratio, the loss vanishes.

Q9. You deposit 1 ETH + 2,000 USDC into a pool. What do you receive?
A) Interest payments B) LP tokens C) More ETH D) Governance rights
Answer: B – LPs receive LP tokens representing their share of the pool.

Q10. In a pool with 0.3% trading fee, who receives the fees?
A) Protocol treasury B) Validators C) Liquidity providers D) Traders
Answer: C – Trading fees accrue to LPs proportional to their pool share.

Quiz Questions (11–15)

Q11. Which AMM variant is optimized for stablecoin trading?
A) Uniswap V2 B) Curve C) Balancer D) Sushiswap
Answer: B – Curve's StableSwap formula minimizes slippage near 1:1 ratios.

Q12. Balancer's weighted pools allow custom ratios like:
A) Only 50/50 B) Only 33/33/33 C) 80/20 or any weights D) Must equal 100
Answer: C – Balancer supports arbitrary weights (e.g., 80/20, 60/40).

Q13. What is the main role of arbitrageurs in AMMs?
A) Provide liquidity B) Pay gas fees C) Align AMM prices with external markets D) Vote on upgrades
Answer: C – Arbitrageurs exploit price differences, keeping AMM prices accurate.

Q14. A sandwich attack involves:
A) Buying before and selling after victim's trade B) Hacking the pool C) Withdrawing all liquidity D) Freezing transactions
Answer: A – Attacker front-runs victim's buy, then back-runs with a sell.

Q15. What does MEV stand for?
A) Maximum Ethereum Value B) Miner Extractable Value C) Maximal Extractable Value D) Market Efficient Value
Answer: C – MEV (formerly "Miner") is now "Maximal Extractable Value".

Quiz Questions (16–20)

Q16. Compared to order books, AMMs have:
A) No liquidity B) Always available liquidity C) Better prices for large trades D) No fees
Answer: B – AMMs provide algorithmic liquidity 24/7 without waiting for makers.

Q17. After buying 10 ETH from a 100 ETH pool with $k = 20{,}000{,}000$, the new ETH reserve is:
A) 90 ETH B) 100 ETH C) 110 ETH D) 80 ETH
Answer: A – Pool loses 10 ETH, leaving 90 ETH (USDC increases to maintain k).

Q18. Uniswap V2's standard fee per trade is:
A) 0.05% B) 0.3% C) 1.0% D) 5.0%
Answer: B – Uniswap V2 charges 0.3% on all trades.

Q19. Which factor INCREASES impermanent loss?
A) Price stability B) High trading fees C) Large price divergence D) Low slippage
Answer: C – Greater price divergence (e.g., 4x) causes higher IL (up to 20%+).

Q20. To withdraw liquidity from an AMM, you must:
A) Wait 30 days B) Burn LP tokens C) Pay a penalty D) Sell all assets
Answer: B – Burning LP tokens returns your share of pool reserves.